

# Multiple quadrilateral detection for projector-camera system applications

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**Abstract**—In this paper, a novel method to detect multiple quadrangles in images for the applications in projected reality, such as a smart office environment, is proposed. The proposed approach is able to detect rectangular surfaces with perspective distortion, which appear as quadrangles in 2D images, in a cluttered background accurately. It can deal with partial occlusion. A new technique called the Q-corner is devised based on randomized Hough transform (RHT). This makes the algorithm become effective and robust in finding multiple rectangular objects like white card-boards in the scene. We present the experiment results with synthetic and real data, showing that our method is more robust to noise than a standard contour-based algorithm and an existing line-based method. The promise of the method for potential applications has been demonstrated in a real scenario.

A key technique involved in these applications is the detection of quadrangles in the camera images. Rectangular regions of arbitrary content and size can be found using algorithms based on Hough transform [3]. In general, they cannot deal with perspective distortion but can be overcome by taking into account vanishing points or lines. Line-based algorithms have been used previously in detecting rectangular signboards [10] or finding the projection boundary [4]. They have shown to be a good alternative for contour-based algorithms [8], which target polygons with exactly four contours. They are useful only if the whole quadrilateral is present in the image and the shape is not occluded in the scene. The OpenCV (Open Source Computer Vision) package provides a contour-based algorithms as described in [9] and is widely used in the computer vision community. Actually, noisy edges, background clutter and obstacle problem are the main challenges in the recognition of rectangular shapes in the real world environment.

## I. INTRODUCTION

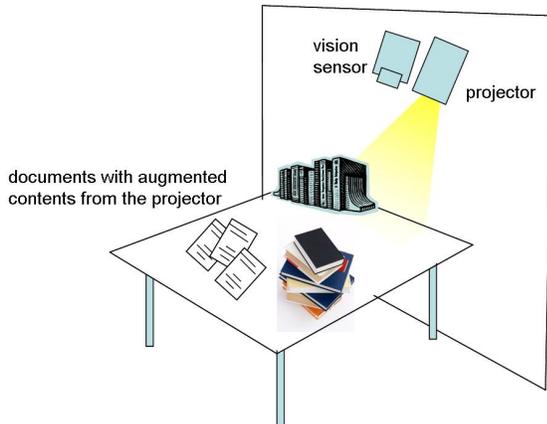


Fig. 1. An intelligence desk scenario proposed by Iwai and Sato [2]. Our multiple quadrangle detection method is suitable for such a kind of applications.

Rectangular object detection is a crucial element in many projector-camera system applications [5]. For example in a smart office environment, documents, memos and books placed on a table are recognized and detected by an overhead camera [1]. Leung *et al.* [4] presented a movable hand-held display which uses a projector to display contents on an ordinary white cardboard. Zhang *et al.* [13] described a vision-based interface system to transform a quadrilateral surface into a touch screen. With these systems, users can interface with the computer more intuitively.

A novel algorithm to detect rectangular shapes, which appears as quadrangles in images, for projector-camera-based smart office applications is proposed. It is devised on the basis of randomized Hough transform (RHT). Firstly, an edge operator is applied. Then, a 2D Hough space for lines is built. A set of vertices, which is probably the vertices of the rectangular objects, is determined from the Hough space. A vertex can be virtual. It means that it is not visible from the image but can be deduced from multiple visible edge lines belonging to the same rectangle. This is a new concept and we name it as the Q-corner. Lastly, the quadrangles are detected from the set of Q-corners using RHT.

The advantages of our method are three-folded. 1. The proposed method is elegant and able to detect multiple quadrangles. 2. It can deal with perspective distortion of rectangular objects in the scene. 3. Our algorithm can find quadrangles that are partially occluded. It works well in a cluttered environment. The proposed algorithm is theoretically better than the methods suggested in [3], [6], [11] as none of them can deal with occlusion nor perspective changes. Experiment results show that our method is better than the contour-based detection method in [9] as well as another line-based detection method in [4]. It is believed that this approach is useful for futuristic applications in spatial augmented reality.

