

On Generalized Arbitrage Pricing Theory Analysis: Empirical Investigation of the Macroeconomics Modulated Independent State-Space Model*

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Abstract: Inception of Markowitz's modern portfolio theory has also fuelled the development of asset pricing models for empirical finance, ranging from linear single-factor models like the capital asset pricing model to fairly complex multi-factor models such as the arbitrage pricing theory (APT). It is well-known in the literature of finance that APT could be used for modelling the underlying security returns generation process. In this paper, we investigate a generalized version of the APT model, called the macroeconomics modulated independent state-space model, in terms of model specification adequacy as well as its performance on prediction. Empirical results reveal that the model is not only well-specified, but also superior to the temporal factor analysis model in stock price and index forecasting, thanks to its salient capabilities of modelling both short-term and long-term market dynamics.

Keywords: Arbitrage pricing theory, macroeconomic factors, white noise test, temporal factor analysis

1 INTRODUCTION

An important milestone of modern financial modelling was laid down by Markowitz's 1952 landmark paper [7] in modern portfolio theory. In the subsequent years, development in formal models of financial asset prices was witnessed by Sharpe's capital asset pricing model (CAPM) [13] in 1964, Merton's intertemporal CAPM [8] in 1973, Ross's arbitrage pricing theory (APT) [12] in 1976, and those of Lucas and Breeden. All such models have one thing in common. They are all based on a single notion of general equilibrium in which demand equals supply across all markets in an uncertain world where individuals and corporations act rationally to optimize their own welfare. It comes out of the result of interaction between prices, preferences and probabilities.

As uncertainties induce risk, one of the central questions of modern finance is the necessity of some tradeoff between risk and expected return. Defining the appropriate measures of risk and reward, determining how they might be linked through fundamental principles of economics and psychology, and then estimating such links empirically using historical data and performing proper statistical inference are

issues that rest on the heart of financial modelling for asset pricing.

Financial asset pricing models are usually divided over two fundamental issues. They are respectively what constitutes and how risk affects security returns. For instance, the CAPM model was structured on the belief that relevant risk measure is related to just one aspect of the macroeconomy-market fluctuation. Although inception of the model was largely attributable to the development of Markowitz's portfolio theory, a major breakthrough of the model was due to the identification and splitting between systematic and diversifiable idiosyncratic risk. Unfortunately, the model was not without its drawbacks. The most critical one was related to the assumptions underlying its derivation. The model was inextricably linked with quite a number of assumptions, and some of them, predominantly homogeneous expectation of investors about the market, were documented in the literature to be empirically unrealistic. To rectify this weaknesses as well as to further extend the capabilities of the CAPM model, the APT was proposed in [12], which assumed that the cross-sectional expected returns of securities follow a multi-factor model characterized by their sensitivities, usually called factor loadings. Both the CAPM and the APT models could be broadly regarded as typical representatives of the class of the more general, linear models. In fact, apart from modelling by linear regression, attempts have been made on nonlinear regression. The aim was to explore any substantially nonlinear hidden relationship. However, there are several disadvantage of nonlinear modelling as compared to the linear one. First, nonlinear modelling involves complicated procedures and it is not rare that the discovered optimum relation turns out to be counter-intuitive. Second, due to the diversity of all possible nonlinear relations, parametric modelling techniques become inappropriate and frequently it is difficult to find a suitable interpretation of the final results.

Previously, we have made an effort to solve the rotation indeterminacy problem encompassing the traditional APT by considering its temporal extension, called the temporal factor analysis (TFA) model. In this paper, we further investigate the macroeconomics modulated independent state-space model, which not only is an extension of the linear TFA model proposed in [16], but also has a salient feature of modelling both short-term and long-term dynamics in the financial market. As implied by its name, the model attempts to relate macroeconomic variables to the stock re-

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turns generation process. However, quite different from [1, 2, 3, 4, 6, 9, 14] which explicitly modelled the relation between asset prices and real economic activities such as production rates, productivity, growth rate of gross national product, unemployment, yield spread, interest rates, inflation, dividend yields and so forth, it models such relation in a subtle way via the independent hidden factors that affect the stock market.

