

# Understanding the Paradoxical Effects of Power Control on the Capacity of Wireless Networks

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**Abstract**—Recent works show conflicting results: network capacity may increase or decrease with higher transmission power under different scenarios. In this work, we want to understand this paradox. Specifically, we address the following questions: (1) Theoretically, should we increase or decrease transmission power to maximize network capacity? (2) Theoretically, how much network capacity gain can we achieve by power control? (3) Under realistic situations, how do power control, link scheduling and routing interact with each other? Under which scenarios can we expect a large capacity gain by using higher transmission power? To answer these questions, firstly, we prove that the optimal network capacity is a non-decreasing function of transmission power. Secondly, we prove that the optimal network capacity can be increased unlimitedly by higher transmission power in some network configurations. However, when nodes are distributed uniformly, the gain of optimal network capacity by higher transmission power is upper-bounded by a positive constant. Thirdly, we discuss why network capacity may increase or decrease with higher transmission power under different scenarios using carrier sensing and the minimum hop-count routing. Extensive simulations verify our analysis.

**Index Terms**—Network capacity, power control, routing, scheduling.

## I. INTRODUCTION

WIRELESS networks have been actively developed for providing ubiquitous network access in the past decades. Recently, wireless mesh networks (WMNs) are considered as a key solution to extend the coverage of the Internet, especially in areas where wired networks are expensive to deploy, e.g., in rural areas. Therefore, improving network capacity is one of the most important issues in the research of wireless networks. Roughly speaking, network capacity is the total end-to-end throughputs, which we will carefully define in Section II. Various techniques ranging from physical layer to network layer have been proposed for this purpose, such as MIMO [1], multi-channel multi-radio [2], high-throughput routing [20]–[23], etc. One way to increase network capacity is by leveraging transmission power. This is effective especially in WMNs where stationary mesh routers usually have sufficient power supply, for example, they can share power supply with street-lamps as cited in [3].

In this paper, we study the impact of power control on the capacity of wireless networks. In particular, we consider wire-

less networks where nodes are stationary and are connected in ad-hoc manner. Under this network setting, power control can significantly affect network capacity via the interactions with the link scheduling and the routing algorithms.

First, many link scheduling algorithms in wireless networks nowadays implement carrier sensing to avoid transmission collisions due to interferences<sup>1</sup>. That is, transmitters sense channel states before transmissions and they can transmit only when the sensed noise strength is below carrier sensing threshold. Power control has a tight relation with carrier sensing. When transmission power increases, the sensed noise strength, mainly due to interference, is more likely beyond carrier sensing threshold, which may reduce spatial reuse, i.e., the number of simultaneous transmissions. Since network capacity decreases with lower spatial reuse, higher transmission power may decrease network capacity. Second, power control has a tight relation with routing. On the one hand, higher transmission power may reduce the number of hops or transmissions that a source needs to reach its destination for a longer transmission range. Since network capacity increases with fewer number of transmissions for an application-layer packet, higher transmission power may increase network capacity. On the other hand, because longer transmission range reduces spatial reuse (see Section II), higher transmission power can decrease network capacity. Considering perfect link scheduling, authors in [4] argued that network capacity decreases with higher transmission power under the minimum hop-count routing. However, some recent works showed that network capacity actually increases with higher transmission power in some scenarios [5] [6].

In this paper, we systematically characterize the impact of power control on network capacity and provide a deep understanding on the interesting *paradox*: why network capacity may increase or decrease with higher transmission power in different scenarios? Specifically, we address the following questions:

- 1) Theoretically, should we increase or decrease transmission power to maximize network capacity?
- 2) Theoretically, how much network capacity gain can we achieve by power control?
- 3) Under realistic situations, how do power control, link scheduling and routing interact with each other? Under which scenarios can we expect a large capacity gain using higher transmission power?

The contributions of this work are as follows:

<sup>1</sup>We do not consider CDMA at the moment, which applies some other techniques for interference cancellation.

