A Stronger Consistency for Soft Global Constraints in Weighted Constraint Satisfaction

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Abstract

Weighted Constraint Satisfaction is made practical by powerful consistency techniques, such as AC*, FDAC* and EDAC*, which reduce search space effectively and efficiently during search, but they are designed for only binary and ternary constraints. To allow soft global constraints, usually of high arity, to enjoy the same benefits, Lee and Leung give polynomial time algorithms to enforce generalized AC* (GAC*) and FDAC* (FDGAC*) for projection-safe soft non-binary constraints. Generalizing the stronger EDAC* is less straightforward. In this paper, we first reveal the oscillation problem when enforcing EDAC* on constraints sharing more than one variable. To avoid oscillation, we propose a weak version of EDAC* and generalize it to weak EDGAC* for non-binary constraints. Weak EDGAC* is stronger than FDGAC* and GAC*, but weaker than VAC and soft k-consistency for \( k > 2 \). We also show that weak EDGAC* can be enforced in polynomial time for projection-safe constraints. Extensive experimentation confirms the efficiency of our proposal.

Introduction

Soft constraints help model preferences and over-constrained problems. Weighted Constraint Satisfaction Problems (WCSPs), a soft CSP framework, is made practical by powerful consistency techniques applied during search. NC*, AC* and FDAC* (Larrosa and Schiex 2003; 2004) and EDAC* (de Givry et al. 2005) are instrumental in solving radio link frequency problems which are binary in nature. Generalizations of these consistencies for ternary constraints (Sanchez, de Givry, and Schiex 2008) help solve Mendelian error detection problems. Zytnicki et al. (2009) introduced BAC\(^2\) for solving RNA gene localization problems.

On the other hand, many real-life problems are complex to model, requiring the use of specialized global constraints which usually have high arities. Lee and Leung (2009) generalize AC* and FDAC* to their non-binary counterparts, GAC* and FDGAC*, and show that the new consistencies can be enforced in polynomial time for projection-safe soft global constraints. A natural next step is to generalize also the stronger consistency EDAC* (de Givry et al. 2005) to EDGAC*, but this turns out to be non-trivial. We identify and analyze an inherent limitation of EDAC*: similar to the case of Full AC* (de Givry et al. 2005), ED(G)AC* enforcement will go into oscillation if two constraints share more than one variable, which is common when a problem involves high arity (soft) constraints. Sanchez et al. (2008) did not mention the oscillation problem but their method for enforcing EDAC* for the special case of ternary constraints would avoid the oscillation problem. In this paper, we give a weak form of EDAC*, which can be generalized to weak EDGAC* for constraints of any arity. Most importantly, weak EDAC* is reduced to EDAC* when no two constraints share more than one variable. Weak EDGAC* is stronger than FDGAC* and GAC* (Lee and Leung 2009), but weaker than VAC (Cooper et al. 2008) and soft k-consistency (Cooper 2005) for \( k > 2 \). We also give an enforcement algorithm for weak EDGAC*, which can be run in polynomial time for projection-safe (Lee and Leung 2009) soft global constraints. Extensive experimentation confirms the efficiency of our proposal both in terms of pruning and running time.

Background

A weighted CSP (WCSP) (Schiex, Fargier, and Verfaillie 1995) is a tuple \((\mathcal{X}, \mathcal{D}, \mathcal{C}, k)\). \( \mathcal{X} \) is a set of variables \( \{x_1, x_2, \ldots, x_n\} \) ordered by their indices. \( \mathcal{D} \) is a set of domains \( D(x_i) \) for \( x_i \in \mathcal{X} \). Each \( x_i \) can only be assigned one value in its corresponding domain. An assignment \( \{x_i \mapsto v_i, \ldots, x_n \mapsto v_n\} \) onto \( \mathcal{S} = \{x_1, \ldots, x_n\} \subseteq \mathcal{X} \) can be represented by a tuple \( \ell \). The notation \( \ell[x_i] \) denotes the value \( v_i \) assigned to \( x_i \in \mathcal{S} \), and \( \mathcal{L}(S) \) denotes a set of tuples corresponding to all possible assignments on variables \( \mathcal{S} \). \( \mathcal{C} \) is a set of soft constraints, each \( C_{ij} \) of which represents a function mapping a tuple \( \ell \in \mathcal{L}(S) \) to a cost in the valuation structure \( V(k) = \{0, \ldots, k\}, \oplus, \leq \). The structure \( V(k) \) contains a set of integers \( \{0, \ldots, k\} \) with standard integer ordering \( \leq \). Addition \( \oplus \) is defined by \( a \oplus b = \min(k, a + b) \), and subtraction \( \ominus \) is defined only for \( a \geq b \), \( a \ominus b = a - b \) if \( a \neq k \) and \( k \ominus a = k \) for any \( a \). To simplify notation, we write \( C_{i_1,i_2,\ldots,i_n} \) as \( C_{s_1,s_2,\ldots,s_n} \) if the context is clear.