The rest of the paper is organized in the following way. Sections 2 briefly reviews the APT and the Gaussian TFA model respectively. Section 3 introduces, with considerable detail, the macroeconomics modulated independent state-space model. Section 4 investigates empirically its model specification adequacy. Section 5 compares its performance on stock indices prediction with the TFA model which is its degenerated variant. Section 6 concludes the paper.

2 Review of Related Models

This section briefly reviews two close ancestors of the macroeconomics modulated independent state-space model.

2.1 The Arbitrage Pricing Theory

The APT begins with the assumption that the $n \times 1$ vector of asset returns, R_t , is generated by a linear stochastic process with k factors [12, 10, 11]:

$$R_t = \bar{R} + \mathbf{A}f_t + e_t \quad (1)$$

where f_t is the $k \times 1$ vector of realizations of k common factors, A is the $n \times k$ matrix of factor weights or loadings, and e_t is a $n \times 1$ vector of asset-specific risks. It is assumed that f_t and e_t have zero expected values so that \bar{R} is the $n \times 1$ vector of mean returns. The model addresses how expected returns behave in a market with no arbitrage opportunities and predicts that an asset's expected return is linearly related to the factor loadings or

$$\bar{R} = R_f + \mathbf{A}p \quad (2)$$

where R_f is a $n \times 1$ vector of constants representing the risk-free return, and p is $k \times 1$ vector of risk premiums. Similar to the derivation of CAPM, (2) is based on the rationale that unsystematic risk is diversifiable and therefore should have a zero price in the market with no arbitrage opportunities.

2.2 Gaussian Temporal Factor Analysis

Suppose the relationship between a state $y_t \in \mathbb{R}^k$ and an observation $x_t \in \mathbb{R}^d$ is described by the first-order state-space equations as follows [15, 16]:

$$y_t = \mathbf{B}y_{t-1} + \varepsilon_t, \quad (3)$$

$$x_t = \mathbf{A}y_t + e_t, \quad t = 1, 2, \dots, N. \quad (4)$$

where ε_t and e_t are mutually independent zero-mean white noises with $E(\varepsilon_i \varepsilon_j) = \Sigma_\varepsilon \delta_{ij}$, $E(e_i e_j) = \Sigma_e \delta_{ij}$, $E(\varepsilon_i e_j) =$

0 , Σ_ε and Σ_e are diagonal matrices, and δ_{ij} is the Kronecker delta function:

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

We call ε_t driving noise upon the fact that it drives the source process over time. Similarly, e_t is called measurement noise because it happens to be there during measurement. The above model is generally referred to as the TFA model.

In the context of APT analysis, equation (1) can be obtained from equation (4) by substituting $(\tilde{R}_t - \bar{R})$ for x_t and f_t for y_t . The only difference between the APT model and the TFA model is the added equation (3) for modelling temporal relation of each factor. The added equation represents the factor series $y = \{y_t\}_{t=1}^T$ in a multi-channel autoregressive process, driven by an i.i.d. noise series $\{\varepsilon_t\}_{t=1}^T$ that are independent of both y_{t-1} and e_t . Specifically, it is assumed that ε_t is Gaussian distributed. Moreover, TFA is defined such that the k sources $y_t^{(1)}, y_t^{(2)}, \dots, y_t^{(k)}$ in this state-space model are statistically independent.

3 The Macroeconomics Modulated Independent State-Space Model

3.1 Overview of the Model

The macroeconomics modulated independent state-space model was proposed by Xu with an aim to model the financial market in a state of general equilibrium. Unlike traditional APT, it further utilizes observed macroeconomic variables and indices to implement the concept of long-term equilibrium in economics. In general, the model takes the following form [16]:

$$y_t = \mathbf{B}y_{t-1} + \mathbf{H}z_{t-1} + \varepsilon_t \quad (6)$$

$$x_t = \mathbf{A}y_t + e_t \quad (7)$$

$$z_t = \mathbf{C}y_t + \mathbf{E}v_t + \epsilon_t \quad (8)$$

where ε_t, e_t and ϵ_t are Gaussian white noises and independent from each other, ε_t is independent of both z_{t-1} and y_{t-1} , e_t and ϵ_t are independent of y_t and v_t .

