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# Strip line detection and thinning by RPCL-based local PCA <sup>☆</sup>

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## Abstract

We solve the tasks of strip line detection and thinning in image processing and pattern recognition with the help of a statistical learning technique called rival penalized competitive learning based local principal component analysis. Due to its model selection and noise resistance ability, the technique is experimentally shown to outperform conventional Hough transform and thinning algorithms.

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*Keywords:* Strip line detection; Strip line thinning; Local PCA; Model selection; RPCL learning

## 1. Introduction

Strip line detection and thinning are two basic problems in image processing and pattern recognition. In this paper, we introduce a novel technique for solving both tasks.

Strip line detection concerns identifying a thick, linear pattern from an image. This can be achieved by detecting the main axis of the strip line concerned. In literature, Hough transform (HT) (Hough, 1962; Ballard, 1981) is an important tool for line detection. Its further advances in term of randomized Hough transform (RHT) (Xu et al., 1990; Xu and Oja, 1993) not only considerably

reduces the time and space complexity of the conventional Hough transform, but also increases the resolution of parameterization and the robustness to outliers. However, neither Hough transform nor RHT can be directly used to detect thick strip lines. The problem is usually resolved by either adopting edge detection for preprocessing or instead, extending Hough transform on strip band detection (Lo and Tsai, 1995). The first approach actually converts the problem of strip line detection to edge line detection and thus make Hough transform applicable. However, what we desire is to detect the main axis of the strip line itself, instead of its edges. Though we may further extend Hough transform to direct the main axis as in (Lo and Tsai, 1995), the resulted performance is still far from satisfactory when used for detecting a set of parallel, thick strip lines, especially under the conventional normal or uniform noises.

On the other hand, image thinning (Davies and Plummer, 1981) is used for eliminating redundant

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pixels in image. In general, thinning is effected via iteratively deleting the successive layers of pixels on the boundary of the pattern until the skeleton remains (Marchand-Maillet and Sharaiha, 2000). However, such a pixel-by-pixel approach has no noise tolerance ability and thus performs poorly in a noisy environment. Moreover, its performance would deteriorate greatly for the blurred pattern that has some deformation on the skeleton.

It is well known that the first principal component in principal component analysis (PCA) gives the main axis of a cluster of samples from a gaussian distribution. Thus, it has been also suggested to perform PCA locally on each of multiple clusters to detect its main axis as a detected line (Xu, 1994). Such local PCA based approaches are more robust under gaussian noise in comparison with the Hough transform related approaches. For the problems of detecting lines or strips in images, however, pixels spreading from its main axis are more likely from a uniform distribution instead of a gaussian distribution. In this paper, we further investigate the problem of detecting the main axis of strip lines. First, in Section 2 we mathematically show that the main axis found by PCA is also the main axis of pixels uniformly distributed on a strip, which thus justifies the direct usage of local PCA for strip line detection. Second, in Section 3 we adopt the RPCL-based local PCA model for implementation. The performance has been shown by experiments in Section 4, and finally we conclude in Section 5.

## 2. A mathematical justification on using local PCA for strip pattern detection and thinning

The term local PCA (Xu, 1994; Hinton et al., 1995; Kambhatla and Leen, 1997; Tipping and Bishop, 1999; Xu, 2001) comes from the extension of PCA. PCA (Jolliffe, 1986) involves a mathematical procedure that linearly transforms a number of correlated variables into a (smaller) number of uncorrelated variables called principal components. It is frequently adopted for dimensionality reduction as the mean square error (MSE) upon reconstruction error is minimized. However, when the data are from multi-modes, performing PCA

can be far from satisfactory and thus its local extension is preferable. Figs. 1 and 2 are two simple examples generated from three gaussian's, where it is better to perform PCA locally on each of multiple clusters to detect its main axis as a detected line (Xu, 1994). A similar concept for curve detection was also discussed by Xu et al. (1993) from the perspective of clustering analysis, with an efficient model selection ability with the help of the RPCL learning. Such local PCA based approaches are more robust under gaussian noise in comparison with the Hough transform related approaches, as shown in Fig. 2, when the first principal component can be described by the main axis of a cluster of samples from a gaussian distribution. However, for a problem of detecting strip line in

