2. Consider any P_3 on vertices u, v and w ($vu, uw \in E$), either put v or u, w into VC. We construct a Bounded Search Tree by branching out at each node with $(G-v, k-1)$ and $(G-\{u,w\}, k-2)$ for each P_3 until $k = 0$ or no P_3 left.

The remaining graph at each leaf node of the search tree is a disjoint union of isolated vertices and edges. We put one endpoint of each edge into VC to form a solution, answer "No" if $k < E$ for all leaf nodes.

$$f(k) \leq f(k-1) + f(k-2) + O(1) \rightarrow f(k) \leq 1.618^k$$

3. By the following lemma, if there is an induced P_3 ($vu, uw \in E$ and $vw \notin E$), we must perform one of delete uv, delete uw or add vw.

Lemma *A graph G is a disjoint union of complete graphs iff it has no induced subgraph P_3 ($vu, uw \in E$ and $vw \notin E$).*

Construct a Bounded Search Tree by branching out at each node if there is an induced P_3 with $(G - uv, k-1)$,$(G - uw, k-1)$ and $(G + vw, k-1)$, until $k = 0$ or no induced P_3 left. Answer "Yes" if some leaf node has no induced P_3, otherwise answer "No".

$$f(k) \leq 3f(k-1) + O(1) \rightarrow f(k) \leq 3^k$$

4. For each edge $e = uv \in E'$, a minimum vertex cover may contain u, v or both. Consider each the 3^k possible subsets S of E', we put S into vertex cover and see that G has a k-VC iff the bipartite graph $G - S$ has a $(k - |S|)$-VC.

Since maximum matching equals minimum vertex cover in bipartite graphs, we can use Hopcroft–Karp algorithm and total time is $O(3^k\sqrt{n}m)$.

5. If a graph G has more than $2k$ edges, it always has a cut with k edges, so answer "Yes"; otherwise, G has at most $2k$ edges and thus at most $4k$ vertices which is a $O(k)$ kernel.

Proof: Suppose G has more than $2k$ edges, let $[S_1, S_2]$ be a largeset cut. For any vertex $v \in S_1$, if v has more neighbours in S_1 than in S_2, then put v into S_2 and we get a larger cut. Similar argument holds for $v \in S_2$. Therefore, any vertex has more edges incident to it in the cut than not and the cut has at least $m/2$ edges.

6. Reduce to Vertex Cover Problem: Construct a graph G by adding a vertex v for each point in S and an edge uv if $dist(u, v) < d$. The answer is "Yes" iff G has a vertex cover of size at most k.

7. Put all points intersecting $> k$ lines into P. Then any point intersects $\leq k$ lines. If there are $> k^2$ lines uncovered, the answer is "No"; otherwise, we have a kernel of $O(k^2)$ lines.

If we require each line covered by two points, then k points can cover at most C_k^2 lines. If $|L| > C_k^2$, then answer is "No"; otherwise, the instance itself is a $O(k^2)$ kernel.

8. We give an FPT reduction from k-CLIQUE problem. Let (G, k) be an instance of k-CLIQUE, we construct a graph G' by subdividing each edge $uv \in E$ with a new vertex w. Denote the set of new vertices as Y and the original vertex set of G as X, then X and Y are the two parts of bipartite graph G'.

We show G has a k-clique iff G' has $k' = k + C_k^2$ vertices inducing $k(k-1)$ edges. Suppose G has a clique S of size k, then S and new vertices for each edge of clique S induces $k(k-1)$ edges in G'. Conversely, suppose S' is a set of k' vertices such that $G'[S']$ has $k(k-1)$ edges. Since $d(w) \leq 2$ for each $w \in Y$, we can see that $S' \cap Y \geq C_k^2$ and $S' \cap X \leq k$. The vertices in $S' \cap X$ has total degree $k(k-1)$, so it is a clique of size k in original graph G.

9. We give an FPT reduction from k-CLIQUE problem. Let (G, k) be an instance of k-CLIQUE, we construct a graph G' as follows. Denote degree of vertex v as $d(v)$ maximum degree of G as Δ. Add a $(kn + 1)$-clique S and for each $v \in G$ connect v with arbitrary $\Delta - d(v)$ vertices in S.

Now we show G' has k vertices covering at most $k\Delta - C_k^2$ edges iff (G, k) is an yes-instance of k-CILQUE. Suppose G has a clique C of size k, then C covers exactly $k\Delta - C_k^2$ edges in G'. On the other hand, suppose C' covers $k\Delta - C_k^2$ edges in G', we can see that $C' \cap S = \emptyset$ because any $v \in S$ covers at least kn edges. Note that $d(v) = k\Delta$ for each $v \in G$. Therefore, $C' \in G$ and is a clique.