Typically, z_t consists of a number of macroeconomic indices and v_t consists of a number of known non-market factors that affect the macroeconomy. Specifically, $\mathbf{H}z_{t-1}$ describes the indirect effect of the macroeconomic indices to the security market via the hidden factors y_t , and $\mathbf{C}y_t$ describes the feedback effect of the market to the macroeconomic indices.

We consider the model describes a capital market via both short-term and long-term dynamics. For short-term dynamics, x_t, y_t and perhaps z_t move to reach an equilibrium in the sense that the series of ε_t, e_t and ϵ_t become stationary white noises, while the parameters $\mathbf{B}, \mathbf{H}, \mathbf{A}, \mathbf{C}, \mathbf{E}$ and the statistics of ε_t, e_t and ϵ_t can be regarded as relatively constant due to slow changing. For long-term dynamics, the parameters $\mathbf{B}, \mathbf{H}, \mathbf{A}, \mathbf{C}, \mathbf{E}$ and the statistics of ε_t, e_t and ϵ_t are all changing to cohere to equilibrium.

3.2 An Algorithm for Implementation

An adaptive algorithm is given in [16] for implementation of the macroeconomics modulated independent state-space model. With \mathbf{H} fixed, $\mathbf{H}z_t$ acts as a constant and can be regarded as a part of the mean of ε_t . The task of estimating \mathbf{H} is a linear regression problem when y_t, y_{t-1} and \mathbf{B} are fixed. In particular, for $G(\varepsilon_t|\mathbf{H}z_{t-1}, \Lambda), G(e_t|0, \Sigma_x)$ and $G(\varepsilon_t|0, \Sigma_z)$, the algorithm consists of four steps shown below.

Step 1 Estimate y_t via \hat{y}_t by maximum likelihood

$$\hat{y}_t = [\mathbf{A}^{-1} + \mathbf{A}^T \Sigma_x^{-1} \mathbf{A} + \mathbf{C}^T \Sigma_z^{-1} \mathbf{C}]^{-1} \cdot [\mathbf{A}^T \Sigma_x^{-1} \bar{x}_t + \mathbf{C}^T \Sigma_z^{-1} (\bar{z}_t - \mathbf{E}v_t) + \mathbf{A}^{-1} (\mathbf{B}y_{t-1} + \mathbf{H}\bar{z}_{t-1})]$$

Step 2 Update parameters of (6)

$$\begin{aligned} \mathbf{B}^{\text{new}} &= \mathbf{B}^{\text{old}} + \eta \text{diag}[\varepsilon_t \hat{y}_t^T], \\ \mathbf{H}^{\text{new}} &= \mathbf{H}^{\text{old}} + \eta [\varepsilon_t \hat{z}_{t-1}^T], \\ \Lambda^{\text{new}} &= (1 - \eta) \Lambda^{\text{old}} + \eta \text{diag}[\varepsilon_t \varepsilon_t^T], \end{aligned}$$

Step 3 Update parameters of (7)

$$\begin{aligned} \mathbf{A}^{\text{new}} &= \mathbf{A}^{\text{old}} + \eta [e_t \hat{y}_t^T], \\ \Sigma_x^{\text{new}} &= (1 - \eta) \Sigma_x^{\text{old}} + \eta \text{diag}[e_t e_t^T], \end{aligned}$$

Step 4 Update parameters of (8)

$$\begin{aligned} \mathbf{C}^{\text{new}} &= \mathbf{C}^{\text{old}} + \eta [\varepsilon_t \hat{y}_t^T], \\ \mathbf{E}^{\text{new}} &= \mathbf{E}^{\text{old}} + \eta [\varepsilon_t \hat{v}_t^T], \\ \Sigma_z^{\text{new}} &= (1 - \eta) \Sigma_z^{\text{old}} + \eta \text{diag}[\varepsilon_t \varepsilon_t^T], \end{aligned}$$

In this paper, due to the absence of known non-market factors affecting the macroeconomy, the term $\mathbf{E}v_t$ is deleted by simply setting $\mathbf{E} = 0$ in (8) and thus the relevant learning rules are omitted.