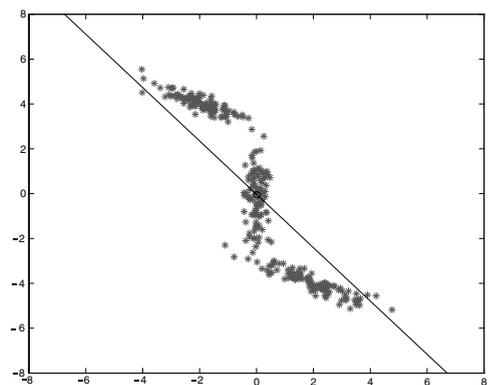


Fig. 1. PCA description.

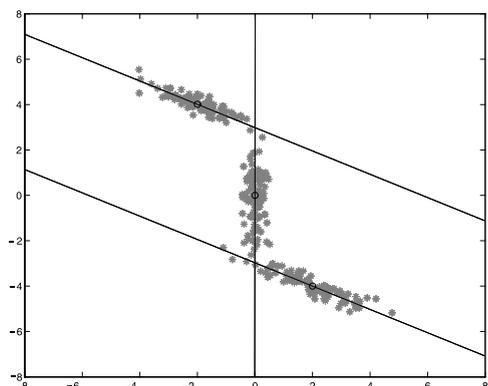


Fig. 2. Local PCA description.

image, pixels spreading from its main axis are more likely from a uniform distribution instead of a gaussian distribution.

Here we mathematically prove the following theorem, which states that the main axis, or the first principal component found by PCA on a strip line with the uniformly distributed pixels is also its main axis, which thus justifies the direct usage of local PCA for strip line detection.

**Theorem 2.1.** *The first principal component obtained by PCA on a strip line with uniformly distributed pixels is its main axis.*

**Proof.** Without loss of generality, assume pixel distribution being bivariate uniform, symmetrical with being  $U(-m, m)$  and  $U(-n, n)$  on  $x$  and  $y$  direction respectively, as shown in Fig. 3.

Let the first principal component pass through origin  $[0, 0]^T$  and in the direction  $[\cos \theta, \sin \theta]^T$ . Denote any pixel  $s = [x, y]^T$ . Transformation via the first principal component results in

$$\tilde{s} = [\cos \theta, \sin \theta][x, y]^T = x \cos \theta + y \sin \theta. \quad (1)$$

This implies the reconstructed signal  $\hat{s}$  is

$$\hat{s} = [\cos \theta, \sin \theta]^T \tilde{s} = \begin{bmatrix} x \cos^2 \theta + y \cos \theta \sin \theta \\ y \sin^2 \theta + x \cos \theta \sin \theta \end{bmatrix}. \quad (2)$$

MSE of the reconstructed data is

$$e = (\hat{s} - s)^T (\hat{s} - s) = x^2 \sin^2 \theta + y^2 \cos^2 \theta - 2xy \cos \theta \sin \theta. \quad (3)$$

Take expectation of (3) over all the reconstructed pixels. The expected MSE is

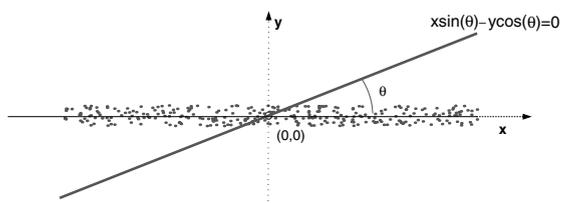


Fig. 3. Uniform distribution of line pixels.

$$\begin{aligned} E &= E_s\{(\hat{s} - s)^T (\hat{s} - s)\} \\ &= E_{x,y}\{x^2 \sin^2 \theta + y^2 \cos^2 \theta - 2xy \cos \theta \sin \theta\} \\ &= \sin^2 \theta \text{Var}(x) + \cos^2 \theta \text{Var}(y) \\ &= \frac{m^2}{3} - \frac{(m^2 - n^2)}{3} \cos^2 \theta. \end{aligned} \quad (4)$$

