

# An Assessment of Testing-Effort Dependent Software Reliability Growth Models

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**Abstract**—Over the last several decades, many Software Reliability Growth Models (SRGM) have been developed to greatly facilitate engineers and managers in tracking and measuring the growth of reliability as software is being improved. However, some research work indicates that the delayed S-shaped model may not fit the software failure data well when the testing-effort spent on fault detection is not a constant. Thus, in this paper, we first review the logistic testing-effort function that can be used to describe the amount of testing-effort spent on software testing. We describe how to incorporate the logistic testing-effort function into both exponential-type, and S-shaped software reliability models. The proposed models are also discussed under both ideal, and imperfect debugging conditions. Results from applying the proposed models to two real data sets are discussed, and compared with other traditional SRGM to show that the proposed models can give better predictions, and that the logistic testing-effort function is suitable for incorporating directly into both exponential-type, and S-shaped software reliability models.

**Index Terms**—Delayed S-shaped model, imperfect debugging, non-homogeneous Poisson process, software reliability growth models, testing-effort function.

## ACRONYM<sup>1</sup>

NHPP	non-homogeneous Poisson process
SRGM	software reliability growth model
MVF	mean value function
MLE	maximum likelihood estimation
TEF	testing-effort function
LOC	lines of code
KLNCSS	thousand lines of non-comment source statements
MSE	mean square of fitting error
K-S	Kolmogorov-Smirnov
KD	Kolmogorov distance

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<sup>1</sup>The singular and plural of an acronym are always spelled the same.

RE	relative error
PE	prediction error
MRE	magnitude of relative error

## NOTATIONS

$m(t)$	expected mean number of faults detected in time $(0, t]$
$m_{remaining}(t)$	number of faults remaining in the software system at time $t$
$\lambda(t)$	failure intensity for $m(t)$
$n(t)$	fault content function
$m_d(t)$	cumulative number of faults detected up to $t$
$m_r(t)$	cumulative number of faults isolated up to $t$ ;
$W(t)$	cumulative testing-effort consumption at time $t$
$W^*(t)$	$W(t) - W(0)$
$W_e(t)$	exponential TEF
$W_r(t)$	Rayleigh TEF
$W_w(t)$	Weibull TEF
$N(t)$	counting process for the total number of failures in $[0, t)$
$a$	expected number of initial faults
$r(t)$	fault detection rate function
$r$	constant fault detection rate
$r_1$	constant fault detection rate in the Delayed S-Shaped model with logistic TEF
$r_2$	constant fault isolation rate in the Delayed S-Shaped model with logistic TEF
$N$	total amount of testing-effort eventually consumed
$b$	scale parameter in the Weibull-type TEF
$\alpha$	testing-effort consumption rate of logistic TEF
$A$	constant parameter in the logistic TEF
$\theta$	scale parameter in the log-logistic TEF
$m, \delta$	shape parameter
$\beta$	fault introduction rate
$m(t_i)$	expected number of faults by time $t_i$
$m_i$	actual observed number of faults by time $t_i$
$\gamma_i$	predicted failure rate
$U(t)$	Laplace trend factor

## I. INTRODUCTION

**S**OFTWARE reliability is defined as the probability of failure-free software operation for a specified period of time in a specified environment [1], [2]. The aim of software reliability engineers is to increase the probability that a designed program will work as intended in the hands of the customers. Typically, failure-based software reliability represents a customer-oriented view of software quality, relating to practical operation rather than simply design of the program [3].

SRGM help measure & track the growth of reliability as software is being improved [4]. There is an extensive body of literature on software reliability growth modeling, with many detailed probability models purporting to represent the probabilistic failure process [5]–[8]. Often SRGM may also yield information on physical properties of the code, such as the number of faults remaining in a software system, etc. For example, Gana & Huang [9] reported that the use of SRGM has greatly enhanced the project’s ability to manage & improve the reliability of the global 5ESS-2000 switch products, used worldwide, to significantly out-perform the downtime objective established by Bellcore for all regional Bell operating companies. In addition, Kruger [10] also reported that an SRGM has demonstrated its applicability to projects ranging in size from 6 KLNCS to 150 KLNCS, and in functions from instrument firmware to application software. In general, the exposure time over which reliability is being assessed may be expressed as calendar time, clock time, CPU execution time, number of test-runs, or some other suitable measures. The measure actually used will depend on the product.

In the context of software testing, the key elements are the testing effort, and effectiveness of the test-cases. Many published models either assume that the consumption rate of testing resources is constant, or do not explicitly consider the testing effort nor its effectiveness [2], [4], [7], [11], [12]. The functions that describe how an effort is distributed over the exposure period, and how effective it is, are referred to by us as testing-effort functions (TEF). To address the issue of the TEF, Musa [2], Yamada *et al.* [13], Bokhari & Ahmad [14], Kapur *et al.* [15], and Huang *et al.* [16]–[18] proposed SRGM describing the relationship among the testing time (calendar time), the amount of testing-effort expended during that time, and the number of software faults detected by testing. Most existing SRGM belong to exponential-type models. Yamada also proposed a delayed S-shaped NHPP model in which the observed growth curve of the cumulative number of detected faults is S-shaped. However, some researchers indicated that the delayed S-shaped model may not fit the observed data well when the testing-effort is not a constant [11].

In this paper, we show how to integrate a logistic TEF into the exponential-type, and S-shaped SRGM [18]. We further discuss how to incorporate logistic TEF into the delayed S-shaped model from two different viewpoints. A method to estimate the model parameters is provided, together with some approaches to obtain the confidence limits for the parameters. We are also concerned with the development of stochastic models for the software failure process considering an imperfect debugging environment. Experimental results from real data applications are

analysed, and compared with other existing models to show that the proposed model gives better predictions.

The remainder of the paper is organized as follows. Section II gives an overview of traditional TEF. The logistic TEF is also presented in this section. In Section III, we show how to incorporate the logistic TEF into the exponential-type, and the S-shaped SRGM. Parameters of the proposed models, as estimated by the method of MLE, together with the application of these models to two real data sets, are discussed in Section IV. We also show how the upper, and the lower bounds of the parameters can be obtained. Finally, Section V discusses the imperfect debugging problem based on the proposed models.

## II. TESTING-EFFORT FUNCTIONS

TEF describes the relationship between the effort expended to test software (e.g., in person-months), and the physical characteristics of the software, such as LOC, exposure time (which can take many forms, and can be expressed either as total effort), etc. [19]–[21]. Yamada *et al.* [13] found that the TEF could be described by a *Weibull*-type distribution with the following three cases.

- (i) *Exponential curve*: the cumulative testing-effort consumed in  $(0, t]$  is

$$W_e(t) = N \times (1 - \exp[-bt]). \quad (1)$$

The exponential curve is used for processes that decline monotonically to an asymptote.

- (ii) *Rayleigh curve*: the cumulative testing-effort consumed is

$$W_r(t) = N \times (1 - \exp[-(b/2)t^2]). \quad (2)$$

The Rayleigh curve first increases to a peak, and then decreases at a decelerating rate. It has been empirically observed that software development projects follow a life-cycle pattern described by the Rayleigh curve [12], [21]. The Rayleigh curve often predicts the costs, and schedules of software development well. It is frequently employed as an alternative to the exponential curve.

- (iii) *Weibull curve*: the cumulative testing-effort consumed is

$$W_w(t) = N \times (1 - \exp[-bt^m]). \quad (3)$$

The tail of the Weibull curve probability density function approaches zero asymptotically, but never reaches it [12].

In fact, Putnam [21] has used the Rayleigh characteristic as the basis for a time-sensitive cost model of software project behavior. He tuned the model using a large sample of project data collected by the Army Computer Systems Command, and found that for large projects the model converged acceptably to the sample data [21]. Here we note that the exponential ( $m = 1$ ), and the Rayleigh ( $m = 2$ ) curves are special cases of the Weibull curve. Actually, the exponential curve is often used when the testing-effort is uniformly consumed with respect to the testing time, while the Rayleigh curve is engaged in other cases [22].

