Week 3 Tutorial Session

1. (a) Write down a regular expression for the following NFA. For this problem, you do not have to go through the procedure described in class.
(b) Convert the following NFA into a DFA.

2. (a) Write down the definition of regular expressions over an alphabet Σ.
(b) Given a string \( w \), define \( w^R \) as the string \( w \) in reverse order. That is, if \( w = w_1w_2 \ldots w_n \), then \( w^R = w_nw_{n-1} \ldots w_1 \). For example, if \( w = \text{live} \), then \( w^R = \text{evil} \).
   Given a language \( L \), define its reversal \( L^R \) as the set of strings in \( L \) in reverse. More precisely, \( L^R = \{ w^R \mid w \in L \} \). For example, if \( L = \{ \text{live, raw, level} \} \), then \( L^R = \{ \text{evil, war, level} \} \).
   If \( L \) is a regular language, prove that \( L^R \) as also regular.

3. Let \( L \) be any language. We say that two strings \( x \) and \( y \) are indistinguishable by \( L \) if for every string \( z \), we have \( xz \in L \) if and only if \( yz \in L \).
   (a) For concreteness, consider \( L_1 = \{ x \in \{0, 1\}^* \mid \text{the number of } 1\text{'s in } x \text{ is divisible by 3} \} \). Prove that \( 1 \) and \( 1111 \) are indistinguishable by \( L_1 \).
   (b) Continuing with (a), which strings are indistinguishable from the string \( 1 \) by \( L_1 \)? The set of all such strings is the equivalence class of the string \( 1 \) and will be denoted by \( [1] \).
   (c) Find a string \( s \) not in \( [1] \). What is the equivalence class of \( s \)? (We will denote this equivalence class by \( [s] \))
   (d) Can you find another string \( t \) not in \( [1] \) or \( [s] \)? What is the equivalence class of \( t \)?
   (e) Can you find yet another string \( u \) not in these equivalence classes?
   (f) Design a DFA for the language \( L_1 \). How are states in your DFA related to the equivalence classes?