As long as  $m > n$ , expected MSE of the reconstructed data equation (4) is minimized when  $\cos^2 \theta = 1$ , which implies that the first principal component is the  $x$ -axis, or the main axis of the strip line.  $\square$

### 3. An algorithm for RPCL-based local PCA

Local PCA can be implemented in several ways. One direct way is to make clustering or estimate a gaussian mixture in the first step and then to perform PCA on each cluster or gaussian in the second step (Kambhatla and Leen, 1997). A better way is to make the two steps coordinately such that the first step and the second step are performed alternatively (Xu, 1994; Hinton et al., 1995; Tipping and Bishop, 1999). Moreover, instead of using the full covariance matrix, each covariance matrix is represented either in a constrained covariance matrix in term of the principal component and the noise variance (Hinton et al., 1995; Tipping and Bishop, 1999) or an equivalent way that directly considers each principal component and the noise variance in a reconstruction cost (Xu, 1994). They are more preferred especially in the case of small sample size, since the free parameters to be estimated are considerably reduced. An adaptive algorithm on how to estimate each principal component is given by Xu (1998) via combining RPCL learning (Xu et al., 1993) and Oja rule. Also, with the help of the RPCL learning a clustering approach was proposed for detecting curve (Xu et al., 1993), which performs exactly local PCA in the 2-dimensional case, but becomes local minor principal analysis (MCA) in high-dimensional case. Recently, an improved RPCL learning based local PCA approach was proposed by Xu (2001).

As preliminarily described by Xu et al. (1993), a salient advantage of RPCL learning is being able to automatically determine the number of clusters (lines) during learning its parameters, as compared with the maximum likelihood learning that does not have the model selection ability. Thus, we choose the RPCL learning based local PCA in (Xu, 2001) for the strip line detection task.

The key idea behind RPCL is that for each sample point, not only the winner cluster center is pulled toward the point, but also the rival (2nd winner) one is pushed slightly away from it. Mathematically, it consists of the following two steps (Xu et al., 1993; Xu, 1998):

*Step 1:* For the sample  $x_t$ , find the winner cluster center  $i_c$  and the rival one  $i_r$  as

$$\begin{aligned} i_c &= \arg \min_i \gamma_i d_i(x_t), \\ i_r &= \arg \min_{i \neq i_c} \gamma_i d_i(x_t), \end{aligned} \quad (5)$$

where  $\gamma_i = n_i / \sum_{j=1}^k n_j$  and  $n_i$  denotes the cumulative times of the cluster  $i$  being winner, and  $d_i(x_t)$  denotes distance between sample  $x_t$  and cluster  $i$ .

*Step 2:*

$$\theta_{i_c}^{\text{new}} = \theta_{i_c}^{\text{old}} - \eta_c \nabla_{\theta_{i_c}} d_{i_c}(x_t), \quad (6)$$

$$\theta_{i_r}^{\text{new}} = \theta_{i_r}^{\text{old}} + \eta_r \nabla_{\theta_{i_r}} d_{i_r}(x_t), \quad (7)$$

where  $\theta_i$  denotes the parameter under consideration, e.g., the cluster center,  $\nabla_{\theta_i} d_i(x_t)$  denotes the derivative of  $d_i(x_t)$  with respect to  $\theta_i$ , and the learning rate  $\eta_c \gg$  de-learning rate  $\eta_r$ .

When the pre-set cluster number is greater than the true one, learning via RPCL will not only possess model selection ability via pushing the redundant cluster centers far away from the data, but also avoid the so-called dead unit problem for clustering due to the introduced conscience and de-learning mechanism (Desieno, 1988; Xu et al., 1993).