Although a *Weibull*-type curve can well fit the data often used in the field of software reliability modeling, it displays a “peak” phenomenon when the shape parameter  $m > 3$ . Hence, Huang *et al.* [16] proposed that a logistic TEF be used instead of the

Weibull-type curve to describe the test-effort patterns during the software development process. Logistic TEF was originally proposed by F. N. Parr [23]. It exhibits similar behavior to the Rayleigh curve, except during the early part of the project. In some two dozen projects studied in the Yourdon 1978–1980 project survey, the logistic TEF appeared to be fairly accurate in describing the expended testing effort [21].

The logistic TEF over time period  $(0, t]$  can be expressed as

$$W(t) = \frac{N}{1 + A \exp[-\alpha t]}. \quad (4)$$

The current testing-effort expenditure rate at testing time  $t$  is

$$w(t) = \frac{NA\alpha \exp[-\alpha t]}{(1 + A \exp[-\alpha t])^2} = \frac{NA\alpha}{(\exp[\alpha t/2] + A \exp[-\alpha t/2])^2} \quad (5)$$

where

$$W(t) = \int_0^t w(t) dt.$$

Rate  $w(t)$  reaches its maximum value at time

$$t_{\max} = \frac{1}{\alpha} \ln A.$$

On the other hand, Gokhale & Trivedi proposed the log-logistic SRGM that can capture the increasing/decreasing nature of the failure occurrence rate per fault [6]. Recently, Bokhari & Ahmad [14] also presented how to use the log-logistic curve to describe the time-dependent behavior of testing effort consumptions during testing. The log-logistic TEF is given by

$$W_{ll}(t) = N \left( \frac{(\theta t)^\delta}{1 + (\theta t)^\delta} \right). \quad (6)$$

### III. SOFTWARE RELIABILITY GROWTH MODELING

#### A. SRGM With Logistic TEF

An SRGM with a logistic TEF is formulated based on the following assumptions [4], [16], [17], [24].

- 1) The fault removal process follows the NHPP.
- 2) The software system is subject to failures at random times caused by faults remaining in the system.
- 3) The mean number of faults detected in the time interval  $(t, t + \Delta t]$  by the current testing-effort is proportional to the mean number of remaining faults in the system.
- 4) The proportionality is a time-dependent fault detection rate function.
- 5) The consumption of testing-effort is modeled by a logistic TEF.
- 6) Each time a failure occurs, the fault that caused it is immediately removed, and no new faults are introduced.

Let  $m(t)$  be the MVF of the expected number of faults detected in time  $(0, t]$ . Then, according to the above assumptions, and from [16], [17], the model can be formulated as

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = r(t) \times [a - m(t)] \quad (a > 0). \quad (7)$$

If  $r(t) = r$  ( $0 < r < 1$ ), we have

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = r \times [a - m(t)]. \quad (8)$$

Solving (8) under the boundary condition  $m(0) = 0$  (i.e.,  $m(t)$  must be equal to zero at time zero), we have

$$\begin{aligned} m(t) &= a \times (1 - \exp[-r(W(t) - W(0))]) \\ &= a \times (1 - \exp[-rW^*(t)]). \end{aligned} \quad (9)$$

Equation (9) can provide testers or developers with an estimate of the time needed to reach a given level of residual faults. It may also be used to determine the appropriate release time for the software to meet the customer's expectations.

In general, the failure intensity function of the NHPP is given by

$$\lambda(t) \equiv \frac{dm(t)}{dt} = a \times r \times w(t) \times \exp[-rW^*(t)], \quad (10)$$

and

$$m(t) = \int_0^t \lambda(x) dx. \quad (11)$$

We can use the failure intensity as an estimate of the software quality [2], [4]. We can also use the number of remaining faults as a measure of quality.

Finally, we can have the expected number of faults remaining in the software system:

$$m_{\text{remaining}}(t) = a \times \exp[-rW^*(t)]. \quad (12)$$

For example, by using (4) & (9), the number of faults remaining in the software system after an infinite amount of test time is

$$\begin{aligned} m_{\text{remaining}}(\infty) &= a - m(\infty) = a \times \exp\left[-r \frac{N}{1+A}\right] \\ &\cong a \times \exp[-rN] \quad (\text{if } N \gg A). \end{aligned}$$

This means that not all the original faults in a software system can be fully detected, even after a long testing (and debugging) period because the total amount of testing-effort to be consumed during the testing phase is limited to  $N$ .

#### B. Delayed S-Shaped Model With Logistic TEF

In this section, we show how to integrate TEF into the delayed S-shaped model. This model was originally proposed by Yamada [25], who described it in terms of a modification of the NHPP to obtain an S-shaped growth curve for the cumulative number of faults detected. That is, it was designed to capture the software fault removal phenomenon. In this case, two phases can be observed within the testing process: fault detection, and fault isolation. There is a time lag between the fault detection, and its reporting. In other words, this model's software fault detection process can be viewed as a learning process because the software testers become familiar with the testing environments and tools as time progresses. It is assumed that testers' skills gradually improve over the testing effort, and then level

TABLE I  
 DATA SET 1 (OHBA [11])

Test time (weeks)	Cumulative execution time (CPU hours)	Cumulative faults	Test time (weeks)	Cumulative execution time (CPU hours)	Cumulative faults
1	2.45	15	11	26.23	233
2	4.90	44	12	27.67	255
3	6.86	66	13	30.93	276
4	7.84	103	14	34.77	298
5	9.52	105	15	38.61	304
6	12.89	110	16	40.91	311
7	17.10	146	17	42.67	320
8	20.47	175	18	44.66	325
9	21.43	179	19	47.65	328
10	23.35	206			

off as the residual faults become more difficult to uncover [4], [11]. However, the assumption that testers learn and improve the testing process may not hold within a single development cycle [5]. Because the original S-shaped model was developed for the analysis of fault isolation data, the testing process contains not only a fault detection process, but also a fault isolation process.

The extended delayed S-shaped model is based on the following assumptions [4], [18], [24], [25].

- 1) The fault removal process follows the NHPP.
- 2) The software system is subject to failures at random times caused by faults remaining in the system.
- 3) The mean number of faults detected in the time interval  $(t, t + \Delta t]$  by the current testing-effort is proportional to the mean number of remaining faults in the system.
- 4) The proportionality of fault detection is constant.
- 5) The mean number of faults isolated in the time interval  $(t, t + \Delta t]$  by the current testing-effort is proportional to the current number of faults not isolated in the system.
- 6) The proportionality of fault isolation is constant.
- 7) The consumption of testing-effort is modeled by a logistic TEF.
- 8) Each time a failure occurs, the fault which caused it is immediately removed, and no new faults are introduced.

Following the similar steps described in Section III-A, the extended S-shaped software reliability model can be formulated as

$$\frac{dm_d(t)}{dt} \times \frac{1}{w(t)} = r_1 \times [a - m_d(t)], \quad (13)$$

and

$$\frac{dm_r(t)}{dt} \times \frac{1}{w(t)} = r_2 \times [m_d(t) - m_r(t)]. \quad (14)$$

Note that here we assume  $r_2 \neq r_1$ . Solving (13) & (14) under the boundary condition  $m_d(0) = m_r(0) = 0$ , we have

$$m_d(t) = a \times (1 - \exp[-r_1 W^*(t)]), \quad (15)$$

and

$$m_r(t) = a \times \left\{ 1 - \frac{r_1 \exp[-r_2 W^*(t)] - r_2 \exp[-r_1 W^*(t)]}{r_1 - r_2} \right\}. \quad (16)$$

If we assume  $r_2 \cong r_1 \cong r$ , then by using L'Hospital's rule, the delayed S-shaped model with TEF is given by

$$m(t) \cong m_r(t) = a \times (1 - (1 + rW^*(t)) \exp[-rW^*(t)]). \quad (17)$$

Details of the derivation can be found in Appendix A.