Without loss of generality, we assume the existence of \( C_i \) for each \( x_i \in \mathcal{D}(x_i) \) and \( C_S \) denoting the minimum cost of the problem. If they are not defined, we assume \( C_i(v) = 0 \) for all \( v \neq v_i \) and \( C_S = -\infty \). Constraints are weak if their domain is the positive semi-definite cone of the valuation structure, and strong if their domain is its interior. In this paper, we focus on the weak version of EDAC* and generalize it to weak EDGAC*. We show that weak EDGAC* is stronger than VAC and soft k-consistency for \( k > 2 \). We also give an enforcement algorithm for weak EDGAC*, which can be run in polynomial time for projection-safe soft global constraints. Extensive experimentation confirms the efficiency of our proposal both in terms of pruning and running time.

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for all \( v \in D(x_i) \) and \( C_\emptyset = 0 \). The cost of a tuple \( \ell \in \mathcal{L}(X) \) is defined as \( \text{cost}(\ell) = C_S \oplus \bigoplus_{C_{ij} \in \mathcal{C}} C_S(\ell[S]) \), where \( \ell[S] \) is the tuple formed by projecting \( \ell \) to \( S \subseteq X \). A tuple \( \ell \in \mathcal{L}(X) \) is feasible if \( \text{cost}(\ell) < k \), and is a solution of a WCSP if \( \text{cost}(\ell) \) is minimum among all tuples in \( \mathcal{L}(X) \).

WCSPs are usually solved with basic branch-and-bound search augmented with consistency techniques which prune infeasible values from variable domains and push costs into \( C_\emptyset \) while preserving the equivalence of the problems. Different consistency notions have been defined such as NC* (Larrosa and Schiex 2004), GAC* (Cooper and Schiex 2004; Lee and Leung 2009), and FD(G)AC* (Lee and Leung 2009).

A variable \( x_i \) is node consistent (NC*) if each value \( v \in D(x_i) \) satisfies \( C_i(v) \oplus C_\emptyset < k \) and there exists a value \( v' \in D(x_i) \) such that \( C_i(v') = 0 \). A WCSP is NC* iff all variables are NC*. Procedure \text{enforceNC*}() in Algorithm 1 enforces NC*, where \text{unaryProject()}() enforces unary constraints towards \( C_\emptyset \) and \text{pruneVal()}() removes infeasible values.