Consider the gaussian mixture model

$$p(x|\theta) = \sum_{i=1}^k \alpha_i G(x|\mu_i, \Sigma_i), \quad (8)$$

where  $\alpha_i > 0$ ,  $\sum_{i=1}^k \alpha_i = 1$ ,  $G(x|\mu_i, \Sigma_i)$  denotes a gaussian density with mean vector  $\mu_i$  and covariance matrix  $\Sigma_i$ . In this case,  $d_i(x_t)$  in the above step 1 is specifically given by the following general distance matrix (Xu, 2001):

$$d_i(x_t) = -\ln[G(x_t|\mu_i, \Sigma_i)\alpha_i]. \quad (9)$$

For the task of strip line detection and thinning where only the first principal component is under consideration, we consider a special case of the elliptic RPCL algorithm (Xu, 2001) by focusing on a covariance matrix as follows:

$$\Sigma_i = \varsigma_i I + \sigma_i \phi_i \phi_i^T, \quad (10)$$

where  $\phi_i^T \phi_i = 1$ ,  $\varsigma_i > 0$ ,  $\sigma_i > 0$ , and the first principal component is  $\phi_i$ .

By such a setting, the parameters to be determined for the local PCA model become  $\Theta = \{\alpha_i, \mu_i, \phi_i, \sigma_i, \varsigma_i\}_{i=1}^k$ . Specifically, the gradients of  $d_i(x_t)$  with respect to  $\Theta$  in the above step 2 have the following detailed forms:

$$\nabla_{\alpha_i} = -1/\alpha_i, \quad \nabla_{\mu_i} = -e_{i,t}, \quad (11)$$

$$\nabla_{\varsigma_i} = 0.5 \text{Tr}(\Sigma_i)^{-1} (I - S_i(\Sigma_i)^{-1}), \quad (12)$$

$$\nabla_{\sigma_i} = 0.5 \phi_i^T (\Sigma_i)^{-1} (I - S_i(\Sigma_i)^{-1}) \phi_i, \quad (13)$$

$$\nabla_{\phi_i} = \sigma_i (\Sigma_i)^{-1} (I - S_i(\Sigma_i)^{-1}) \phi_i, \quad (14)$$

where  $S_i = e_{i,t} e_{i,t}^T$  with  $e_{i,t} = x_t - \mu_i$ ,  $\varsigma_i = e^{q_i}$  and  $\sigma_i = e^{o_i}$  can be introduced to constrain  $\varsigma_i > 0$  and  $\sigma_i > 0$  respectively, and the constraints  $\sum_{i=1}^k \alpha_i = 1$  and  $\phi_i^T \phi_i = 1$  can be satisfied via no more than one step of normalization.

#### 4. Simulations

We present four experiments to illustrate the RPCL-based local PCA for strip line detection and thinning. The first experiment aims to illustrate the model selection ability of the RPCL-based local PCA, the second one to demonstrate the strip line thinning, the third one to demonstrate the strip line detection, and last one to illustrate how the

RPCL-based local PCA can be used to aid the container recognition.

4.1. On model selection

This experiment aims to illustrate the importance of model selection for the strip line detection and thinning. We base our comparison on two algorithms. They are respectively the maximum likelihood learning based local PCA algorithm (Tipping and Bishop, 1999) and the RPCL-based local PCA. The original image is extracted from an image of striped shirt (McCafferty, 1990, Fig. 9.25(a)). For both local PCA algorithms, we initialize the same set of 8 cluster centers.

Figs. 4 and 5 respectively illustrate the line detection results of the maximum likelihood learning based local PCA and RPCL-based local PCA. Without model selection, the maximum likelihood learning based local PCA cannot drive the redundant cluster centers away. Consequently, not only the 6 lines in the original image are incorrectly detected as 8, but also the position of the 2 lines are misplaced. On the other hand, pushing force on the rival resulted from the de-learning of the RPCL learning can be traced from the paths of the redundant cluster centers in Fig. 5. As a result, the RPCL-based local PCA succeeds in line detection via an efficient model selection ability.