On the other hand, if we let

$$r(t) = \frac{r^2 t}{1 + rt}, \quad (18)$$

and substitute it into (7), we obtain

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = \frac{r^2 t}{1 + rt} \times [a - m(t)]. \quad (19)$$

Solving (19) under the boundary condition  $m(0) = 0$ , we can also obtain the MVF that is the same as (17) [8], [24].

The failure intensity function for the delayed S-shaped model with TEF is given by

$$\lambda(t) = a \times r^2 \times w(t) \times W^*(t) \times \exp[-rW^*(t)]. \quad (20)$$

Finally, the expected number of faults remaining in the software system is

$$m_{remaining}(t) = a \times \{(1 + rW^*(t)) \exp[-rW^*(t)]\}. \quad (21)$$

#### IV. NUMERICAL EXAMPLES, AND EXPERIMENTS

##### A. Data Description, and Laplace Test

The first data set employed (DS1) was from the paper by Ohba [11] for a PL/I database application software system consisting of approximately 1 317 000 LOC. Over the course of 19 weeks, 47.65 CPU hours were consumed, and 328 software faults were removed. Although this is an old data-set, we feel it is instructive to use it because it allows direct comparison with the work of others who have used it. In addition, we use a second data set (DS2) presented by Wood from a subset of products for four separate software releases at Tandem Computers Company [26]. Wood reported that the specific products & releases are not identified, and the test data sets have been suitably transformed in order to avoid confidentiality issues. Here we only use Release 1 for illustrations. Over the course of 20 weeks, 10 000 CPU hours were consumed, and 100 software faults were removed. Tables I and II list the data sets DS1, and DS2, respectively.

Software reliability studies are usually based on the application of different SRGM to obtain various measures of interest.

TABLE II  
DATA SET 2 (WOOD [26])

Test time (weeks)	Cumulative execution time (CPU hours)	Cumulative faults	Test time (weeks)	Cumulative execution time (CPU hours)	Cumulative faults
1	519	16	11	6539	81
2	968	24	12	7083	86
3	1430	27	13	7487	90
4	1893	33	14	7846	93
5	2490	41	15	8205	96
6	3058	49	16	8564	98
7	3625	54	17	8923	99
8	4422	58	18	9282	100
9	5218	69	19	9641	100
10	5823	75	20	10000	100

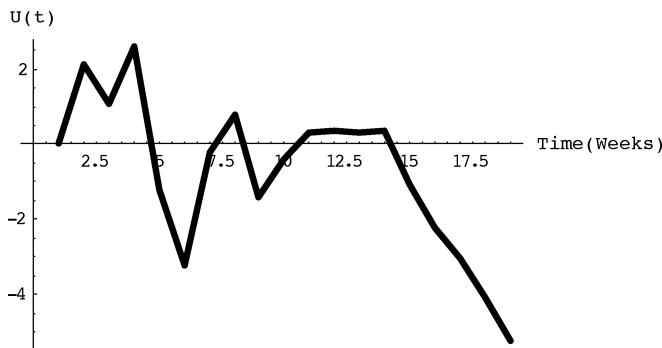


Fig. 1. Laplace trend test for the first data set.

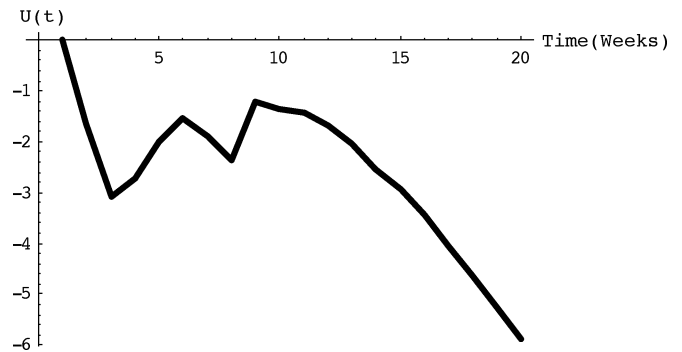


Fig. 2. Laplace trend test for the second data set.

Reliability growth can be analysed by trend tests. Blindly applying SRGM may not lead to meaningful results when the trend indicated by the data differs from that predicted by the model. If the model is applied to the software failure data, and shows a trend in accordance with its assumption, the results can be greatly improved [1]. Various statistical tests have been published for identifying trends in grouped data or time-series. Trend tests include graphical tests, and analytical tests. Among the analytical tests, the Laplace test is the most commonly used because it is often found to be the most appropriate one when failures & fault-removal follow NHPP [27]. Here we calculate  $U(t)$ . If the value of  $U(t)$  is negative, it indicates a decreasing failure intensity, and thus a reliability growth. On the other hand, if the value of  $U(t)$  is positive, it depicts an increasing failure intensity, and thus the reliability decreases [1], [28], [29].

For example, with reference to the above two data sets, the Laplace trend test results are shown in Figs. 1 and 2 for DS1, and DS2, respectively. As seen from Fig. 1, we find that in about the first 75% of the time, the value of  $U(t)$  is between  $-2$ , and  $+2$ ; this indicates a stable reliability. Thereafter, the value of  $U(t)$  is negative, and this means a decreasing failure intensity. Thus, in this case, our proposed models can be applied. Similarly, from Fig. 2, we can see that the value of  $U(t)$  is always negative; this means a growth in reliability.

### B. Criteria for Model Comparison

In general, a model can be analysed according to its ability to reproduce the observed behavior of the software, and to predict the future behavior of the software from the observed failure

data. The two data sets listed in Section IV-A are failure counts. The three comparison criteria are:

#### 1) The Goodness-of-Fit Criterion.

To quantitatively compare long-term predictions, we use MSE because it provides a well-understood measure of the differences between actual, and predicted values. The MSE is defined as [4], [16], [24], [30]

$$\text{MSE} = \frac{\sum_{i=1}^k [m(t_i) - m_i]^2}{k}. \quad (22)$$

A smaller MSE indicates a smaller fitting error, and better performance.

After the proposed model is fitted to the actual observed data, the deviation between the observed and the fitted values is evaluated by using K-S test, or the Chi-Square test. The K-S test is generally considered to be more effective compared with the Chi-Square test [31]. Therefore, we will present the results of the K-S test for each selected model. Here KD will be calculated and it is defined as the maximum vertical derivation between the plot, and the line of unit slope.

#### 2) The Predictive Validity Criterion.

The capability of the model to predict failure behavior from present & past failure behavior is called *predictive validity*. This approach, which was proposed by Musa [2], can be represented by computing RE for a data set

$$\text{RE} = \frac{m(t_q) - q}{q}. \quad (23)$$

Assuming we have observed  $q$  failures by the end of test time  $t_q$ , we employ the failure data up to time  $t_e (t_e \leq t_q)$  to estimate the parameters of  $m(t)$ . Substituting the estimates of these parameters in the MVF yields the estimate of the number of failures  $m(t_q)$  by time  $t_q$ . The estimate is compared with the actual number  $q$ . The procedure is repeated for various values of  $t_e$ . We can check the predictive validity by plotting the relative error for different values of  $t_q$ . Numbers closer to zero imply more accurate prediction. Positive values of error indicate overestimation; negative indicate underestimation [2].

### 3) The Noise Criterion.

The Noise is defined as [32]

$$\sum_{i=1}^n \left| \frac{\gamma_i - \gamma_{i-1}}{\gamma_{i-1}} \right|. \quad (24)$$

Small values represent less noise in the model's prediction behavior, indicating more smoothness.

Finally, in order to check the performance of the logistic TEF, and make a comparison with the Rayleigh TEF, here we also select some comparison criteria for our evaluations [16], [33]–[36]:

$$PE_i = Actual(\text{observed})_i - Predicted(\text{estimated})_i. \quad (25)$$

$$Bias = \sum_{i=1}^n \frac{PE_i}{n}. \quad (26)$$

$$Variation = \sqrt{\frac{\sum_{i=1}^n (PE_i - Bias)^2}{n - 1}}. \quad (27)$$

$$MRE = \left| \frac{M_{estimated} - M_{actual}}{M_{actual}} \right|. \quad (28)$$

### C. Model Performance Analysis

In this section, we present our evaluation of the performance of the proposed models when applied to DS1, and DS2.