```
Procedure \text{enforceNC*}()
1 foreach \( x_i \in X \) do \text{unaryProject}(x_i);
2 foreach \( x_i \in X \) do \text{pruneVal}(x_i);

Procedure \text{unaryProject}(x_i)
3 \( \alpha := \min(C_i(v)|v \in D(x_i)); \)
4 \( C_\emptyset := C_\emptyset \oplus \alpha; \)
5 foreach \( v \in D(x_i) \) do \( C_i(v) := C_i(v) \oplus \alpha; \)

Function \text{pruneVal}(v_i); Boolean
6 flag := false;
7 foreach \( v \in D(x_i) \) s.t. \( C_i(v) \oplus C_\emptyset = k \) do
8 \( D(x_i) := D(x_i) \setminus \{ v \}; \)
9 return flag;

Flag: Algorithm 1: Enforce NC*

A variable \( x_i \in S \) is (G)AC* with respect to a non-unary constraint \( C_S \) if it is NC* and all values \( v \in D(x_i) \) have a tuple \( \ell \in \mathcal{L}(S) \) with \( \ell[x_i] = v \) such that \( C_S(\ell) = 0 \). Such a tuple is a simple support of \( v \in D(x_i) \) with respect to \( C_S \). A WCSP is (G)AC* iff all variables are (G)AC* with respect to all non-unary constraints. The procedure \text{findSupport()} in Algorithm 2 enforces simple supports for values in \( D(x_i) \) with respect to \( C_S \), which requires time complexity exponential in \( |S| \) in general. However, if the constraints are projection-safe, enforcing (G)AC* requires only polynomial time (Lee and Leung 2009).

Suppose variables are ordered by their indices. A full support of a value \( v \in D(x_i) \) with respect to \( C_S \) with \( x_i \in S \) and a set of variables \( U \subseteq X \) is a tuple \( \ell \in \mathcal{L}(S) \) with \( \ell[x_i] = v \) such that \( C_S(\ell) \oplus \bigoplus_{x_j \in U \setminus \{ x_i \}} C_j(v_j) = 0 \). A variable \( x_i \in S \) is directional (generalized) arc consistent star (D(G)AC*) with respect to a non-unary constraint \( C_S \) if it is NC* and all values \( v \in D(x_i) \) have full supports with respect to \( C_S \) and any \( \{ x_j | j > i \} \cap S \). A WCSP is full directional (generalized) arc consistent star (FD(G)AC*) if all variables are D(G)AC* and (G)AC* with respect to all non-unary constraints. The procedure \text{findFullSupport()} in Algorithm 2 enforces full supports for values in \( D(x_i) \) with respect to \( C_S \) and \( U \subseteq X \), which requires time complexity exponential in \( |S| \) in general. Again, if the constraints are projection-safe, enforcing D(G)AC* requires only polynomial time (Lee and Leung 2009).

An Inherent Limitation of EDAC*

A variable is existential arc consistent (EAC*) if it is NC* and there exists a value \( v \in D(x_i) \) with zero unary cost such that it has full supports with respect to all constraints \( C_{ij} \) and \( \{ x_j \} \). A WCSP is existential directional arc consistent (EDAC*) if it is FDAC* and all variables are EAC* (de Givry et al. 2005). Enforcing EAC* on a variable \( x_i \) requires two main operations: (1) compute \( \alpha = \min_{v \in D(x_i)} \{ C_i(a) \oplus \bigoplus_{C_{ij} \in \mathcal{C}} \min_{v \in D(x_j)} \{ C_{ij}(a, b) \oplus C_j(b) \} \} \), which determines whether enforcing full supports breaks the NC* requirement, and (2) if \( \alpha > 0 \), enforce full supports by invoking \text{findFullSupport}(x_i, C_i, \{ x_j \}) for each \( C_{ij} \in \mathcal{C} \), which NC* is no longer satisfied and hence \( C_S \) can be increased by enforcing NC*.

EAC* enforcement will oscillate with constraints sharing more than one variable. The situation is similar to Example 3 by de Givry et al. (2005). We demonstrate by the example in Figure 1(a), which shows a WCSP with two soft constraints \( C_{13} \) and \( C_{23} \). It is FDAC* but not EDAC*. If \( x_2 \) takes the value \( a \), \( C_{13}(v, a) \oplus C_1(v) \geq 1 \) for all values \( v \in D(x_1q) \); if \( x_2 \) takes the value \( b \), \( C_{23}(v, b) \oplus C_1(v) \geq 1 \) for all values \( v \in D(x_1q) \). Thus, by enforcing full supports of each value in \( D(x_2) \) with respect to all constraints and
\{x_1\}, NC* is broken and \( C_\emptyset \) can be increased. To increase \( C_\emptyset \), we enforce full supports: the cost of 1 in \( C_1(a) \) is extended (lines 12 to 14 in Algorithm 2) to \( C_{12} \), resulting in Figure 1(b). No cost in \( C_1 \) can be extended to \( C_{12}^2 \). Performing projection (lines 5 to 7 in Algorithm 2) from \( C_{12}^1 \) to \( C_2 \) results in Figure 1(c). The WCSP is now EAC* but not FDAC*. Enforcing FDAC* converts the problem state back to Figure 1(a).

