

TOPICS IN GRAPH ALGORITHMS (CSCI5320-22S)

Homework 4

Due: 5pm April 19, 2022

1. Consider the following VERTEX COVER EXTENSION problem: Given a k -vertex cover V' of $G - e$ for some edge e of G , determine whether G contains a k -vertex cover.
 - (a) Prove that VERTEX COVER EXTENSION admits no polynomial-time algorithm unless $P = NP$.
 - (b) Design an FPT algorithm for the problem.
2. Let $G = (V, E_b \cup E_r)$ be an edge-bicolored graph where E_b and E_r are blue and red edges respectively. Use the iterative compression method to design an FPT algorithm for the problem of deleting at most k edges E' from G so that the resulting graph contains no *conflict pair*, i.e., a pair of adjacent edges with different colors.

Hint: Consider a similarity with VERTEX COVER.
3. Let G be a weighted graph where each edge e has a positive integer $w(e)$ as its weight. Design an FPT algorithm to find a k -path of minimum weight in G .
4. Design an FPT algorithm to determine whether a graph G contains a given k -vertex tree T as a partial subgraph.
5. A 2-regular graph consists of disjoint union of cycles. Design FPT algorithms for finding a 2-regular subgraph H with k vertices in a cubic graph G for H being
 - (a) an induced subgraph, and
 - (b) a partial subgraph respectively.
6. Consider the INDUCED MATCHING problem that asks whether a graph G contains an induced matching with k edges.
 - (a) Prove that INDUCED MATCHING is $W[1]$ -hard.
 - (b) Design an FPT algorithm for INDUCED MATCHING on planar graphs.