1) *DS1*: Fitting a proposed model to actual data involves estimating the model parameters from the real failure data. We employ the method of MLE to estimate the parameters of different SRGM. Computational details can be found in Appendix B. Similarly, all the parameters of the logistic, and Rayleigh TEF are also estimated by MLE. Firstly, the three unknown parameters  $N$ ,  $A$ , and  $\alpha$  of the logistic TEF are solved by MLE, giving the estimated values  $N = 54.84$  (CPU hours),  $A = 13.03$ , and  $\alpha = 2.26 \times 10^{-1}/\text{week}$ . Correspondingly, the estimated parameters of the Rayleigh TEF are  $N = 49.32$  (CPU hours), and  $b = 1.37 \times 10^{-2}/\text{week}$ . Fig. 3 plots the comparisons between the observed failure data, and the data estimated by the logistic, and Rayleigh TEF. The PE, Bias, Variation, and MRE for the logistic, and Rayleigh TEF are listed in Table III. From Table III, we see that the logistic TEF has lower values of *PE*, *Bias*, *Variation*, and *MRE* than the Rayleigh TEF. On average, the logistic TEF yields a better fit for this data set.

Table IV lists the estimated values of parameters of different SRGM, including the Goel-Okumoto model, and the traditional

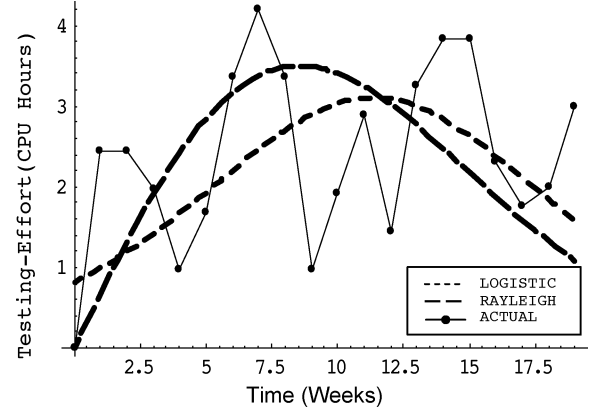


Fig. 3. Observed/estimated logistic & Rayleigh TEF for DS1.

TABLE III  
COMPARISON RESULTS FOR DIFFERENT TEF APPLIED TO DS1

TEF	Bias	Variation	MRE	PE <sub>end of testing</sub>
logistic TEF	-0.10	1.31	0.02	1.06
Rayleigh TEF	0.83	2.17	0.05	2.50

Yamada delayed S-Shaped model. We also give the values of MSE, RE, Noise, and KD in Table IV. It is observed that the SRGM with logistic TEF has the smallest value of MSE, and KD when compared with other SRGM. Because parameters of SRGM are estimated based on a limited amount of data, confidence estimation is necessary [2], [37]. The 90 percent confidence limits for all the models are given in Table V. The relevant calculation details can be found in Appendix B.

Finally, Fig. 4 depicts the RE curves for different selected models. It is worth noting that, in the study in [11], the author reported that the Yamada delayed S-shaped model may not fit the observed data well when the testing-effort spent on fault detection is not a constant. However, from Table IV, we see that the delayed S-shaped model with the logistic TEF achieves lower MSE, RE, and KD than the traditional Yamada delayed S-Shaped model, and the delayed S-Shaped model with the Rayleigh TEF. Overall, the delayed S-Shaped model with the logistic TEF predicts more accurately than these two S-shaped software reliability models.

2) *DS2*: Similarly, the maximum likelihood estimates of the parameters for the logistic TEF in the case of DS2 are  $N = 10\,505$  (CPU hours),  $A = 15.24$ , and  $\alpha = 2.85 \times 10^{-1}/\text{week}$ . Also, the estimated parameters for the Rayleigh TEF are  $N = 10\,534$  (CPU hours), and  $b = 1.49 \times 10^{-2}/\text{week}$ . The computed Bias, Variation, PE, and MRE for the logistic, and Rayleigh TEF are listed in Table VI. Fig. 5 graphically illustrates the comparisons between the observed failure data, and the data estimated by the logistic, and Rayleigh TEF. As seen from Fig. 5, and Table VI, similar to DS1, the logistic TEF yields a better fit than the Rayleigh TEF for DS2. Table VII shows the estimated values of parameters of different SRGM, and the values of MSE, RE, Noise, and KD. From Table VII, we find that the SRGM with the logistic TEF has the smallest value of MSE compared with the other SRGM. Besides, we also see that the values of MSE, and RE of the delayed S-Shaped model with the logistic TEF are still lower than those of the traditional Yamada

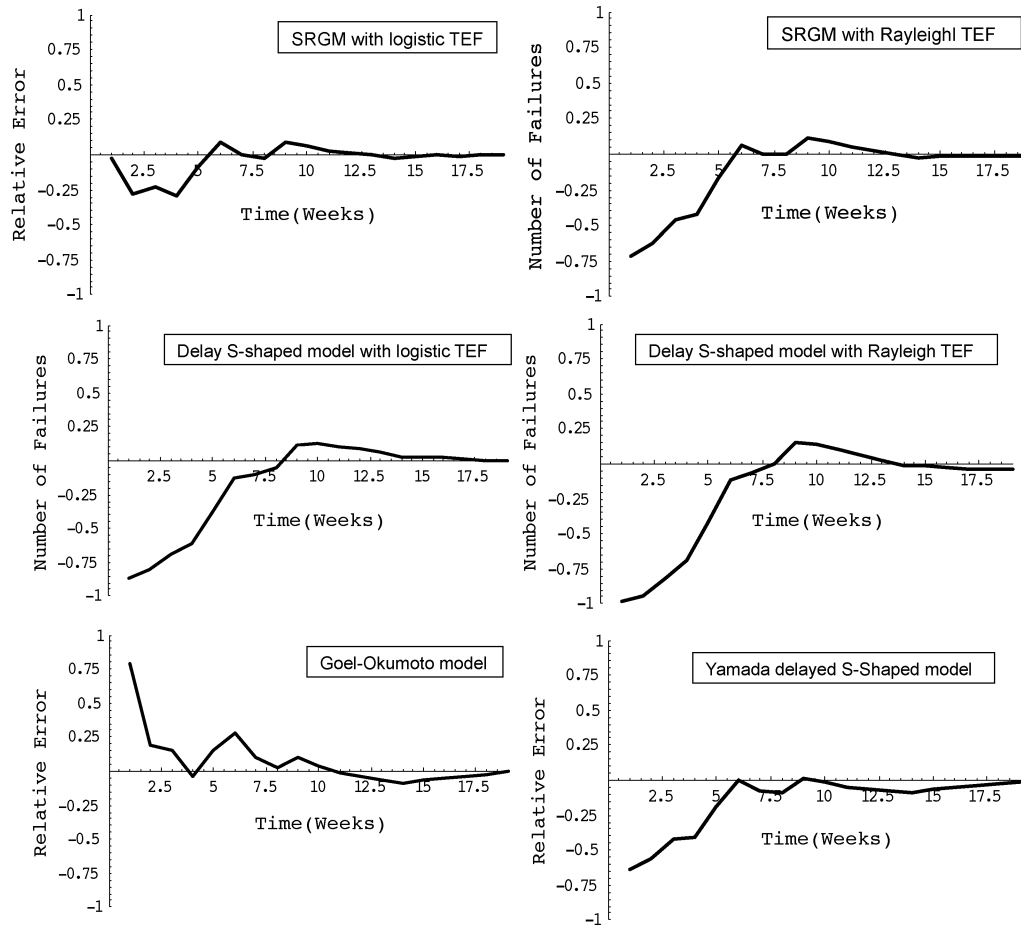


Fig. 4. RE curves of selected models compared with actual failure data (DS1).