## II. QUADRANGLE FINDING ALGORITHM

In this section, we describe how a potential quadrilateral can be determined based on randomized Hough transform

(RHT) [12] with the use of the proposed "Q-corners" concept. The overview of our proposed algorithm is depicted in Fig. 2.

#### A. Q-corner Extraction

A Q-corner is defined as a point feature that uses line information instead of color information as its descriptor. When a 2D Hough space for lines is built, a set of vertices that are probably the vertices of the rectangular objects is determined from the Hough space. A vertex may be virtual. It means that it is not visible due to occlusion but can be induced from multiple visible edge lines belonging to the same rectangular object. This is called the "Q-corner". In other words, a set of visible edge lines that belongs to the same rectangular object intersected at a 2D point is a Q-corner. The 2D intersection point may be occluded. Each Q-corner represents a virtual vertex, which is the intersection point of at least 2 line segments or at most 2 line equations. In this paper, we assume that only one line segment is possible to be detected from a line equation in the Hough Space. To simplify the explanation, we also assume a bijection relationship between a line equation and a line segment. So it is always possible to find a Q-corner when any line segment pair  $(l_1, l_2)$ , line 1  $l_1$  and line 2  $l_2$  constitute a potential Q-corner, and at the same time they are not parallel and do not contain the intersection point of the line segment pair. Suppose that line 1 is defined by two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , line 2 is defined by two points  $(x_3, y_3)$  and  $(x_4, y_4)$ , and the virtual vertex is defined by point  $(x_q, y_q)$ . Line 1 and line 2 do not contain the virtual vertex if

$$(x_1 - x_q)(x_2 - x_q) \geq 0 \quad (1)$$

$$(y_1 - y_q)(y_2 - y_q) \geq 0 \quad (2)$$

If line 1 and line 2 intersect with each other, the intersection point  $p_q$  of line 1 and line 2, which is also named the virtual vertex of the Q-corner, can be found by the cross product of line 1 and line 2.

$$p_q = l_1 \times l_2 \quad (3)$$

where  $l_1$  and  $l_2$  are defined as

$$l_1 = [a_1, b_1, c_1]^T = [x_1, y_1, 1]^T \times [x_2, y_2, 1]^T \quad (4)$$

$$l_2 = [a_2, b_2, c_2]^T = [x_3, y_3, 1]^T \times [x_4, y_4, 1]^T \quad (5)$$

For any Q-corner at  $(x_q, y_q)$ , there exists a corner angle  $\theta_q$  describing the relationship between two lines and the corner. If neither one of line 1 and line 2 defined previously contains the virtual vertex, the vertex angle is equal to

$$\theta_q = \tan^{-1}\left(\frac{y_3 - y_q}{x_3 - x_q}\right) - \tan^{-1}\left(\frac{y_1 - y_q}{x_1 - x_q}\right) \quad (6)$$

The ordering of two lines are considered to satisfy the less-than- $\pi$  constraint if

$$0 \leq \theta_q < \pi \quad (7)$$

Otherwise, the order of two lines must be reversed. There are five different categories according to the properties of