The problem is caused by the first step, which does not tell how the unary costs are distributed to increase \( C_\emptyset \). Although an increment is predicted, the unary cost \( C_1(a) \) has a choice of moving the cost to \( C_{12} \) or \( C_{12}^2 \). During computation, we obtain no information on how the unary costs are moved. As shown, a wrong movement breaks DAC* without incrementing \( C_\emptyset \), resulting in oscillation.

This problem does not occur in existing solvers which handle only up to ternary constraints. The solvers allow only one binary constraint for every pair of variables. If there are indeed two constraints for the same two variables, the constraints can be merged into one, where the cost of a tuple in the merged constraint is the sum of the costs of the same tuple in the two original constraints. However, if we allow high arity global constraints, sharing of more than one tuple in the two original constraints. A straightforward generalization of EDAC* for non-binary constraints would inherit the same oscillation problem. For example, Figure 2(a) shows a WCSP with two ternary constraints \( C_{124} \) and \( C_{134} \). Each unit-cost ternary tuple is represented by three lines joined by a black dot. The WCSP is FDGAC*.

With a similar argument, a lower bound of 1 should be deduced by finding full supports of \( x_1 \) with respect to \( C_{124} \) and \( \{x_1, x_2\} \), and \( C_{134} \) and \( \{x_1, x_3\} \). Applying full support enforcement would result in the state in Figure 2(b), but enforcing FDGAC* again will convert the problem back to the state in Figure 2(a).

In the case of ternary constraints, Sanchez et al. (2008) cleverly avoid the oscillation problem by re-defining full supports to include not just unary but also binary constraints. During EDAC enforcement, unary costs are distributed through extension to binary constraints. However, the method is only designed for ternary constraints. In the following, we define a weak version of EDAC*, which is based on the notion of cost-providing partitions.

**Cost-Providing Partitions and Weak EDGAC**

A cost-providing partition \( B_{x_i} \) for variable \( x_i \in \mathcal{X} \) is a set of sets \( \{B_{x_i,c} | x_i \in S\} \) such that:

- \( |B_{x_i}| \) is the number of constraints related to \( x_i \);
- \( B_{x_i,c} \subseteq S \);
- \( B_{x_i,c} \cap B_{x_i,c' \in \mathcal{C}} = \emptyset \) for any two different constraints \( C_{S_1}, C_{S_2} \in \mathcal{C} \) and;
- \( \bigcup_{B_{x_i,c} \in B_{x_i}} B_{x_i,c} = (\bigcup_{C_{S} \in C} C_{S} \setminus \{x_i\}) \) \( \cup \{x_i\} \).

Essentially, \( B_{x_i} \) forms a partition of the set containing all variables related to \( x_i \). If \( x_j \in B_{x_i} \), the unary costs in \( C_j \) can only be extended to \( C_j \) when enforcing EAC* for \( x_i \). This avoids the problem of determining how the unary costs of \( x_j \) are distributed when there exists more than one constraint on \( \{x_i, x_j\} \).

Based on the cost-providing partitions, we define weak EDGAC*. Given a WCSP \( P(\mathcal{X}, \mathcal{D}, \mathcal{C}, k) \) and the cost-providing partitions \( B_{x_i} \) for each variable \( x_i \in \mathcal{X} \). A weak fully supported value \( v \in D(x_i) \) of a variable \( x_i \in \mathcal{X} \) is a value with zero unary cost and for each variable \( x_j \) and a constraint \( C_{ij}^m \), there exists a value \( b \in D(x_j) \) such that \( C_{ij}^m(v, b) = 0 \) if \( B_{x_i,c} \wedge v \neq \emptyset \), and \( C_{ij}^m(v, b) \oplus C_j(b) = 0 \) if \( B_{x_i,c} = \{x_i\} \). A variable \( x_i \) is weak existential arc consistent (weak EAC*) if it is NC* and there exists at least one
weak fully supported value in its domain. \( P \) is weak existential directional arc consistent (weak EDAC*) if it is FDAC* and each variable is weak EAC*. Weak EDAC* collapses to AC when WCSPs collapse to CSPs. Moreover, weak EDAC* is reduced to EDAC* (de Givry et al. 2005) when the binary soft constraints share at most one variable.