TABLE IV  
ESTIMATED PARAMETER VALUES, AND MODEL COMPARISONS FOR DS1

Models	$a$	$r$	MSE	RE	Noise	KD
SRGM with logistic TEF	395.57	$4.16 \times 10^{-2}$	114.06	$2.14 \times 10^{-3}$	2.12	$9.44 \times 10^{-2}$
SRGM with Rayleigh TEF	459.09	$2.73 \times 10^{-2}$	268.42	$-7.77 \times 10^{-3}$	3.45	$1.30 \times 10^{-1}$
Delayed S-Shaped model with logistic TEF	339.96	$1.21 \times 10^{-1}$	634.83	$7.11 \times 10^{-7}$	5.47	$1.03 \times 10^{-1}$
Delayed S-Shaped model with Rayleigh TEF	333.18	$1.00 \times 10^{-1}$	798.49	$-4.46 \times 10^{-2}$	11.74	$2.14 \times 10^{-1}$
Goel-Okumoto model	513.15	$5.37 \times 10^{-2}$	222.14	$-2.90 \times 10^{-5}$	0.94	$9.52 \times 10^{-2}$
Yamada delayed S-Shaped model	384.05	$2.19 \times 10^{-1}$	640.67	$1.15 \times 10^{-1}$	2.33	$1.12 \times 10^{-1}$

delayed S-Shaped model, and the delayed S-Shaped model with the Rayleigh TEF. The 90% confidence limits for the proposed models are also given in Table VIII. And Fig. 6 depicts the RE curves for all selected models.

Finally, software reliability depends on the pattern of operation of the software, and the performance of SRGM strongly depends on the kind of data set. If the software development project managers plan to employ SRGM for estimation of reliability growth of products during software development processes, the software developers or reliability engineers need to select several representative models, and apply them in parallel. Although models sometimes give good results, there is no single model that can be trusted to give accurate results in all circumstances, nor is there a way in which the most suitable model can

be chosen *a priori* for a particular situation [1], [3], [4]. From our results, we can conclude that the logistic TEF may be a good approach to providing a more accurate description of resource consumption during the software development phase than previous approaches. By incorporating the logistic TEF into both exponential-type, and S-shaped software reliability models, the modified SRGM become more powerful, and more informative in the software reliability engineering process.

## V. IMPERFECT DEBUGGING MODELING

In general, different SRGM make different assumptions, and therefore can be applied to different situations. Most SRGM published in the literature assume that each time a failure occurs, the fault that caused it is immediately removed, and no

TABLE V  
 90% CONFIDENCE LIMITS FOR DIFFERENT SELECTED MODELS (DS1)

Models	$a$		$r$	
	Upper	Lower	Upper	Lower
SRGM with logistic TEF	440.84	350.31	$4.96 \times 10^{-2}$	$3.36 \times 10^{-2}$
SRGM with Rayleigh TEF	531.35	386.82	$3.43 \times 10^{-2}$	$2.03 \times 10^{-2}$
Delayed S-Shaped model with logistic TEF	371.13	308.79	$1.31 \times 10^{-1}$	$1.11 \times 10^{-1}$
Delayed S-Shaped model with Rayleigh TEF	364.10	302.27	$1.09 \times 10^{-1}$	$9.19 \times 10^{-2}$
Goel-Okumoto model	617.73	408.57	$7.07 \times 10^{-2}$	$3.67 \times 10^{-2}$
Yamada delayed S-Shaped model	420.84	347.26	$2.39 \times 10^{-1}$	$1.98 \times 10^{-1}$

 TABLE VI  
 COMPARISON RESULTS FOR DIFFERENT TEF APPLIED TO DS2

TEF	Bias	Variation	MRE	PE <sub>end of testing</sub>
logistic TEF	-49.90	290.03	$3.82 \times 10^{-15}$	$3.82 \times 10^{-11}$
Rayleigh TEF	193.33	405.45	$0.50 \times 10^{-13}$	$0.50 \times 10^{-9}$

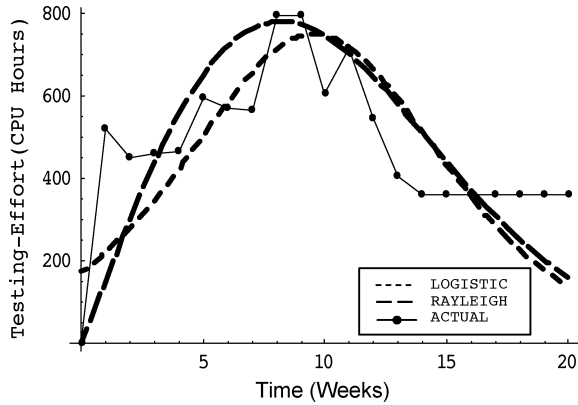


Fig. 5. Observed/estimated logistic &amp; Rayleigh TEF for DS2.

new faults are introduced. Besides, some people also assume that the correction of a fault takes only negligible time, and the detected fault is removed with certainty [38]–[40]. These assumptions help to reduce the complexity of modeling software reliability growth; however, in reality, developers experience cases where they fix one bug, but create another new one. Debugging is in fact a complex cognitive activity because it consists of locating & correcting the faults that generated the observed failures. Therefore, imperfect debugging could occur in the real world.

There are many papers that have addressed the problem of imperfect debugging [40]–[48]. For instance, Ohba & Chou [44] reported that, in their study, about 14 percent of the faults detected & removed during an observation period would introduce new faults as a result of imperfect debugging. They also demonstrated that, in such cases, SRGM are still applicable, although imperfect debugging caused some variation in the parameter values of the engaged models. On the other hand, Goel & Okumoto [48] showed that an imperfect debugging model provided a good fit to the software failure data from a real-time control system for a land-based radar system developed by the Raytheon Company. Xie & Yang [41] tried to investigate the effect of imperfect debugging on software development costs. They extended a commonly used cost model to the case of imperfect debugging. In addition, Zhang *et al.* [45] also proposed

a method to integrate fault removal efficiency, and fault introduction rate into SRGM. Therefore, we have to consider the imperfect debugging problem when we propose a new SRGM, as it provides an essential, valuable insight into the debugging process.

In this section, we investigate a relaxation of the perfect debugging assumption. We modify the assumption 6 presented in Section III-A to be as follows: *When a fault is detected & removed, new faults may be generated. Besides, when removing or fixing a detected fault, the probability of introducing another fault is a constant  $\beta$ .* Based on assumptions 1–5 described in Section III-A, we can describe in detail the SRGM with the logistic TEF within an imperfect debugging environment. According to these assumptions, we rewrite (7) as

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = r(t) \times [n(t) - m(t)]. \quad (29)$$

Note that  $n(t)$  is generally defined as the sum of the expected number of initial software faults, and introduced faults as a function of time  $t$  [4], [45].