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- We prove that the *optimal* network capacity is a *non-decreasing* function of transmission power when the network is using the optimal link scheduling and routing.
- We prove that under some specific configurations, the optimal network capacity can be increased unlimitedly by higher transmission power. However, when nodes are distributed uniformly over a space, the gain of the optimal network capacity by higher transmission power is upper-bounded by some positive constant. To the best of our knowledge, we are the first to prove this property.
- We provide a qualitative analysis on the interactions of power control, carrier sensing and the minimum hop-count routing. The later two are the key features commonly used in the link scheduling and routing algorithms nowadays. Through this analysis, we can explain the paradoxical effects of power control on increasing network capacity. The essential reason is that carrier sensing and the minimum hop-count routing are not optimal. We also provide a taxonomy of different scenarios where network capacity (may) increase or decrease with higher transmission power.
- Our work offers some important implications to network designers. First, one can redesign the link scheduling and routing algorithms so as to increase network capacity under high transmission power. Second, we observe from simulation that high transmission power can significantly increase network capacity in the networks whose diameters are within a few hops, which can find applications in small WMNs.

The paper structure is as follows. In Section II, we present a model of wireless networks and define network capacity. In Section III, we prove the theoretical network capacity gain of power control. In Section IV, we discuss why network capacity may increase or decrease with higher transmission power under carrier sensing and the minimum hop-count routing. In Section V, we do simulations to verify our analysis in the previous section. In Section VI, we present related works. In Section VII, we conclude our paper.

## II. SYSTEM MODEL

In this section, we first present a physical model commonly used in the research of wireless networks [8]. Then we define performance measures and some notations used throughout this paper.

In this paper, we consider a *static* network of  $n$  nodes which are located on a 2D plane. Nodes are connected in ad-hoc manner. We use  $(A, B)$  to denote a link transmitting from node  $A$  to node  $B$ , and use  $|A - B|$  to denote the Euclidean distance between  $A$  and  $B$ . We make the following assumptions for the wireless physical model: 1) *Common transmission power*. All nodes use the same transmission power. This assumption simplifies our discussions. Actually, the authors of the COMPOW (COMmon POWer) protocol showed that per-node (or per-link) power control can only improve network capacity marginally than *common power control* [4]. 2) *Single ideal channel*. All nodes transmit on an *ideal* channel without channel fading. This assumption simplifies our analysis so that we can focus on understanding

this paradox. In practice, there are some physical technologies such as MIMO which can greatly mitigate channel fading by using smart antennas [1]. 3) *Single transmission rate*. All nodes transmit at the same data rate of  $W$  bps. 4) *Correct packet reception based on signal-to-noise (SNR) threshold*.

Let  $P_t$  be the transmission power. For a link  $e$ , the received signal strength  $P_r$  at  $e$ 's receiver is

$$P_r = \frac{c_p P_t}{d^\alpha}, \quad (1)$$

where  $c_p$  is a constant determined by some physical parameters, e.g. antenna height,  $\alpha$  is the path loss exponent, varying from 2 to 6 depending on the environment [10], and  $d$  is the distance from  $e$ 's transmitter to its receiver (we call it the *length* of link  $e$ ). We assume all  $c_p$ 's are equal. Thus, by letting  $P_t$  denote  $c_p P_t$ , we can simplify Eq. (1) as

$$P_r = \frac{P_t}{d^\alpha}. \quad (2)$$

For link  $e$ , its signal-to-noise (SNR) is defined at its receiver side, which is

$$SNR = \frac{P_r}{\sum_{i \neq e} I_i + N_0}, \quad (3)$$

where  $P_r$  is the signal strength at  $e$ 's receiver,  $I_i$  is the interference strength from some other transmitting link  $i$  to  $e$ , and  $N_0$  is the white noise.  $I_i$  is also calculated by Eq.(2) except that  $d$  here is the distance from  $i$ 's transmitter to  $e$ 's receiver. The accumulative interference strength and  $N_0$  are treated as *noise* by  $e$ 's receiver. Note that  $N_0$  is usually small comparing with interference strength so that we can ignore it.

To successfully receive a packet, the following two conditions should both be satisfied:

$$P_r \geq H_r, \quad (4)$$

and

$$SNR \geq \beta, \quad (5)$$

where  $H_r$  is the receiving power threshold and  $\beta$  is the SNR threshold for decoding packets correctly.

From the above equations, one can derive  $r$ , the maximum distance between a transmitter and a receiver for successful packet receptions (the maximum is achieved when interference is zero),

$$r = \min \left\{ \left( \frac{P_t}{N_0 \beta} \right)^{1/\alpha}, \left( \frac{P_t}{H_r} \right)^{1/\alpha} \right\}. \quad (6)$$

We refer to  $r$  as *transmission range*. Two nodes can form a link when they are within a distance of  $r$ .