4 White-Noise Test on Model Specification Adequacy

It is common in the literature of statistics that test for model adequacy should immediately follow parameter estimation of the model under consideration. Usually a model is considered adequate only if the residual component consists of white noise. Since ε_t in (6) is likely to be temporally correlated, we require the estimated residual to be substantially serially uncorrelated, i.e., autocorrelation of its lags should not be significantly different from zero. On the other hand, for both e_t of (7) and ε_t of (8) to be adequate, the estimated residuals should be largely uncorrelated among its components.

4.1 Data Consideration

We carry out the analysis using past stock price and return data of Hong Kong. Daily closing prices of 30 actively trading stocks covering the period from January 1, 1998 to December 31, 1999 are used. The number of trading days throughout this period is 522. These stocks belong to the Hang Seng Index (HSI) constituents. HSI is the most representative index in the Hong Kong stock market. For macroeconomic indices, we use the 1 month Hong Kong Inter-Bank

Middle Rate, the Hang Seng Index and the Dow Jones Industrial Average (DJIA) respectively. The DJIA is used as a proxy in view of its significant co-integration effect [5] with the HSI.

4.2 Data Preprocessing

Before carrying out the analysis, the stock prices should be converted to stationary stock returns. The transformation applied can be described in four steps as shown below.

Step 1 Transform the raw prices to returns by

$$R_t = \frac{p_t - p_{t-1}}{p_{t-1}}.$$

Step 2 Calculate the mean return \bar{R} by $\frac{1}{N} \sum_{t=1}^N R_t$.

Step 3 Subtract \bar{R} from R_t to get the zero-mean return.

Step 4 Let the result of above transformation be the adjusted return \tilde{R}_t .

4.3 Test Statistics

To check if the residual ε_t behave as a white-noise process, we adopt the Ljung-Box modified Q -statistic shown below. The Q -statistic can be used to test whether a group of autocorrelations is significantly different from zero.

$$Q = N(N+2) \sum_{k=1}^s \frac{r_k^2(\hat{\varepsilon})}{N-k} \quad (9)$$

where N is the effective number of observations and s is the lag order. If the sample value of Q calculated above exceeds the critical value of χ^2 with $s-1$ degrees of freedom at $\alpha = 5\%$, then we can conclude that at least one value of r_k is statistically different from zero at 5% level of significance and suspect the residuals are serially correlated and not white.

On the other hand, to investigate whether each cross correlation coefficient of the observation noise residuals e_t and ε_t is not significantly different from zero, we apply the t -test with the test statistic given by

$$t = \frac{r}{\sqrt{1-r^2}} \cdot \sqrt{n-2}, \quad (10)$$

where r is the correlation coefficient of a sample of n points (x_i, y_i) as given by

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{[\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2]^{\frac{1}{2}}}. \quad (11)$$

Assume that the x and y values originate from a bivariate Gaussian distribution, and that the relationship is linear, it can be shown that t follows Student's t -distribution with $n-2$ degrees of freedom. Again the predefined level of significance is set at $\alpha = 5\%$.

4.4 Empirical Test Results

Results of Q statistics and p -values for the residual ε_t at lags from order 1 to 15 are shown in Table 1. At 5% level of

significance, autocorrelations of all residuals are not significantly different from zero. It implies that the specification of (6) is adequate. As for e_t , partial results showing t -test on correlation coefficients of the first two stocks with respect to the 30 HSI constituents are shown in Table 2. Results of the other 28 stocks are omitted due to space constraint. Out of 435 cross correlation coefficients, only 11 of them, or 2.53% are statistically significant at $\alpha = 5\%$. As the percentage is quite small, the results are satisfactory and we accept the null hypothesis that the residuals e_t are substantially white. Similar results for ϵ_t are shown in Table 3.