Mathematically, assuming that

$$n(t) = a + \beta \times m(t), \quad (30)$$

solving (29) by substituting (30) into it, and assuming  $m(0) = 0$  &  $r(t) = r$ , we obtain the MVF

$$m(t) = \frac{a}{1 - \beta} (1 - \exp[-r(1 - \beta)W^*(t)]). \quad (31)$$

We also have

$$n(t) = \frac{a}{1 - \beta} (1 - \beta \exp[-r(1 - \beta)W^*(t)]). \quad (32)$$

It is noted that  $\lambda(t)$  is given by

$$\lambda(t) = a \times r \times w(t) \times \exp[-r(1 - \beta)W^*(t)]. \quad (33)$$

In this case, we have

$$m_{\text{remaining}}(t) = n(t) - m(t) = a \times (\exp[-r(1 - \beta)W^*(t)]). \quad (34)$$

Similarly, we can modify the assumption 8 presented in Section III-B. For example, by using (19), we have

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = \frac{r^2 t}{1 + rt} \times [n(t) - m(t)]. \quad (35)$$



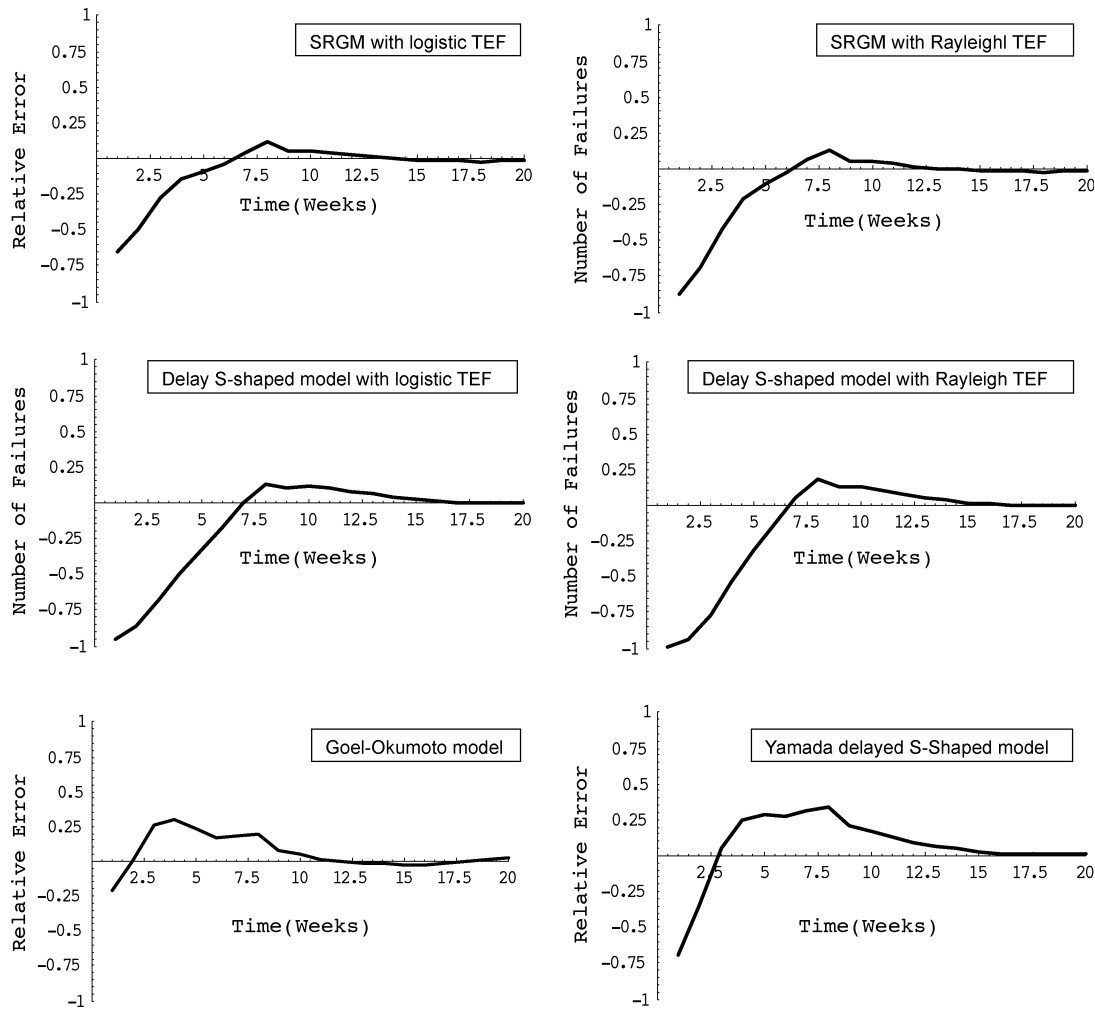


Fig. 6. RE curves of selected models compared to actual failure data (DS2).

TABLE VII  
ESTIMATED PARAMETER VALUES, AND MODEL COMPARISONS FOR DS2

Models	$a$	$R$	MSE	RE	Noise	KD
SRGM with logistic TEF	107.66	$2.66 \times 10^{-4}$	22.76	$-1.27 \times 10^{-2}$	3.44	$1.18 \times 10^{-1}$
SRGM with Rayleigh TEF	110.61	$2.26 \times 10^{-4}$	39.69	$-9.51 \times 10^{-3}$	4.24	$1.65 \times 10^{-1}$
Delayed S-Shaped model with logistic TEF	101.86	$6.35 \times 10^{-4}$	92.66	$1.22 \times 10^{-6}$	6.79	$1.86 \times 10^{-1}$
Delayed S-shaped model with Rayleigh TEF	102.14	$5.78 \times 10^{-4}$	107.97	$-1.67 \times 10^{-6}$	12.05	$2.25 \times 10^{-1}$
Goel-Okumoto model	112.48	$1.20 \times 10^{-1}$	30.27	$2.20 \times 10^{-2}$	2.14	$9.79 \times 10^{-2}$
Yamada delayed S-Shaped model	102.26	$3.45 \times 10^{-1}$	94.99	$1.49 \times 10^{-2}$	3.99	$1.88 \times 10^{-1}$

Solving (35) under the boundary condition  $m(0) = 0$ , the delayed S-Shaped model with logistic TEF under imperfect debugging is given by

$$m(t) = \frac{a}{1-\beta} \left\{ 1 - (1 + rW^*(t))^{1-\beta} \exp[-r(1-\beta)W^*(t)] \right\}. \quad (36)$$

These above equations (31) & (36) can represent the case where a fault is not successfully removed, and new faults are introduced during the testing/debugging phase.

Due to the space limitations, here we use only DS1 (i.e., the Ohba data) to discuss the issue of imperfect debugging. Simi-

larly, the parameters  $a$ ,  $r$ , and  $\beta$  in (31), and (36) can be solved numerically by the method of MLE. Moreover, as discussed in [44], we can apply the extended Goel-Okumoto model by taking account of imperfect debugging with MLE for parameter estimation. Table IX gives the estimated parameters of selected models under imperfect debugging, and results of model comparisons. It is observed that the values of MSE and KD of the SRGM with the logistic TEF are the lowest among all the models considered. Besides, we also see that the values of MSE, RE, Noise, and KD of the delayed S-Shaped model with the logistic TEF are still lower than those of the delayed S-Shaped model with Rayleigh TEF.

TABLE VIII  
COMPARISONS OF 90% CONFIDENCE LIMITS FOR DIFFERENT SELECTED MODELS (DS2)

Models	<i>a</i>		<i>r</i>	
	Upper	Lower	Upper	Lower
SRGM with logistic TEF	126.44	88.87	$3.36 \times 10^{-4}$	$1.97 \times 10^{-4}$
SRGM with Rayleigh TEF	130.54	90.68	$2.89 \times 10^{-4}$	$1.62 \times 10^{-4}$
Delayed S-Shaped model with logistic TEF	118.67	85.06	$7.19 \times 10^{-4}$	$5.50 \times 10^{-4}$
Delayed S-Shaped model with Rayleigh TEF	119.00	85.27	$6.56 \times 10^{-4}$	$4.99 \times 10^{-4}$
Goel-Okumoto model	132.53	92.44	$1.53 \times 10^{-1}$	$8.63 \times 10^{-2}$
Yamada delayed S-Shaped model	119.13	85.46	$3.89 \times 10^{-1}$	$3.02 \times 10^{-1}$

TABLE IX  
ESTIMATED PARAMETER VALUES, AND MODEL COMPARISONS UNDER IMPERFECT DEBUGGING (DS1)