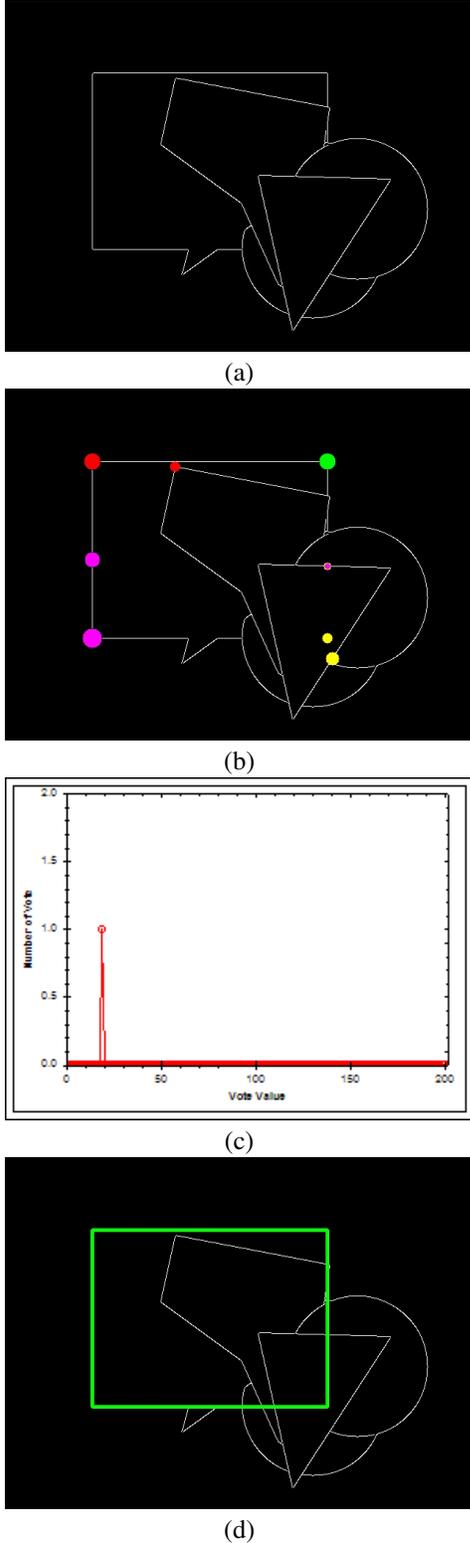


Fig. 2. Overview of the proposed detection algorithm. (a) Step 1: Apply an edge operator. (b) Step 2: Extract Q-corners. (c) Step 3: Vote by randomized Hough transform. (d) Step 4: The detection result. The detected quadrangle is superimposed on the original image.

the Q-corner. They are (i)upper-left, (ii)upper-right, (iii)lower-left, (iv)lower-right, and (v)undefined. We describe a Q-corner whose category is to be predicted by a classifier using a feature vector of the Q-corner. A single Q-corner can belong to multiple categories. In summary, we hypothesize that any set of four points which respectively present in four different Q-corner categories could be the set of vertices of a quadrilateral shapes in the image.

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**Algorithm 1** Randomized Hough Transform (RHT)

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1: while still enough edgels in  $P$  do
2:   randomly select an edgel  $p_i$  from edgel space  $P$ 
3:   for all  $q \in$  Q-corner space of different types do
4:     if  $q$  contains  $p_i$  then
5:       increment the weight of  $q$ 
6:     end if
7:   end for
8: end while
9: construct cumulative density functions (cdf) from the
  weight of Q-corners
10:  $N \leftarrow 0$ 
11: while  $N < epoch_{RHT}$  do
12:   randomly select Q-corners  $q_1, q_2, q_3,$  and  $q_4$  according
  to the cdf
13:   increment  $N$ 
14:   if  $q_1, q_2, q_3,$  and  $q_4$  fulfill the quadrilateral criterion
  then
15:     increment cell  $A(q_1, q_2, q_3,$  and  $q_4)$  in the accumu-
  lator space
16:     if the counter  $|A(q_1, q_2, q_3,$  and  $q_4)|$  equals threshold
   $t$  then
17:        $(q_1, q_2, q_3, q_4)$  parameterize the detected quadri-
  lateral
18:       delete all Q-corners close to  $(q_1, q_2, q_3, q_4)$  from
  Q-corner space
19:       reset the accumulator
20:     end if
21:   end if
22: end while

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*B. Quadrilateral Detection*

In computer vision, the term edgel is commonly used to refer to a pixel in an image that has the characteristics of an edge. It originates from an abbreviation of the term "edge pixel". Each edgel belongs to one or more Q-corners. For detecting convex quadrilaterals, four edgels from the input space that belong to different Q-corner types are mapped onto one point in the Hough Space. This point is the one corresponding to the plane spanned by the four edgels  $p_1, p_2, p_3$  and  $p_4$  from the edgel space. Eventually the cells corresponding to actual quadrilaterals receive more votes and are distinguishable from the other cells. If Q-corners have no or little relevant edgels, they most likely do not belong to a quadrilateral. The basic algorithm is described in **Algorithm 1**.