The *interference range*  $r_I$  of a link  $e$  is defined as the minimum distance between an interfering transmitter and  $e$ 's receiver so that  $e$ 's transmissions are not corrupted. Let  $d$  be the length of  $e$ . From Eq. (2)-(3) and ignoring  $N_0$ , we have

$$r_I = \beta^{1/\alpha} \cdot d \quad (7)$$

We observe that  $r_I$  is a constant times of  $d$  and is independent of transmission power. Another observation is that the *silence area* for successful transmissions of a link is proportional to the link length. This suggests that spatial reuse,

i.e. the number of simultaneous transmissions, will decrease with the lengths of links.

Next, we define network capacity according to [9]<sup>2</sup>. We consider a network  $G$  and a set of flows  $F$ . Each flow  $i$  has a traffic workload  $u_i$  (bytes). Suppose the network delivers all traffic workload in time  $T$ , then the network capacity is

$$\frac{\sum_{i=1 \dots |F|} u_i}{T}. \quad (8)$$

Finally, we define the network capacity gain of power control. Given the wireless network and the traffic pattern, let  $C_P(R, S)$  be the network capacity when  $P_t = P$  under the routing algorithm  $R$  and the link scheduling algorithm  $S$ .  $R$  defines the routes of each flow, and  $S$  defines whether a link can transmit at any time  $t$ . We use  $C_P^*(R^*, S^*)$  or  $C_P^*$  to denote the optimal network capacity when  $P_t = P$  under the optimal routing algorithm  $R^*$  and the optimal link scheduling algorithm  $S^*$ .

Let  $P$  and  $KP$  ( $K > 1$ ) be the minimal and the maximal transmission power, respectively. Note that  $P$  should guarantee *network connectivity*; Otherwise, network capacity is meaningless since some flows may not be able to find routes to reach their destinations. We define *network capacity gain of power control* ( $G_K(R, S)$ ) by using the routing algorithm  $R$  and the link scheduling algorithm  $S$  as

$$G_K(R, S) = \frac{C_{KP}(R, S)}{C_P(R, S)}. \quad (9)$$

Furthermore, we define the *theoretical network capacity gain of power control* ( $G_K^*$ ), i.e.,

$$G_K^* = \frac{C_{KP}^*}{C_P^*}. \quad (10)$$

Unless stated otherwise, we use  $K$  to denote the ratio of maximal transmission power to minimal transmission power.

### III. THEORETICAL NETWORK CAPACITY GAIN OF POWER CONTROL

In this section, we derive the theoretical capacity gain of power control based on the information-theoretic perspective. In order to derive the optimal network capacity, we assume that nodes transmit in a synchronous time-slotted mode and each transmission occupies one time slot. From now on we will use the phrase "with high probability" abbreviated as "whp" to stand for "with probability approaching 1 as  $n \rightarrow \infty$ " where  $n$  is the number of nodes in the network. The following theorem states the relationship between the optimal network capacity and transmission power.

**Theorem 1:** *Given the network topology and the traffic pattern, the optimal network capacity is a non-decreasing function of the common transmission power. Therefore,  $G_K^* \geq 1$ .*

**Proof:** Let  $S_P^*(t)$  denote the set of transmitting links at time slot  $t$  when  $P_t = P$ . For any link  $e \in S_P^*(t)$ , its SNR satisfies

$$\frac{P_r}{\sum_{i \in S_P^*(t), i \neq e} I_i + N_0} \geq \beta, \quad (11)$$

<sup>2</sup>We adopt this definition of network capacity because it isolates the capacity definition from fairness concerns

where  $P_r$  is the signal strength of  $e$  and  $I_i$  is the interference strength from some other transmitting link  $i$  to  $e$ . Now we set  $P_t = KP$  ( $K > 1$ ) and use the *same* routes and the *same* link scheduling sequence as  $P_t = P$ . We can see that at time slot  $t$ ,  $e$ 's SNR is

$$\frac{KP_r}{\sum_{i \in S_P^*(t), i \neq e} KI_i + N_0} > \frac{P_r}{\sum_{i \in S_P^*(t), i \neq e} I_i + N_0} \geq \beta, \quad (12)$$

where we use the fact that  $P_r$  and  $I_i$  are proportional to  $P_t$ . So  $S_P^*(t)$  can be scheduled at  $t$  when  $P_t = KP$  for any  $t$ . Since  $R^*$  and  $S^*$  are optimal routing and link scheduling, we have  $C_{KP}^* \geq C_P^*$  by optimality. ■

**Remarks:** The theorem seems counter intuitive but is easy to understand. Basically, given a set of simultaneous links, SNR does not decrease with higher transmission power because both signal strength and interference strength increase at the same ratio. Network capacity can be further improved if we can find *better* routes under higher transmission power. Therefore, theoretically, it is desirable to use higher transmission power to increase network capacity.