Models	<i>a</i>	<i>r</i>	$\beta$	MSE	RE	Noise	KD
SRGM with logistic TEF	391.62	$4.20 \times 10^{-2}$	$1.16 \times 10^{-2}$	114.09	$2.83 \times 10^{-3}$	2.12	$9.48 \times 10^{-2}$
SRGM with Rayleigh TEF	399.02	$3.16 \times 10^{-2}$	$1.23 \times 10^{-1}$	268.55	$-1.00 \times 10^{-2}$	3.46	$1.29 \times 10^{-1}$
Delayed S-Shaped model with logistic TEF	335.39	$1.24 \times 10^{-1}$	$1.15 \times 10^{-2}$	634.60	$-4.12 \times 10^{-14}$	5.48	$1.09 \times 10^{-1}$
Delayed S-Shaped model with Rayleigh TEF	346.09	$9.88 \times 10^{-2}$	$1.39 \times 10^{-2}$	880.49	$-5.30 \times 10^{-13}$	11.73	$2.10 \times 10^{-1}$
Extended Goel-Okumoto model [44]	365.85	$7.53 \times 10^{-2}$	$2.87 \times 10^{-1}$	222.09	$-2.20 \times 10^{-13}$	0.94	$9.52 \times 10^{-2}$

TABLE X  
90% CONFIDENCE LIMITS FOR SRGM UNDER IMPERFECT DEBUGGING (DS1)

Models	<i>a</i>		<i>R</i>	
	Upper	Lower	Upper	Lower
SRGM with logistic TEF	436.56	346.69	$5.01 \times 10^{-2}$	$3.39 \times 10^{-2}$
SRGM with Rayleigh TEF	460.48	337.55	$3.95 \times 10^{-2}$	$2.37 \times 10^{-2}$
Delayed S-Shaped model with logistic TEF	370.02	308.57	$1.33 \times 10^{-1}$	$1.14 \times 10^{-1}$
Delayed S-Shaped model with Rayleigh TEF	378.44	313.73	$1.08 \times 10^{-1}$	$9.02 \times 10^{-2}$
Extended Goel-Okumoto model [44]	440.43	291.28	$9.91 \times 10^{-2}$	$5.14 \times 10^{-2}$

From Table IX, we observe that the fault removal process in the software development & testing environment may not be a pure perfect debugging process because the estimated values of  $\beta$  are all close but not equal to zero. For example, we can see that the fault introduction rate of the SRGM with the logistic TEF is  $1.16 \times 10^{-2}$ . This means that, on the average, one fault will be introduced per about 100 removed faults. Hence, we see that the introduction of new faults during the correction process tends to be a minor effect in the development process if we apply the software reliability models listed in Table IX. Actually, Yin *et al.* [43] reported that, from a statistical point of view, the number of introduced faults is less significant when the total number of remaining faults is relatively large. They postulated that the imperfect debugging should be taken into account when the software product is reaching the mature stage, where the number of remaining faults, and the number of introduced faults are of the same order of magnitude [43], [49]. In addition to (30), there may be other useful fault content functions [24], [45], but further discussion of this topic is beyond the scope of this paper. The 90% confidence limits for the selected models are given in Table X, while Fig. 7 illustrates the RE curves for different models. Altogether, these metrics can provide engineers with insightful information about software development & testing efforts, and help project managers make the best decisions in allocating testing resources.

APPENDIX A

From (16), we note that, if  $r_2$  is approximately the same as  $r_1$ , the right hand side of this equation will approach negative infinity. In this case, we let

$$f(r_2) = r_1 \exp[-r_2 W^*(t)] - r_2 \exp[-r_1 W^*(t)], \quad (37)$$

and

$$g(r_2) = r_1 - r_2. \quad (38)$$

From L'Hospital's Rule, we know

$$\begin{aligned} \lim_{r_2 \rightarrow r_1} \frac{f(r_2)}{g(r_2)} &= \lim_{r_2 \rightarrow r_1} \frac{f(r_2) - f(r_1)}{g(r_2) - g(r_1)} \\ &= \lim_{r_2 \rightarrow r_1} \frac{\frac{f(r_2) - f(r_1)}{r_2 - r_1}}{\frac{g(r_2) - g(r_1)}{r_2 - r_1}} \\ &= \lim_{r_2 \rightarrow r_1} \frac{f'(r_2) - f'(r_1)}{g'(r_2) - g'(r_1)} \\ &= \frac{f'(r_1)}{g'(r_1)}. \end{aligned} \quad (39)$$

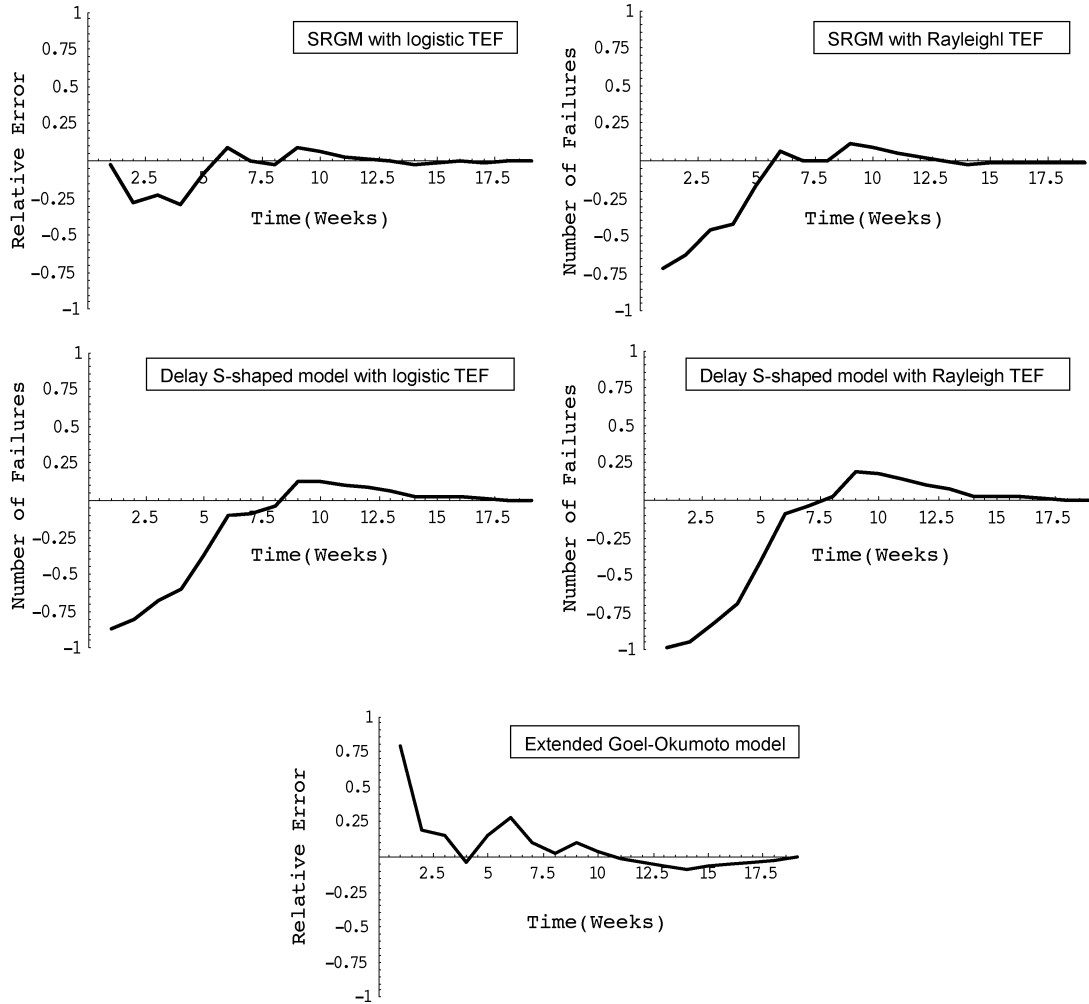


Fig. 7. RE curves of selected models under imperfect debugging compared to actual failure data (DS1).