We further generalize weak EDAC* to weak EDGAC* for \( n \)-ary soft constraints. Given a WCSP \( P(\mathcal{X}, \mathcal{D}, \mathcal{C}, k) \) and any cost-providing partition \( B_{x_i} \) for each variable \( x_i \in \mathcal{X} \). A weak fully supported value \( v \in D(x_i) \) of a variable \( x_i \) is a value with zero unary cost and full supports with respect to all constraints \( C_{x_i} \) with \( x_i \in S \) and \( B_{x_i} \). A variable \( x_i \) is weak existential generalized arc consistent (weak EGAC*) if it is NC* and each variable is weak EAC*. We further generalize weak EDAC* to weak EDGAC* (weak EGAC*) if it is FDGAC* and each variable is weak EGAC*. For example, in Figure 2(a), the WCSP is not weak EGAC* with the cost-providing partition \( B_{x_1} = \{B_{x_1}, C_{x_1}, B_{x_2}, C_{x_2}\} = \{\{x_2\}, \{x_1, x_3\}\} \). If we enforce full supports on \( x_1 \) with respect to \( C_{1, 2, 3} \) and \( \{x_2\} \) (Figure 2(c)), and \( C_{1, 2, 3} \) and \( \{x_1, x_3\} \) (Figure 2(d)), enforcing NC* on \( x_1 \) increases \( C_{1, 2, 3} \) by 1. Given any cost-providing partition, weak EDGAC* is reduced to GAC when WCSPs collapse to CSPs.

To compute the cost-providing partition \( B_{x_i} \) of a variable \( x_i \), we apply Algorithm 3, which is a greedy approach to partition the set \( Y \) containing all variables related to \( x_i \) defined in line 1, hoping to maximize \( \max\{|B_{x_i, C_{x_i}}|\} \). Whether such choice deduces the highest lower bound in weak EDGAC* requires further studies.

**Procedure** findCostProvidingPartition(\( x_i \))

1. foreach \( C_{x_i} \in C \) s.t. \( x_i \in S \) do
2.   \( B_{x_i, C_{x_i}} = Y \cap S \);
3.   \( Y = Y \setminus S \);

**Algorithm 3:** Finding \( B_{x_i} \)

The procedure enforceWeakEDGAC*() in Algorithm 4 enforces weak EDGAC* of a WCSP. The cost-providing partitions are first computed in lines 1 and 2. The procedure makes use of four propagation queues \( \mathcal{P}, \mathcal{Q}, \mathcal{R} \) and \( S \). If \( x_i \in \mathcal{P} \), the variable \( x_i \) is potentially not weak EDGAC* due to a change in unary costs or a removal of values in some variables. If \( x_j \in \mathcal{R} \), the variables \( x_i \) involving in the same constraints as \( x_j \) are potentially not GAC*. The propagation queue \( S \) helps build \( \mathcal{P} \) efficiently. The procedure consists of three inner-while loops and one for-loop. The first inner-while loop from lines 6 to 10 enforces weak EGAC* on each variable by the procedure findExistentialSupport(\( x_i \)) in line 8. If the procedure returns true, a projection from some constraints to \( C_{x_i} \) has been performed. The weak fully supported values of other variables may be destroyed. Thus, the related variables are pushed back to \( \mathcal{P} \) for revision in line 10. The second inner-while loop from lines 12 to 19 enforces DGAC*, while the third inner-while loop from lines 20 to 26 enforces GAC*. A change in unary cost requires re-examining DGAC* and weak EGAC*, which is done by pushing the variables into the corresponding queues in lines 9 and 10, and lines 18 and 19. In the last step, NC* is enforced by the for-loop from lines 28 to 31. Again, if a value in \( D(x_i) \) is removed, GAC*, DGAC* or weak EGAC* may be destroyed, and \( x_i \) is pushed into the corresponding queues for re-examination.
