

# Digital Circuit Equations

## Diode

$$I_D = I_S(e^{V_D/\phi_T} - 1) = Q_D/\tau_T$$

$$C_j = \frac{C_{j0}}{(1 - V_D/\phi_0)^m}$$

$$K_{eq} = \frac{-\phi_0^m}{(V_{high} - V_{low})(1 - m)} \times [(\phi_0 - V_{high})^{1-m} - (\phi_0 - V_{low})^{1-m}]$$

## MOS Transistor

$$V_T = V_{T0} + \gamma(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|})$$

$$I_D = \frac{k'_n W}{2 L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \quad (sat)$$

$$I_D = v_{sat} C_{ox} W \left( V_{GS} - V_T - \frac{V_{DSAT}}{2} \right) (1 + \lambda V_{DS}) \quad (velocitysat)$$

$$I_D = k'_n \frac{W}{L} \left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right) \quad (triode)$$

$$I_D = I_S e^{\frac{V_{GS}}{n k T/q}} \left( 1 - e^{-\frac{V_{DS}}{k T/q}} \right) \quad (subthreshold)$$

## Deep Submicron MOS Unified Model

$$I_D = 0 \text{ for } V_{GT} \leq 0$$

$$I_D = k' \frac{W}{L} \left( V_{GT} V_{min} - \frac{V_{min}^2}{2} \right) (1 + \lambda V_{GS}) \text{ for } V_{GT} \geq 0$$

with  $V_{min} = \min(V_{GT}, V_{DS}, V_{DSAT})$   
and  $V_{GT} = V_{GS} - V_T$

## MOS Switch Model

$$R_{eq} = \frac{1}{2} \left( \frac{V_{DD}}{I_{DSAT}(1 + \lambda V_{DD})} + \frac{V_{DD}/2}{I_{DSAT}(1 + \lambda V_{DD}/2)} \right)$$

$$\approx \frac{3}{4} \frac{V_{DD}}{I_{DSAT}} \left( 1 - \frac{5}{6} \lambda V_{DD} \right)$$

## Inverter

$$V_{OH} = f(V_{OL})$$

$$V_{OL} = f(V_{OH})$$

$$V_M = f(V_M)$$

$$t_p = 0.69 R_{eq} C_L = \frac{C_L (V_{swing}/2)}{I_{avg}}$$

$$P_{dyn} = C_L V_{DD} V_{swing} f$$

$$P_{stat} = V_{DD} I_{DD}$$

## Static CMOS Inverter

$$V_{OH} = V_{DD}$$

$$V_{OL} = GND$$

$$V_M \approx \frac{r V_{DD}}{1 + r} \text{ with } r = \frac{k_p V_{DSATp}}{k_n V_{DSATn}}$$

$$V_{IH} = V_M - \frac{V_M}{g} \quad V_{IL} = V_M + \frac{V_{DD} - V_M}{g}$$

with  $g \approx \frac{1 + r}{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n - \lambda_p)}$

$$t_p = \frac{t_{pHL} + t_{pLH}}{2} = 0.69 R C_L \left( \frac{R_{eqn} + R_{eqp}}{2} \right)$$

$$P_{av} = C_L V_{DD}^2 f$$

## Interconnect

Lumped RC:  $t_p = 0.69 RC$

Distributed RC:  $t_p = 0.38 RC$

RC-chain:  $\tau_N = \sum_{i=1}^N R_i \sum_{j=i}^N C_j = \sum_{i=1}^N C_i \sum_{j=1}^i R_j$

Transmission line reflection:

$$\rho = \frac{V_{refl}}{V_{inc}} = \frac{I_{refl}}{I_{inc}} = \frac{R - Z_0}{R + Z_0}$$