Thus, we obtain

$$\begin{aligned} f'(r_1) &= -r_1 W^*(t) \exp[-r_1 W^*(t)] - \exp[-r_1 W^*(t)] \\ &= -(1 + r_1 W^*(t)) \exp[-r_1 W^*(t)], \end{aligned} \quad (40)$$

and

$$g'(r_1) = -1. \quad (41)$$

That is,

$$\frac{f'(r_1)}{g'(r_1)} = (1 + r_1 W^*(t)) \exp[-r_1 W^*(t)]. \quad (42)$$

Therefore,

$$\begin{aligned} \lim_{r_2 \rightarrow r_1} m_r(t) &= \lim_{r_2 \rightarrow r_1} a \left\{ 1 - \frac{r_1 \exp[-r_2 W^*(t)] - r_2 \exp[-r_1 W^*(t)]}{r_1 - r_2} \right\} \\ &= a - \lim_{r_2 \rightarrow r_1} a \left( \frac{r_1 \exp[-r_2 W^*(t)] - r_2 \exp[-r_1 W^*(t)]}{r_1 - r_2} \right) \\ &= a \times \{1 - (1 + r_1 W^*(t)) \exp[-r_1 W^*(t)]\}. \end{aligned} \quad (43)$$

#### APPENDIX B

Fitting a proposed model to actual fault data involves estimating the model parameters from the real test data sets. Here

we employ the method of MLE to estimate the parameters  $a$ , and  $r$  [1], [2]. All parameters of different TEF can be estimated by the method of MLE. For example, suppose that  $a$ , and  $r$  are determined for the  $n$  observed data pairs:

$$(t_0, m_0), (t_1, m_1), (t_2, m_2), (t_3, m_3), (t_4, m_4), \dots, (t_n, m_n).$$

Then the likelihood function for the parameters  $a$ , and  $r$  in the NHPP model with  $m(t)$  in (9) is given by

$$\begin{aligned} L &\equiv P_r \{N(t_1) = m_1, N(t_2) = m_2, \dots, N(t_n) = m_n\} \\ &= \prod_{k=1}^n \frac{\{m(t_k) - m(t_{k-1})\}}{(m_k - m_{k-1})!} \exp[-(m(t_k) - m(t_{k-1}))], \end{aligned} \quad (44)$$

where  $m_0 \equiv 0$  for  $t_0 \equiv 0$ .

Taking the natural logarithm of the likelihood function in (44), we have

$$\begin{aligned} \ln L &= \sum_{k=1}^n (m_k - m_{k-1}) \ln [m(t_k) - m(t_{k-1})] \\ &\quad - \sum_{k=1}^n ((m(t_k) - m(t_{k-1})) - \sum_{k=1}^n \ln [(m_k - m_{k-1})!]). \end{aligned} \quad (45)$$

From (9), we know that  $m(tk) - m(tk - 1) = a(\exp[-rW^*(tk - 1)] - \exp[-rW^*(tk)])$ . Thus,

$$\begin{aligned} \ln L = & \sum_{k=1}^n (m_k - m_{k-1}) \ln a + \sum_{k=1}^n (m_k - m_{k-1}) \\ & \times \ln [(\exp[-rW^*(t_{k-1})] - \exp[-rW^*(t_k)])] \\ & - a(1 - \exp[-rW^*(t_n)]) - \sum_{k=1}^n \ln [(m_k - m_{k-1})!]. \end{aligned} \tag{46}$$

Consequently, the maximum likelihood estimates of  $a$ , and  $r$  can be obtained by solving

$$\frac{\partial \ln L}{\partial a} = \frac{\partial \ln L}{\partial r} = 0. \tag{47}$$

Therefore, we obtain

$$a = \frac{\sum_{k=1}^n (m_k - m_{k-1})}{1 - \exp[-rW^*(t_n)]} = \frac{m_n}{1 - \exp[-rW^*(t_n)]}, \tag{48}$$

and (49) at the bottom of the page. Therefore,  $a$ , and  $r$  can be solved by numerical methods.

Similarly, for the delayed S-Shaped model with logistic TEF, we can get

$$a = \frac{m_n}{1 - (1 + rW^*(t_n)) \times \exp[-rW^*(t_n)]}, \tag{50}$$

and (51) at the bottom of the page.

Finally, if the sample size  $n$  of  $(t_k, m_k)$  is sufficiently large, then the maximum-likelihood estimates  $\hat{a}$ , and  $\hat{r}$  of the SRGM's parameters  $a$ , and  $r$  asymptotically follow a bivariate  $s$ -normal (BVN) distribution [13], [15], [16], [22]

$$\begin{pmatrix} \hat{a} \\ \hat{r} \end{pmatrix} \sim BVN \left( \begin{pmatrix} a \\ r \end{pmatrix}, V \right) \quad (n \rightarrow \infty) \tag{52}$$

The mean values of  $\hat{a}$ , and  $\hat{r}$  are the true values of  $a$ , and  $r$ , respectively; and the variance-covariance matrix  $V$  is given by the inverse matrix of the Fisher information matrix [50]. The Fisher information matrix  $F$  for the two-parameter NHPP model (i.e.,  $\hat{a}$ , and  $\hat{r}$ ) can be derived from  $\ln L$  as

$$F = \begin{bmatrix} E \left[ -\frac{\partial^2 \ln L}{\partial a^2} \right] & E \left[ -\frac{\partial^2 \ln L}{\partial a \partial r} \right] \\ E \left[ -\frac{\partial^2 \ln L}{\partial a \partial r} \right] & E \left[ -\frac{\partial^2 \ln L}{\partial r^2} \right] \end{bmatrix} \tag{53}$$

Applying  $\hat{a}$ , and  $\hat{r}$  to the above equation, and calculating  $F^{-1}$ , the large sample asymptotic variance-covariance matrix  $V$  is given by

$$V = F^{-1} = \begin{bmatrix} Var\{\hat{a}\} & Cov\{\hat{a}, \hat{r}\} \\ Cov\{\hat{a}, \hat{r}\} & Var\{\hat{r}\} \end{bmatrix} \tag{54}$$

The variance-covariance matrix  $V$  is useful in quantifying the variability of the estimated parameters. The two-sided approximate  $100\alpha\%$  confidence limits for  $a$ , and  $r$  can then be obtained in a standard way [13], [37], [50]. For example, the two-sided approximate  $100\alpha\%$  confidence limits for the parameters  $a$ , and  $r$  are

$$a_U = \hat{a} + Z_{\alpha/2} \sqrt{Var(\hat{a})}, \tag{55}$$

$$a_L = \hat{a} - Z_{\alpha/2} \sqrt{Var(\hat{a})}, \tag{56}$$

$$r_U = \hat{r} + Z_{\alpha/2} \sqrt{Var(\hat{r})}, \tag{57}$$

and

$$r_L = \hat{r} - Z_{\alpha/2} \sqrt{Var(\hat{r})}, \tag{58}$$

where  $Z_{\alpha/2}$  is the  $(1 - \alpha/2)$  quartile of the standard  $s$ -normal distribution.

#### ACKNOWLEDGMENT

The authors would like to thank Editor Prof. M. A. Vouk for his constructive, insightful suggestions, which led to a significant improvement of this paper.

$$\begin{aligned} a \times (W^*(t_n)) \times \exp[-rW^*(t_n)] \\ = \sum_{k=1}^n (m_k - m_{k-1}) \times \frac{(-W^*(t_{k-1})) \times \exp[-rW^*(t_{k-1})] + W^*(t_k) \times \exp[-rW^*(t_{k-1})]}{\exp[-rW^*(t_{k-1})] - \exp[-rW^*(t_k)]} \end{aligned} \tag{49}$$

$$\begin{aligned} a \times (W^*(t_n))^2 \times \exp[-rW^*(t_n)] \\ = \sum_{k=1}^n (m_k - m_{k-1}) \times \frac{\{(W^*(t_k))^2 \times \exp[-rW^*(t_k)] - (W^*(t_{k-1}))^2 \times \exp[-rW^*(t_{k-1})]\}}{(1 + rW^*(t_{k-1})) \times \exp[-rW^*(t_{k-1})] - (1 + rW^*(t_k)) \times \exp[-rW^*(t_k)]} \end{aligned} \tag{51}$$

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