An interesting question is how much network capacity gain we can achieve by using higher transmission power. To answer this question, let us analyze it based on the information-theoretic perspective [8]. Without loss of generality, we scale space and suppose that  $n$  nodes are located in a disc of unit area.

**Theorem 2:** *In general,  $G_K^*$  can be unbounded when  $n \rightarrow \infty$ .*

**Proof:** We prove it by constructing a specific network. There are  $2m+1$  vertical links each with a length of  $d$ . The horizontal distance between any two adjacent vertical links is  $2d$ . Fig. 1 illustrates five vertical links where  $(A_1, A_2)$  is the middle link of the network.  $A_3$  evenly separates the line between  $A_1$  and  $A_2$ . Also, there are two nodes evenly separating the line between any two horizontally neighboring nodes. So there are totally  $n = 12m + 3$  nodes in the network. There is a flow along each vertical link from the top node to the bottom node. Let  $\alpha = 4$  and  $\beta = 10$  in the physical model.

The maximal transmission power  $KP$  is set large enough that the transmission range  $r$  is much larger than  $d$  and  $N_0$  can be neglected. Thus, the  $2m+1$  vertical links can transmit simultaneously for any  $m$ . To see this, we can check the SNR of the middle link  $(A_1, A_2)$  which suffers the most interference, i.e.,

$$\text{SNR}_{(A_1, A_2)} \geq \frac{\frac{KP}{d^4}}{2 \cdot \sum_{i=1}^m \frac{KP}{(\sqrt{d^2 + (2id)^2})^4}}.$$

$\text{SNR}_{(A_1, A_2)} \approx 11 > \beta$  when  $m \rightarrow \infty$ . Therefore,  $C_{KP}^*$  is  $(2m+1)W$  or  $(\frac{1}{6}n + \frac{1}{2})W$ . The minimal transmission power  $P$  is set so that  $d > r > \frac{2}{3}d$ . Thus all flows have to go through  $A_1, A_3$  and  $A_2$  to reach their destinations. For example, the route from  $E_1$  to  $E_2$  is through  $C_1, A_1, A_3, A_2$  and  $C_2$ . So  $C_P^*$  is at most  $\frac{1}{2}W$  since  $(A_1, A_3)$  and  $(A_3, A_2)$  are the bottleneck links for all flows. Therefore,  $G_K^*$  is at least  $(\frac{1}{3}n + 1)$ , which is unbounded when  $n \rightarrow \infty$ . ■

**Remarks:** The above theorem shows that network capacity can be increased unboundedly by using higher transmission power

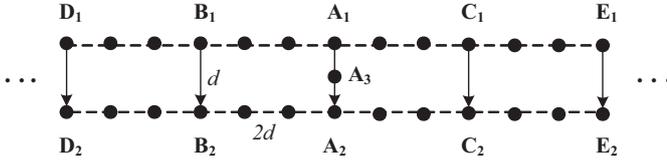


Fig. 1. A network having unbounded  $G_K^*$ .

in some network configurations.

However, nodes placement is approximately random in many real networks. We will show that  $G_K^*$  is upper-bounded by a constant *whp* for networks with uniform node distribution. Before we finally prove this result, we have the following lemmas. We first cite a lemma which was proved in [8].

**Lemma 1:** For any two simultaneous links  $(A, B)$  and  $(C, D)$ , we have  $|B - D| \geq \frac{\Delta}{2}(|A - B| + |C - D|)$ , where  $\Delta = \beta^{1/\alpha} - 1$ .

**Remarks:** From this lemma, if we draw a disc for each link where the center of the disc is the link's receiver and the radius is  $\frac{\Delta}{2}$  times the link length, all such discs are disjoint. Note that  $\Delta > 0$  because we usually have  $\beta > 1$  in practice.