III. EXPERIMENT RESULTS

In this section, we tested our algorithm using synthetic and real images. A computer with 2.7 Ghz dual core processors and 2 GB memory was used. The value of maximum iteration

$epoch_{RHT}$  of the proposed method was set to a large enough value, say 200.

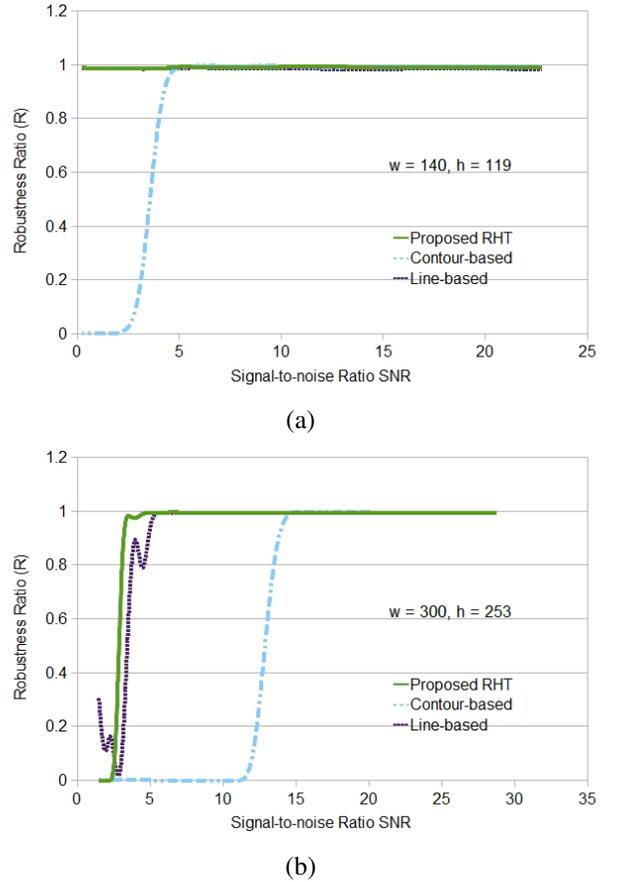


Fig. 3. Plots of the mean value of the robustness ratio  $R$  as a function of the SNR. Continuous lines denote results using the proposed method, while dotted dashed lines denote results of the contour-based method in [9] and dotted lines denote results of the line based method in [4]. The width of the quadrangle: (a)  $w = 140$ ; (b)  $w = 300$ . Both line-based method in [4] and our proposed method work fairly well for low SNR, i.e.  $SNR < 10$ .

*A. Experiments with Synthetic Data*

Robustness to noise in the determination of the boundary of quadrangles is important in real applications. Here, the degree of robustness of our method with increasing noise in comparison with the contour-based method in [9] as well the line-based method in [4] is investigated. To carry out the study, we use the robustness ratio  $R$  and is defined as

$$R = \frac{v_{overlap}}{v_{truth}} \tag{8}$$

where  $v_{overlap}$  is the overlapping area between the detected quadrangle and the actual quadrangle, and  $v_{total}$  is the area of the actual quadrangle. Obviously,  $R$  is equal to 1 if and only if the detected quadrangle has exactly the same area and position as the actual one. Otherwise, it is a non-negative number that is less than 1.  $SNR$  [7] is calculated by the following equation

$$SNR = 20 \log_{10} \frac{v_c}{v_\sigma} \tag{9}$$

$c$  is the contrast and  $\sigma$  is the standard deviation of the noise. The results are plotted in Fig. 3. The second issue