Theorem 1 The procedure enforceWeakEDGAC*() in Algorithm 4 requires \( O(\max\{nd, k\}(f_{\text{EDGAC}} + r^2 e_{\text{DGAC}} + nd) + r^2 d_{\text{DGAC}})) \), where \( n = |X|, \; d = \max\{|D(x)|\}, \; e = |C|, \; \text{and} \; r = \max_{C \in C^e}{|S|} \). Thus, enforceWeakEDGAC*() must terminate.

Proof: As lines 1 and 2 require only \( O(nr) \), we only analyze the time complexity of each inner while-loop and compute the overall time complexity.

A variable is pushed into \( S \) if a value is removed or weak EGAC* is violated. The former happens \( O(nd) \) times, while the latter occurs \( O(k) \) times (each time weak EGAC* is violated, \( C_{\emptyset} \) increases). Since \( \mathcal{P} \) is built on \( S \), the number of iterations caused by \( \mathcal{P} \) is \( O(\max\{nd, k\}) \). Thus, the first inner while-loop in line 6 requires \( O(\max\{nd, k\})f_{\text{EDGAC}} \) (Lee and Leung 2009).

A variable is pushed into \( R \) if either a value is removed, or unary costs are moved by GAC* or weak EGAC* enforcement. The number of iterations due to \( R \) is \( O(\max\{nd, k\}) \). Consider the second inner while-loop in line 11. Once a variable is popped out in line 13, it is not pushed back into \( R \) again by line 19. Thus, the loop only iterates \( O(n) \) times. It follows that the second inner while-loop in line 12 requires \( O(\max\{nd, k\})^2 e_{\text{DGAC}} \) (Lee and Leung 2009).

A variable is pushed into \( Q \) only if a value is removed. Thus, the number of iterations caused by \( Q \) is \( O(nd) \). Thus the third while-loop in line 20 requires \( O(r^2 d_{\text{DGAC}}) \) (Lee and Leung 2009).

The outer while-loop in line 4 terminates when all propagation queues are empty. Thus, the main while-loop iterates \( O(\max\{nd, k\}) \) times. The last for-loop in line 28 requires \( O(\max\{nd, k\}nd) \) times in total.

By summing up all time complexity results, the global time complexity is \( O(\max\{nd, k\}(f_{\text{EGAC}} + r^2 e_{\text{DGAC}} + nd) + r^2 d_{\text{DGAC}})) \) (Lee and Leung 2009).

Theorem 2 Weak EDGAC* with any cost-providing partition is strictly stronger than FDGAC*, which is in turn strictly stronger than GAC* (Lee and Leung 2009)\footnote{http://carlit.toulouse.inra.fr/cgi-bin/awki.cgi/ToolBarIntro}. In other words, enforcing FDGAC* on a problem which is already weak EDGAC* cannot further improve \( C_{\emptyset} \) or remove more values.

In addition, VAC (Cooper et al. 2008) is strictly stronger than EDGAC*. So is soft k-consistency (Cooper 2005) for \( k > 2 \). Since EDGAC* is stronger than weak EDGAC*, we have VAC and soft k-consistency \((k > 2)\) strictly stronger than weak EDGAC*.

Theorem 3 VAC and soft k-consistency \((k > 2)\) are strictly stronger than weak EDGAC* with any cost-providing partition.

Experimental Results

To test the efficiency of weak EDGAC*, we perform the following experiments and compare it with FDGAC* (Lee and Leung 2009). Weaker than FDGAC*, GAC* and strong \( \square \)C (Lee and Leung 2009) are omitted due to the space limitation. VAC and soft k-consistency are omitted as they have not been implemented efficiently for general n-ary constraints. Weak EDGAC* enforcement is implemented in ToolBar2\footnote{http://www.emn.fr/x-info/sdemasse/gccat/}.

