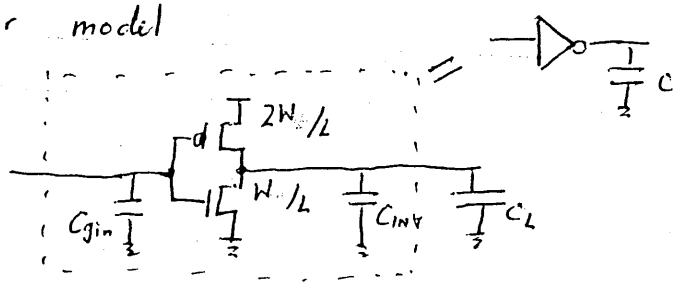
Inverter model



Suppose $L_{min} = 0.25\mu m$
$W_p = 2W_n = 2W$

Assuming equal rise & fall times
$R_p = R_n = R_W$

Since $C_{gin} \propto W$ and $C_{INT} \propto W$, $C_{gin} \propto C_{INT}$
let $C_{INT} = \gamma C_{gin}$ (turns out $\gamma \approx 1$)

Delay
$$t_p = 0.69 R_W (C_{INT} + C_L)$$
$$= k R_W C_{INT} \left(1 + \frac{C_L}{C_{INT}}\right)$$

Inverter chain

$$t_p = \sum_{i=1}^{N} k R_{W,i} C_{INT,i} \left(1 + \frac{C_{L,i}}{C_{INT,i}}\right)$$

But
$$k R_{W,i} C_{INT,i} = t_{p0} \quad (\uparrow W \text{ causes } R_W\downarrow \text{ \& } C_{INT,i}\uparrow \text{ proportionally})$$

$$C_{L,i} = C_{gin,i+1}$$
$$C_{INT,i} = \gamma C_{gin,i}$$

$$t_p = t_{p0} \sum_i \left(1 + \frac{C_{gin,i+1}}{\gamma C_{gin,i}}\right) \qquad C_{gin,N+1} = C_L$$

Given N, optimal tapering which minimises t_p is
$$\frac{C_{gin,j+1}}{C_{gin,j}} = \frac{C_{gin,j}}{C_{gin,j-1}} = f$$

each stage has the same delay
$$t_{p0}(1 + f/\gamma)$$
entire chain has delay
$$t_p = N t_{p0} (1 + f/\gamma)$$

If $F = \frac{C_L}{C_{gin,1}}$ then $f = \sqrt[N]{F}$

Optimal value of N found by $\frac{\partial t_p}{\partial N} = 0$
$$f = e^{1+\gamma/f} \quad (\text{typ } \gamma \approx 1, \text{ set } f = 4)$$