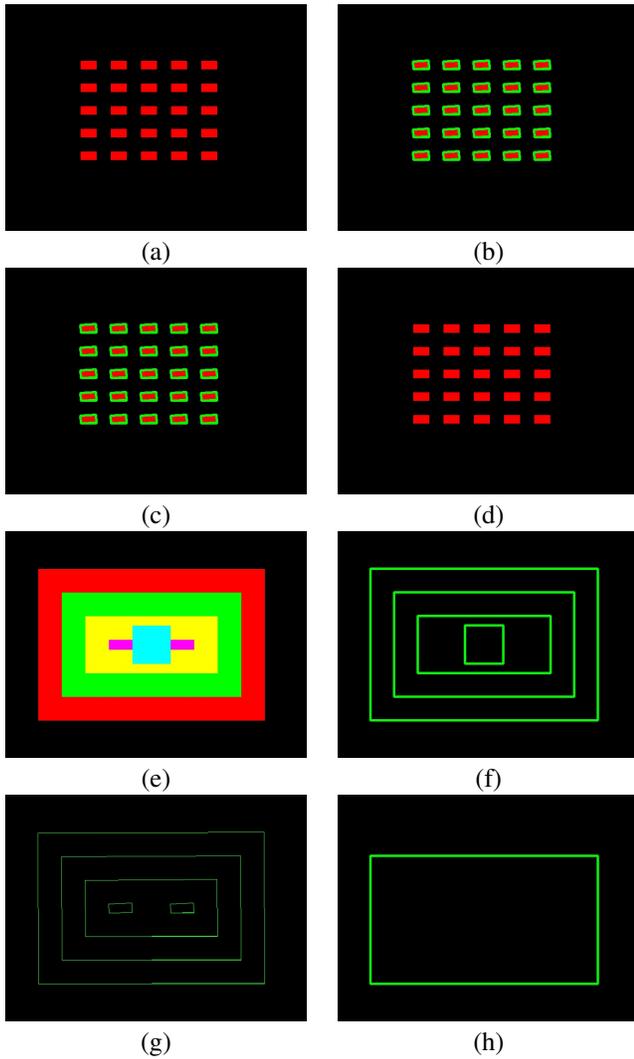


Fig. 4. The detection results of 25 non-occluded quadrangles and 4-layer nested inscribed quadrilaterals in synthetic tests are shown. The detected quadrilaterals are superimposed on the original image in order to show the fitting. (a), (e) Original images. (b), (f) Results of the proposed method. (c), (g) Results of the contour-based method in [9]. (d), (h) Results of the line-based method in [4].

that we would like to study is the detection of multiple non-occluded quadrangles and nested inscribed quadrilaterals. The results are plotted in Fig. 4. The detected quadrilaterals are marked by green borders and are superimposed on the original images. Both the proposed method and the contour-based method in [9] were able to obtain perfect results whereas the line-based methods suggested by Zhang *et al.* [13] and Leung *et al.* [4] failed to detect any valid quadrangle. The third issue that we would like to investigate is the detection of the occluded quadrangle. The results are shown in Fig. 5. The proposed method performed the best in the experiment.

### B. Experiments with Real Data

In the real experiment, the images were captured while at least one rectangular object was moving around in the space. Both the proposed method and the line-based method were able to detect the textbook in Fig. 6. In the scene of Fig. 7, a user took two pieces of cardboards. The contour-based and the