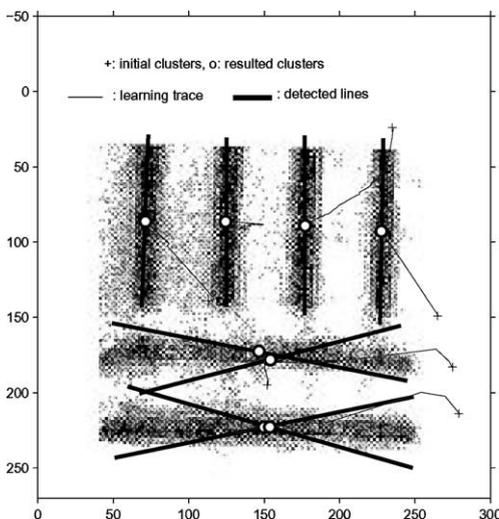


Fig. 4. ML-based local PCA with wrong number of clusters.

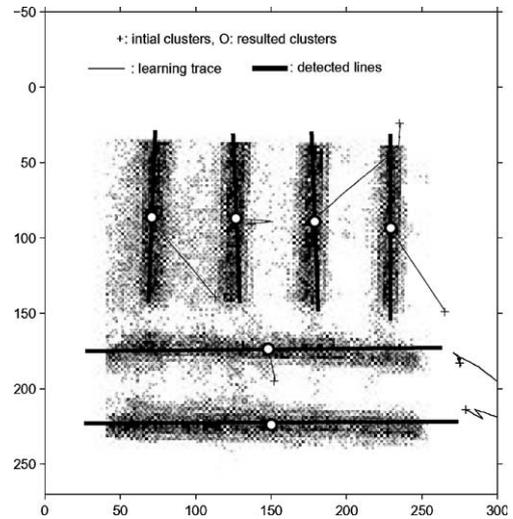


Fig. 5. Result by RPCL-based local PCA with model selection.

4.2. On strip lines detection

This simulation aims at comparing the effectiveness of RPCL-based local PCA and Hough transform for strip line detection. The effectiveness performance of the RHT is similar on such a task. Fig. 6 shows the original image taken in the campus. The image threshold is set according to the grey-level histogram approach (Otsu, 1979). Here we use Hough transform in two ways for comparison, i.e., first directly on the image and then with preprocessing by edge detection. We initialize 10 random cluster centers for the RPCL-based local PCA algorithm at the beginning of the simulation.

Results by Hough transform without edge detection are shown in Figs. 8 and 11. The 10 “peaks” in Fig. 11 correspond to the 10 lines in Fig. 8, which is inconsistent with the original seven strip lines. Results of Hough transform after edge

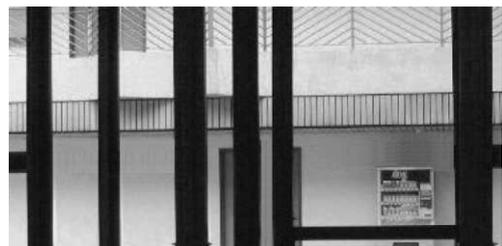


Fig. 6. The original real image.

detection are shown in Figs. 9, 10 and 12 respectively. The 14 “peaks” in Fig. 12 correspond to the 14 lines in Fig. 10, from which it is also hard to obtain the seven original strip lines. The results can be compared with the successful one by RPCL-based local PCA shown in Fig. 7.

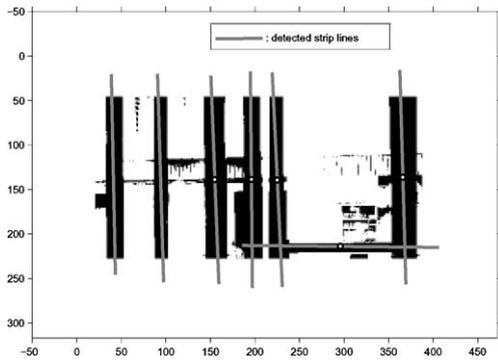


Fig. 7. Result by RPCL-based local PCA.

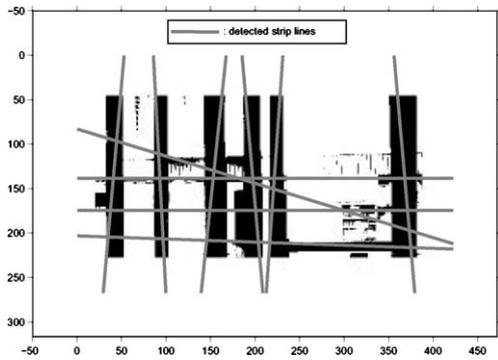


Fig. 8. Result by Hough transform.