**Lemma 2:** Consider a set of simultaneously transmitting links where the length of any link is at least  $d$ . Given a region whose diameter is  $2R$ , the number of links intersecting the region is upper-bounded by  $\frac{1}{\Delta^2}(4(\Delta + 1)\frac{R}{d} + \Delta + 2)^2$ , where  $\Delta = \beta^{1/\alpha} - 1$ .

**Proof:** See Appendix A-1 in our technical report [7]. ■

We define  $r_c$  as the *critical* transmission range for network connectivity *whp*. From [8], we know that  $r_c = \sqrt{\frac{\log n + k_n}{\pi n}}$  for  $n$  nodes uniformly located in a disc of unit area, where  $k_n \rightarrow \infty$  as  $n \rightarrow \infty$ .

**Lemma 3:** Assume transmission power is sufficiently large so that  $r > 4r_c$ . For a network with uniform node distribution, there exists a route between any two nodes  $A$  and  $B$  which satisfies the following conditions *whp*: (a) for any relay link on the route, its length is smaller than or equal to  $4r_c$ ; (b) the vertical distance from any relay node to the straight-line segment of  $(A, B)$  is at most  $r_c$ ; (c) the number of hops between any two relay nodes  $a_1$  and  $a_2$  is not more than  $\frac{|a_1 - a_2|}{2r_c} + 1$ .

**Proof:** See Appendix A-2 in our technical report [7]. ■

**Remarks:** Intuitively, the lemma shows that there exists a route which can "approximate" the straight-line segment of any two nodes *whp* for a network with uniform node distribution.

**Theorem 3:** Assume  $\alpha > 2$  and transmission power is sufficiently large so that  $r > 4r_c$ . For a network with uniform node distribution,  $G_K^*$  is bounded by a constant  $c$  *whp*, where  $c$  is not depending on  $K$  or traffic pattern.

**Proof:** Let  $P$  and  $KP$  ( $K > 1$ ) be the minimal and maximal transmission power, respectively. Let  $S_{KP}^*(t)$  be the set of simultaneously transmitting links at time slot  $t$  when  $P_t =$

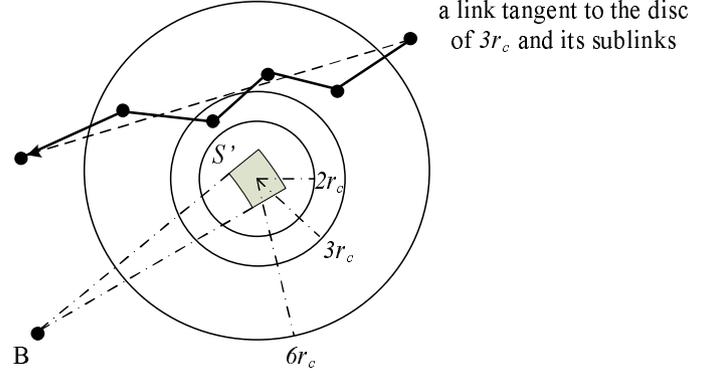


Fig. 2. Illustration of the worst case for a link whose sublinks can intersect  $S'$ .

$KP$ . To prove this theorem, it is sufficient to prove that for any  $t$  we can schedule the traffic in  $S_{KP}^*(t)$  in at most  $c$  time slots when  $P_t = P$ . By optimality, we have  $G_K^* \leq c$ . We will construct such  $c$ .

To avoid confusion here, we use "link" to denote a link when  $P_t = KP$  and use "sublink" to denote a link when  $P_t = P$ . Note that we construct all sublinks from their corresponding links in this proof according to Lemma 3. That is, suppose  $P$  is sufficiently large so that  $r > 4r_c$ , we can find the *relay sublinks* which satisfy the conditions of Lemma 3 for each link in  $S_{KP}^*(t)$  *whp* when  $P_t = P$ .

First, we will show that such a sublink is interfered by at most  $c_0$  sublinks, where  $c_0$  is a constant not depending on  $K$  or traffic pattern. Note that we only consider the links in  $S_{KP}^*(t)$  with a length larger than or equal to  $r_c$  here, since we can schedule the links in  $S_{KP}^*(t)$  with a length smaller than  $r_c$  using another time slot.