Table 1: Results showing Q -statistic and p -value of residual ϵ_t for 30 HSI constituents.

Lag	Q -Stat	p -value	Q -Stat	p -value
	Residual 1		Residual 2	
1	0.0056	0.9403	0.0077	0.9299
2	0.4153	0.8125	0.0081	0.9960
3	1.8558	0.6029	0.6221	0.8914
4	2.6812	0.6125	0.7548	0.9444
5	4.3332	0.5025	1.1613	0.9485
6	6.0460	0.4181	3.7385	0.7120
7	7.8827	0.3431	9.7960	0.2005
8	7.8827	0.4450	9.8682	0.2744
9	8.0167	0.5325	9.9039	0.3584
10	8.7007	0.5607	11.6716	0.3077
11	9.1603	0.6071	11.7318	0.3842
12	9.2222	0.6838	14.9980	0.2416
13	11.6317	0.5581	17.0208	0.1984
14	13.3091	0.5024	17.2598	0.2427
15	13.3499	0.5753	19.1754	0.2060
	Residual 3		Residual 4	
1	0.0318	0.8585	0.1188	0.7303
2	0.2229	0.8945	3.2264	0.1993
3	0.2250	0.9735	7.1968	0.0659
4	0.4433	0.9788	7.2105	0.1252
5	0.5247	0.9912	7.2607	0.2020
6	3.0091	0.8077	8.9177	0.1783
7	4.3104	0.7434	9.4983	0.2189
8	4.3568	0.8236	10.4737	0.2334
9	5.0294	0.8317	11.5217	0.2417
10	5.2900	0.8710	16.7004	0.0813
11	5.3412	0.9135	16.7067	0.1169
12	6.1249	0.9096	17.0780	0.1468
13	7.6111	0.8680	19.6578	0.1041
14	7.6424	0.9071	22.9557	0.0611
15	7.8130	0.9310	23.2691	0.0787

5 Simulations of Stock Price and Index Prediction

One possible application of the macroeconomics modulated independent state-space model is in stock price and index forecasting. Specifically, after setting up the model (6)-(8), we can use x_t , z_t and z_{t-1} to get \hat{y}_t . The predicted \hat{x}_{t+1} can be obtained by $\mathbf{A}\hat{y}_{t+1}$ where \hat{y}_{t+1} is approximately given by $\mathbf{B}\hat{y}_t + \mathbf{H}z_t$.

To explore the relative merit of modelling both short-term as well as long-term dynamics, we would compare the performance of the macroeconomics modulated independent state-space model with that of the TFA model. It is worth noting that the TFA model is a degenerated variant of the

Table 2: Partial results of t -test on the residual e_t for 30 HSI constituents. Only correlation coefficients the first two stocks with respect to 30 constituents are shown. Results of the other 28 stocks are omitted due to space constraint.

Stock #	ρ	t -stat.	p -value
	Stock #1		
1	1.0000	-	-
2	-0.0704	1.6055	0.1090
3	0.0710	1.6188	0.1061
4	-0.0120	0.2728	0.7851
5	-0.0111	0.2533	0.8001
6	0.0587	1.3340	0.1828
7	-0.0143	0.3249	0.7454
8	0.0417	0.9509	0.3421
9	-0.0074	0.1667	0.8677
10	0.0255	0.5795	0.5625
11	0.0491	1.1161	0.2649
12	0.0076	0.1730	0.8627
13	-0.0270	0.6123	0.5406
14	0.0023	0.0512	0.9592
15	-0.0316	0.7180	0.4731
16	-0.0318	0.7228	0.4701
17	-0.0244	0.5546	0.5794
18	-0.0836	1.9053	0.0573
19	-0.0687	1.5667	0.1178
20	0.0090	0.2047	0.8379
21	0.0641	1.4618	0.1444
22	-0.0472	1.0754	0.2827
23	-0.0480	1.0932	0.2748
24	0.1017	2.3317	0.0201
25	0.1057	2.4055	0.0165
26	0.0205	0.4666	0.6410
27	-0.0451	1.0257	0.3055
28	0.0817	1.8625	0.0631
29	0.0457	1.0407	0.2985
30	0.0299	0.6799	0.4969