The following six benchmarks are used in our experiments:

- The Latin square problem (CSPLIB003) of order \( n \) is to fill an \( n \times n \) table using numbers from \( \{0, \ldots, n-1\} \) such that each number occurs only once in every row and every column.

- The generalized round robin tournament problem (modified from CSPLIB026), parameterized by \((N, P, W)\), is to schedule a tournament of \( N \) teams over \( W \) weeks, with each week divided into \( P \) periods, such that: (1) every team plays at least once a week, (2) every team plays at most twice in the same period over the tournament, and (3) every team plays every other team.

- The fair scheduling problem, suggested by the Global Constraint Catalog\footnote{http://www.emn.fr/x-info/sdemasse/gccat/}, is to schedule \( n \) persons into four shifts over five days such that each person should be assigned the same number of the \( i^{th} \) shift.

- The people-mission scheduling problem, extending the doctor-nurse rostering problem described by Beldiceanu
et al. (2004) is to schedule three groups of \( n \) people into six missions such that each mission is done by a team containing exactly one person in each group. In this problem, we also place a table constraint on each team, restricting some combinations.

The nurse scheduling problem is to schedule a group of \( n \) nurses into four shifts: PM shift, AM shift, Overnight, and Day-Off, over four days such that (1) each nurse must have at most three AM shifts, at least two PM shifts, at least one Overnight, and at least one Day-Off, (2) each AM shift must have two nurses, each PM shift must have one nurse, and each Overnight must have one nurse, and (3) AM-shifts are preferred to be packed together, and the same preference is also posted on Day-Offs.

The stretch modeling problem consists of a sequence of variables \( \{x_1, \ldots, x_n\} \) with domains \( D(x_i) = \{a, b\} \). Each subsequence \( \{x_i, \ldots, x_{i+5}\} \), where \( 1 \leq i \leq 5 \), is required to contain \( a \)-stretches of length 2 and \( b \)-stretches of length 2 or 3, restricted using stretch constraints (Pesant 2001) modeled by regular constraints (Pesant 2004).

The above benchmarks are originally hard in nature and modeled using global constraints. We soften these problems by introducing random unary costs ranging from 0 to 9 on each variable. The hard global constraints GC are also replaced by their projection-safe soft variants softGC (Lee and Leung 2009) with different violation measures \( \mu: \text{var}, \text{val}, \text{edit} \) (van Hoeve, Pesant, and Rousseau 2006).

In the experiments, variables are assigned in lexicographic order. Value assignments start with the value with the smallest index. The nurse scheduling problem is usually compensated for the extra effort.

<table>
<thead>
<tr>
<th>Problem</th>
<th>SoftGC</th>
<th>WeakEDGAC*</th>
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<tbody>
<tr>
<td>Latin Square</td>
<td>78.2</td>
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<td>Crew Scheduling</td>
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<td>23.8</td>
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<tr>
<td>Modeling stretch</td>
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</table>

Table 1: Experimental results: time (in seconds) and number of nodes

Conclusion

Our contributions are three-fold. First, we discover and give an example of a limitation of EDAC*. When constraints share more than one variable, oscillation similar to the one demonstrated in Full AC* (de Givry et al. 2005) will occur. Second, we introduce cost-providing partitions, which restrict the distribution of the cost when enforcing EDAC*. Based on cost-providing partitions, we define weak EDAC*, which can be enforced in polynomial time for projection-safe soft global constraints (Lee and Leung 2009).
Third, we perform extensive experiments to compare weak EDGAC* and FDGAC*, and confirm the pruning and execution efficiency of our proposal.

One immediate future work is to investigate the effect of cost-providing partitions. It is unclear how different variable arrangement in the cost-providing partitions affect domain pruning as well as lower bound deduction. Another possible direction is to investigate if other even stronger consistencies, such as VAC (Cooper et al. 2008), can also benefit from projection safety to make their enforcement practical. Such work can help enrich the applicability of soft constraints to real-life problems.

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