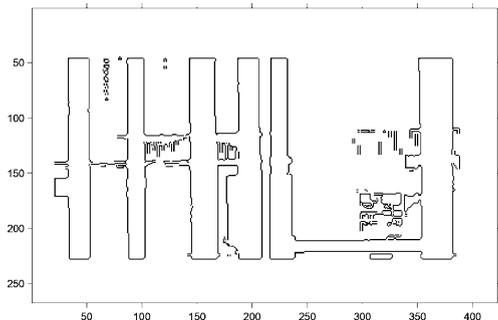


Fig. 9. Resulted image via edge detection.



Fig. 10. Result by Hough transform.

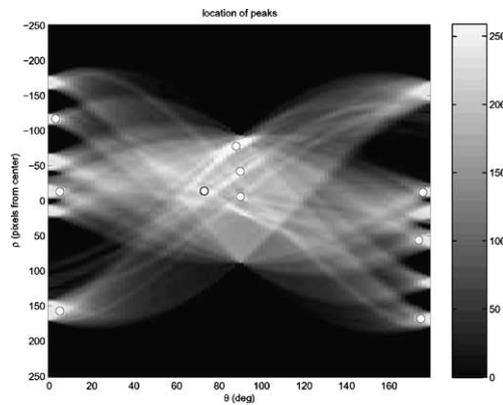


Fig. 11. “Peaks” of Hough transform without edge detection.

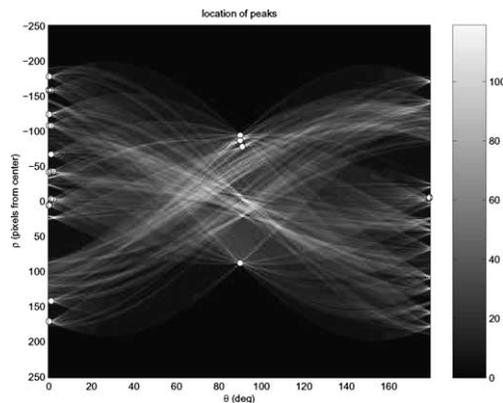


Fig. 12. “Peaks” of Hough transform after edge detection.

### 4.3. On strip lines thinning

This experiment compares the performance of RPCL-based local PCA and conventional thinning

algorithm (Davies and Plummer, 1981) on strip lines thinning when image is affected by uniform background noise, as the *table* image shown in Fig. 13, or is blurred due to scanning, as the Chinese characters shown in Fig. 17.

Result of thinning for the *table* by the conventional approach is shown in Fig. 14 and that by the RPCL-based local PCA in Figs. 15 and 16. On the other hand, result of thinning for the Chinese characters by the conventional approach is shown in Figs. 17 and 18 and that by the RPCL-based local PCA in Figs. 19 and 20.

From Fig. 16 we can see that the RPCL-based local PCA not only can outline the main skeleton of the original *table* image, but also remove the background noise. On the contrary, as shown in

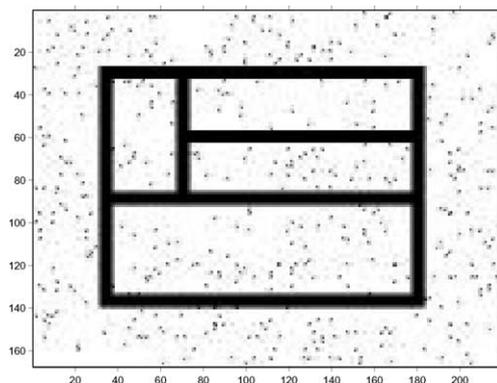


Fig. 13. Original table image with background noise.