macroeconomics modulated independent state-space model and could be obtained by setting $H = 0$ in (6), thus ignoring the feedback of z_{t-1} on y_t .

We demonstrate the results using the HSI and a representative stock, the Hong Kong and Shanghai Banking Corporation (HSBC) Holding. The HSBC holding is a constituent stock of the HSI and was ranked the first in market capitalization during the period. We use the first 400 data points for training and the remaining 120 data points for test. Both training and test are carried out in an adaptive fashion.

To illustrate stock index prediction, results by the TFA model and the macroeconomics modulated independent state-space model are shown in Figure 1 and 2 respectively.

Similarly, results for stock price prediction by the TFA model and the macroeconomics modulated independent state-space model are shown in Figure 3 and 4 respectively.

The performance of each model can be compared quantitatively by their respective root mean square errors (RMSE) between the predicted prices \hat{p}_t and the desired prices p_t . As shown in Table 4, the macroeconomics modulated independent state-space approach consistently outperforms the TFA approach by having smaller RMSE for the HSI index and HSBC Holding. The results may be attributable to merit of modelling both short-term and long-term dynamics by the

Table 2: *Continued.*

Stock #	ρ	t -stat.	p -value
		Stock #2	
1	-0.0704	1.6055	0.1090
2	1.0000	-	-
3	0.0219	0.4988	0.6181
4	0.0981	2.2356	0.0258
5	0.0163	0.3699	0.7116
6	-0.0398	0.9031	0.3669
7	0.0079	0.1781	0.8587
8	0.0338	0.7689	0.4423
9	-0.0178	0.4049	0.6857
10	0.0600	1.3648	0.1729
11	-0.0533	1.2140	0.2253
12	0.0152	0.3460	0.7295
13	0.0381	0.8662	0.3868
14	-0.0319	0.7277	0.4671
15	-0.0229	0.5216	0.6022
16	-0.0134	0.3052	0.7603
17	0.0324	0.7391	0.4602
18	-0.0453	1.0321	0.3025
19	0.0212	0.4833	0.6291
20	0.0217	0.4946	0.6211
21	0.0054	0.1226	0.9025
22	-0.0602	1.3709	0.1710
23	0.0143	0.3249	0.7454
24	-0.0111	0.2521	0.8011
25	-0.0053	0.1200	0.9045
26	0.0180	0.4076	0.6837
27	0.1005	2.2973	0.0220
28	0.0830	1.8976	0.0583
29	-0.1061	2.4262	0.0156
30	-0.0493	1.1231	0.2619

Table 3: Results of t -test on the residual ϵ_t for 30 HSI constituents.

Stock #	corr.	t -stat.	p -value
ρ_{12}	-0.0205	0.4652	0.6420
ρ_{13}	-0.0052	0.1188	0.9055
ρ_{23}	0.0498	1.1326	0.2579

macroeconomics modulated independent state-space model.

Table 4: Root mean square error (RMSE) for the two approaches

Model Type	HSI	HSBC
the TFA model	267.9210	1.5168
the macroeconomics modulated independent state-space model	218.8671	1.4745

6 Conclusion

In this paper, we empirically explore the macroeconomics modulated independent state-space model in two main aspects. First, we carry out white noise test to ensure model specification adequacy. Second, its performance in stock price and index forecasting is compared with that of the TFA model to illustrate the relative merit of simultaneously modelling both the short-term and long-term dynamics.

