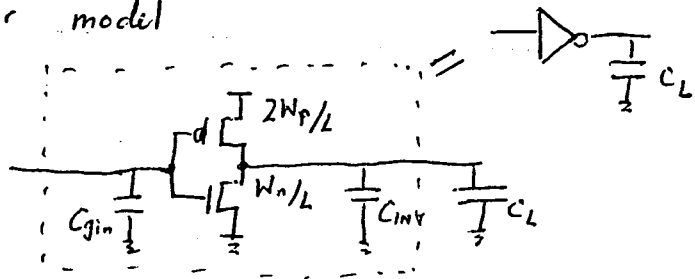


Inverter model



Suppose $L_{min} = 0.25 \mu m$
 $W_p = 2W_n = 2W$

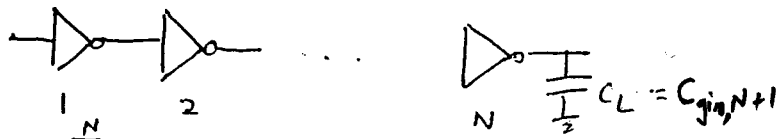
Assuming equal rise & fall times

$$R_p = R_n = R_w$$

Since $C_{gin} \propto W$ and $C_{INT} \propto W$, $C_{gin} \propto C_{INT}$
 let $C_{INT} = \gamma C_{gin}$ (turns out $\gamma \approx 1$)

$$\begin{aligned} \text{Delay } t_p &= 0.69 R_w (C_{INT} + C_L) \\ &= k R_w C_{INT} \left(1 + \frac{C_L}{C_{INT}}\right) \end{aligned}$$

Inverter chain



$$t_p = \sum_{i=1}^N k R_{w,i} C_{INT,i} \left(1 + \frac{C_{L,i}}{C_{INT,i}}\right)$$

But

$k R_{w,i} C_{INT,i} = t_{p0}$ ($\uparrow W$ causes $R_w \downarrow$ & $C_{INT,i} \uparrow$ proportionally)

$$C_{L,i} = C_{gin,i+1}$$

$$C_{INT,i} = \gamma C_{gin,i}$$

$$t_p = t_{p0} \sum_i \left(1 + \frac{C_{gin,i+1}}{\gamma C_{gin,i}}\right) \quad C_{gin,N+1} = C_L$$

Given N , optimal tapering which minimises t_p is

$$\frac{C_{gin,j+1}}{C_{gin,j}} = \frac{C_{gin,j}}{C_{gin,j-1}} = f$$

each stage has the same delay

$$t_{p0} (1 + f/\gamma)$$

entire chain has delay

$$t_p = N t_{p0} (1 + f/\gamma)$$

$$\text{If } F = \frac{C_L}{C_{gin,1}} \text{ then } f = \sqrt[N]{F}$$

Optimal value of N found by $\frac{\partial t_p}{\partial N} = 0$

$$f = e^{1+\gamma/F} \quad (\text{typ } \gamma \approx 1, \text{ set } f=4)$$