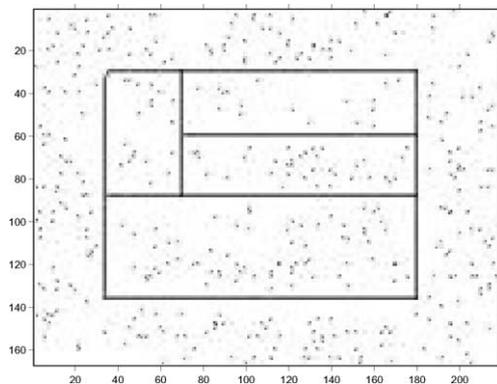


Fig. 14. Result by thinning algorithm.

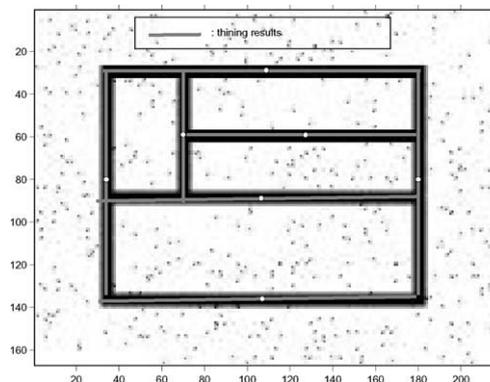


Fig. 15. Result by RPCL-based local PCA.

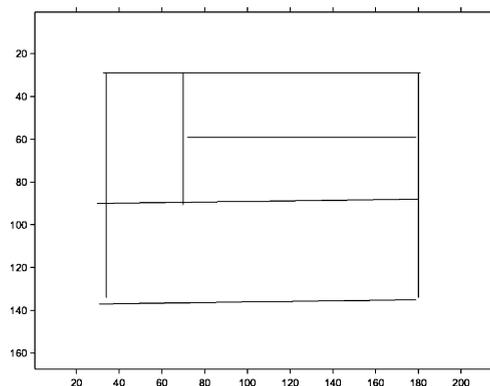


Fig. 16. Skeleton by RPCL-based local PCA.

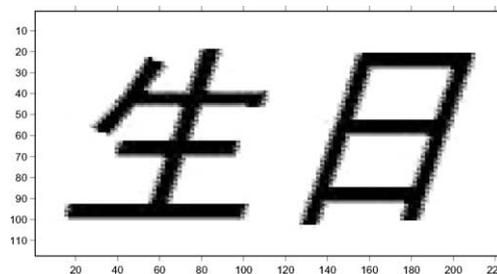


Fig. 17. Original blurred Chinese character image.

Fig. 14, the conventional pixel-by-pixel thinning approach cannot remove the background. Also, when the skeleton of Chinese characters is blurred as shown in Fig. 17, the conventional thinning algorithm cannot obtain the main axis via *magnifying*

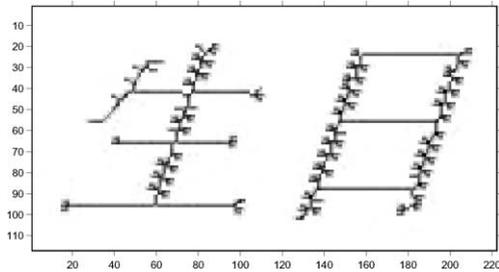


Fig. 18. Result by thinning algorithm.

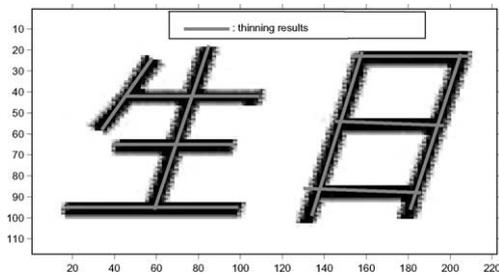


Fig. 19. Result by RPCL-based local PCA.

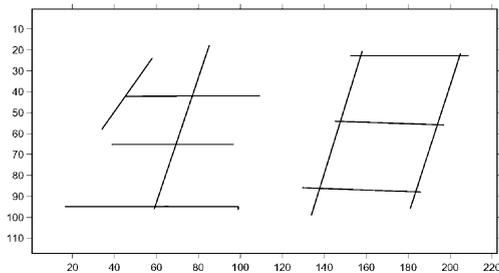


Fig. 20. Skeleton by RPCL-based local PCA.

its distorted outliers. This can be contrasted with the success by RPCL-based local PCA by capturing its main structure as shown in Fig. 20.

