

TOPICS IN GRAPH ALGORITHMS (CSCI5320-22S)

Homework 3

Due: 5pm March 22, 2022

1. Define a parameterized problem Π that interests you, and generate at least 9 different parameterized problems related to Π .
2. For k -VERTEX COVER, design an FPT algorithm faster than $O^*(2^k)$ by considering paths P_3 on three vertices, where $O^*(f(k))$ is a shorthand for $f(k)n^{O(1)}$.
3. Design an FPT algorithm to determining whether it is possible to obtain a disjoint union of complete graphs from the input graph G by adding or deleting a total of at most k edges.

Hint: Consider a characterization of disjoint union of complete graphs.

4. Let G be a graph that contains k edges E' such that $G - E'$ is a bipartite graph. Design an FPT algorithm for finding a minimum vertex cover of G .
5. Find a kernel of the following problem: determine whether a graph G contains a cut $[V', V - V']$ with at least k edges.
6. Find a kernel of the following problem: determine whether a set S of n points on the plane contains k points P so that the closest point pair in $S - P$ has distance at least d .
7. Find a kernel of the following problem: Given a set L of n lines on the plane, determine whether there are $\leq k$ points P that hit all lines in L . What happens if we require that every line in L contains at least two points in P ?
8. Prove that it is W[1]-hard to find k vertices V' in a bipartite graph G to maximize the number of edges in $G[V']$.
9. Prove that it is W[1]-hard to find k vertices V' in a graph to cover the minimum number of edges.