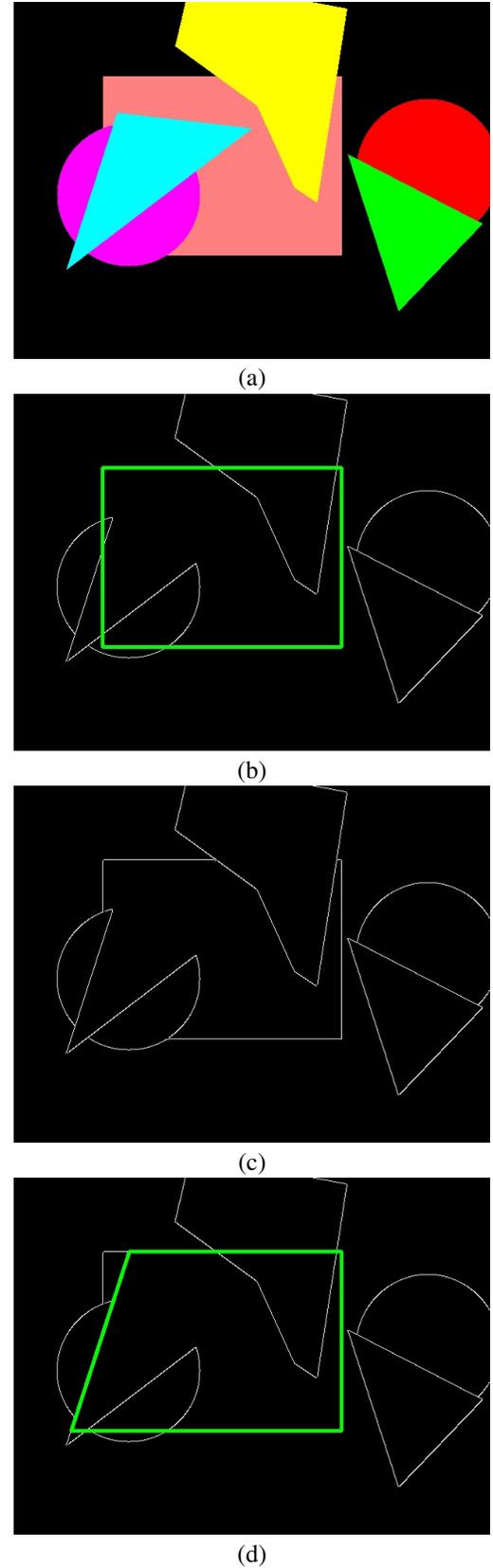


Fig. 5. The detection results of an occluded quadrangle in the synthetic test. (a) Original image. (b) Result of the proposed method. (c) Result of the contour-based method in [9]. (d) Result of the line-based method in [4].



(a)



(b)



(c)

Fig. 6. Results of the real single object experiment. The detected quadrilaterals are superimposed on the original images. (a) Result of our method. (b) Result of the contour-based method in [9]. (c) Result of the line-based method in [4].

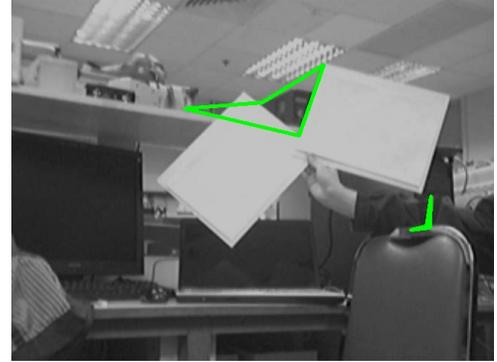
line-based method failed to detect any valid cardboard. The proposed method successfully detected all white cardboards in the scene. Experiment results show that our algorithm outperformed the widely used contour-based detection method in [9] as well as the line-based detection method in [4].

#### IV. CONCLUSION

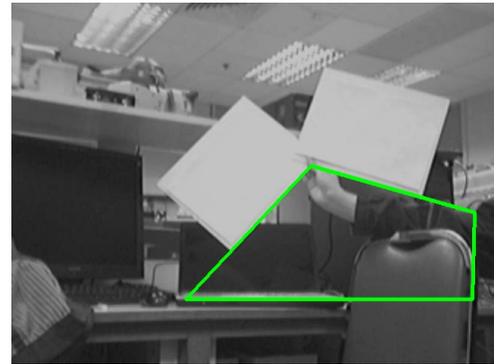
In this paper, we proposed an effective rectangular plane detection method for projector-camera-system-based applications. Our algorithm is able to find multiple rectangular surfaces, which appear as quadrilaterals in the images, with partial occlusion in a cluttered environment. The approach is based on a new idea called the Q-corners. It enables the search of line intersections even if some of them are



(a)



(b)



(c)

Fig. 7. Results of the real multiple object experiment. The detected quadrilaterals are superimposed on the original images. (a) Result of our proposed method. (b) Result of the contour-based method in [9]. (c) Result of the line-based method in [4].

occluded. The randomized Hough transform is employed in the detection process. The proposed method can deal with perspective distortion of rectangular objects in the scene in an elegant way. It is theoretically better than the methods suggested in [3], [6], [11] as none of them can deal with occlusion nor perspective changes. Experiment results show that our algorithm is better than the widely used contour-based method [9] as well as another popular line-based detection method [4] in terms of both accuracy and capability.

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