#### 4.4. On container recognition

Automatic container recognition system is very useful for customs or logistic management. Here we illustrate how the RPCL-based local PCA can be employed in such a system.

Container recognition is usually based on the captured container number located at the back of

the container. For example, the container in Fig. 21 can be recognized by the number “OCLU 1522770”. The whole process can be roughly broken down into two subtasks. The first one involves locating and extracting a rectangular area of the raw image that contains the number, and the second one concerns actually recognizing the number via some image processing and pattern recognition techniques.

Using the container in Fig. 21 as an example, we first preprocess the image by sharpening, erosion, and threshold, with the resulted image shown in Fig. 22. For the first subtask, we adopt the RPCL-based local PCA to roughly locate the



Fig. 21. The original container image.

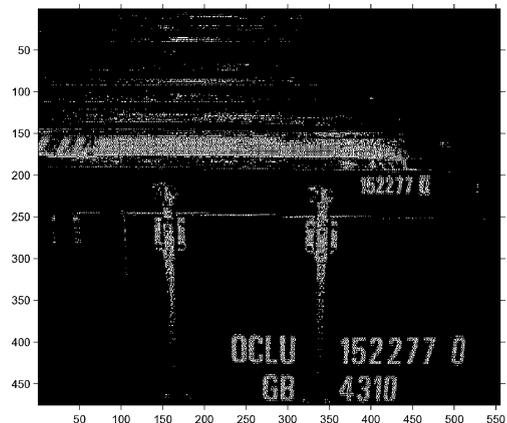


Fig. 22. Pre-processed image.

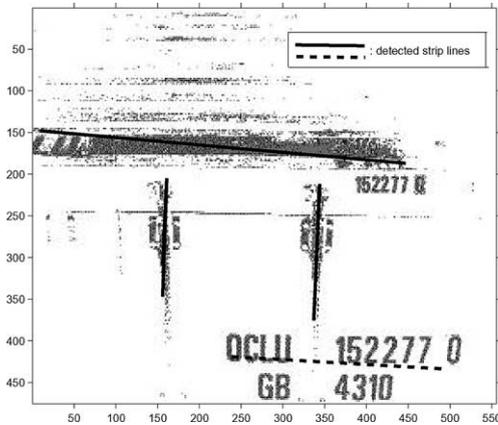


Fig. 23. Four detected strip lines.

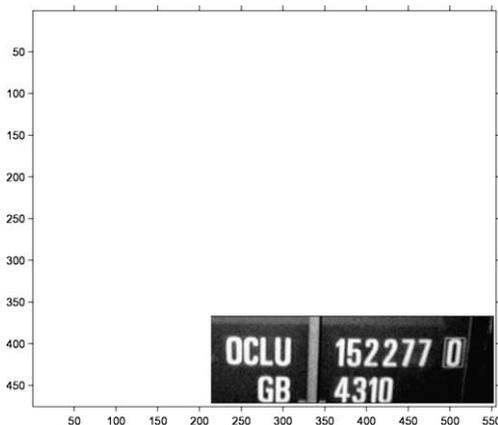


Fig. 24. Detected interesting area.

container number via detecting several (four in this example) standard strip lines as shown in Fig. 23. Based on the lowest detected strip line (the dashed one in Fig. 23), area with roughly a rectangular shape as shown in Fig. 24 can be extracted from the raw image. Then, the number can be subsequently recognized using some other popular image processing and pattern recognition techniques.

## 5. Conclusions and future work

In this paper, we introduce how the RPCL-based local PCA model can be novelly applied to

two traditional image processing tasks, i.e., strip lines detection and thinning, respectively. In particular, its model selection property is beneficial for automatically determining the line number and its noise resistance property is helpful for thinning. Because the focus of this paper is on the study of linear structure, future research effort may be directed to its nonlinear extension.

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