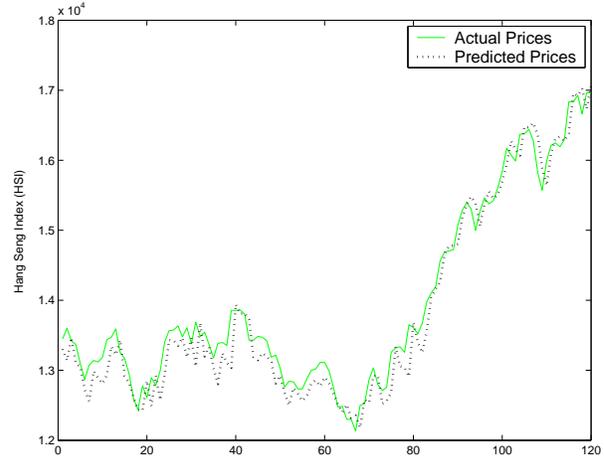


Figure 1: Predicted prices of HSI by the TFA model

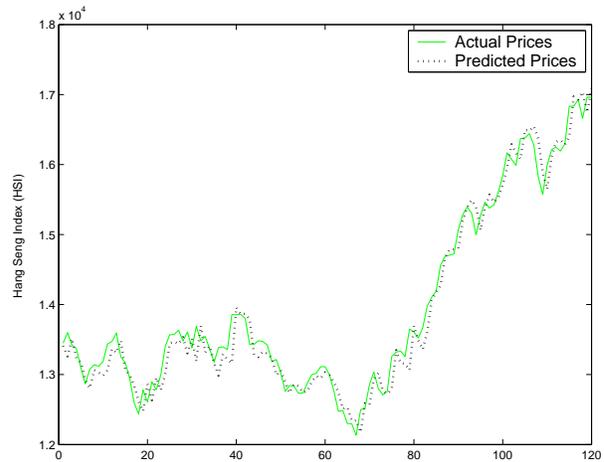


Figure 2: Predicted prices of HSI by the macroeconomics modulated independent state-space model

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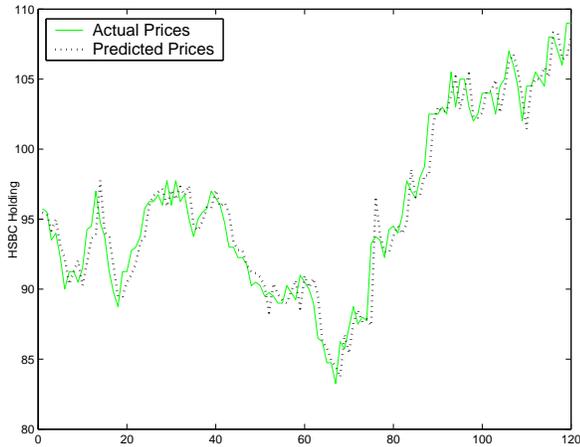


Figure 3: Predicted prices of HSBC Holding by the TFA model

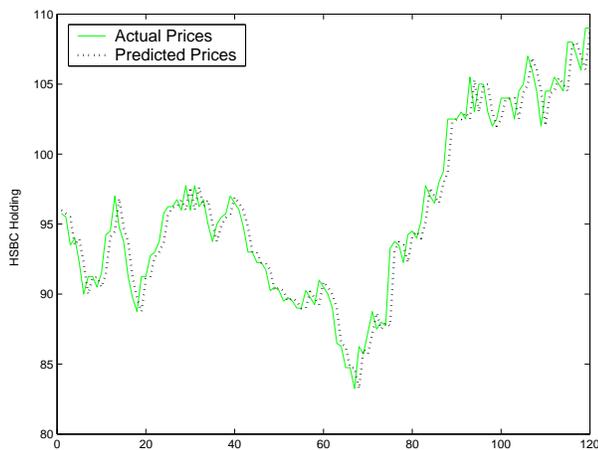


Figure 4: Predicted prices of HSBC Holding by the macroeconomics modulated independent state-space model

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