We consider some relay sublink  $(A, B)$ . In the preparatory step, we count the number of sublinks intersecting the annulus  $U(i)$  of all points lying within a distance between  $ir_c$  and  $(i + 1)r_c$  from  $B$ , where  $i \geq m$  ( $m$  is a constant which we will determine later). We evenly divide  $U(i)$  into  $\lceil 2\pi(i + 1) \rceil$  sectors, each of which has a central angle of at most  $\frac{1}{i+1}$ . Consider such a sector  $S$ . It is easy to see that its diameter is not more than  $2r_c$ . So we can draw a disc of radius  $2r_c$ , named  $S'$ , to cover  $S$ . From Lemma 3, a relay sublink deviates from its corresponding link by a distance of not more than  $r_c$ . Therefore, if a sublink intersects  $S'$ , the shortest distance between its corresponding link and  $S'$  is at least  $r_c$ . Fig. 2 illustrates the worst case for a link (denoted by the directional dashed line) whose sublinks intersect  $S'$ , where the link should at least intersect a disc of radius  $3r_c$ . Since we consider the links with a length not less than  $r_c$ , from Lemma 2, the number of links whose sublinks intersect  $S'$  is upper-bounded by  $\frac{1}{\Delta^2}(4(\Delta + 1)\frac{3r_c}{r_c} + \Delta + 2)^2 = \frac{1}{\Delta^2}(13\Delta + 14)^2$ .

A sublink cannot intersect  $S'$  if the shortest distance between its transmitter (or receiver) and  $S'$  is larger than  $4r_c$ , since its length is not more than  $4r_c$  according to Lemma 3. Therefore, for any link, the number of its corresponding sublinks intersecting  $S'$  is upper-bounded by  $\frac{2(2+4)r_c}{2r_c} + 1 = 7$ .

From the above results, the number of sublinks intersecting the annulus  $U(i)$  is upper-bounded by  $\lceil 2\pi(i + 1) \rceil \cdot \frac{1}{\Delta^2}(13\Delta + 14)^2 \cdot 7 < c_1(i + 2)$ , where  $c_1 = \frac{14\pi}{\Delta^2}(13\Delta + 14)^2$ . Besides, for

a sublink intersecting  $U(i)$ , the distance from its transmitter to  $B$  is not less than  $(i-4)r_c$ . As a result, the total interference to  $B$  contributed by the sublinks intersecting  $U(i)$  is upper-bounded by  $c_1(i+2) \cdot \frac{P}{((i-4)r_c)^\alpha}$ .

Consider the disc  $C(B, mr_c)$  of all points lying within a distance  $mr_c$  from  $B$ . Suppose that *no simultaneous* transmissions of the sublinks intersecting  $C(B, mr_c)$  are allowed, the SNR of  $(A, B)$  is lower-bounded by

$$\begin{aligned} & \frac{\frac{P}{(4r_c)^\alpha}}{\sum_{i=m}^{\infty} c_1(i+2) \cdot \frac{P}{((i-4)r_c)^\alpha} + N_0} \\ = & \frac{\frac{P}{(4r_c)^\alpha N_0}}{\left(\frac{6}{\alpha-1}m^{1-\alpha} + \frac{1}{\alpha-2}m^{2-\alpha}\right) \cdot \frac{c_1 P}{r_c^\alpha N_0} + 1}. \end{aligned} \quad (13)$$

We see that the denominator of the last term above approaches 1 when  $m \rightarrow \infty$  for  $\alpha > 2$  (In practice, we usually have  $\alpha > 2$  [10]. And  $\alpha = 2$  corresponds to the free-space path loss model). Suppose  $P$  is sufficiently large so that  $r > 4r_c$ , then we have  $\frac{P}{(4r_c)^\alpha N_0} > \beta$ . So there must exist some constant  $m$  making Eq. (13) larger than or equal to  $\beta$ . Clearly,  $m$  only depends on  $c_1$ . Therefore,  $(A, B)$  is *only* interfered by the sublinks intersecting  $C(B, mr_c)$ . So the number of sublinks interfering  $(A, B)$  is upper-bounded by

$$\begin{aligned} c_0 &= \frac{1}{\Delta^4} (4(\Delta+1) \frac{(m+1)r_c}{r_c} + \Delta+2)^2 \cdot \left(\frac{2(m+4)r_c}{2r_c} + 1\right) \\ &= \frac{m+5}{\Delta^4} ((4m+5)\Delta + 4m+6)^2, \end{aligned} \quad (14)$$

following the similar arguments above. Note that  $c_0$  is not depending on  $K$  or traffic pattern.

Second, we can consider each sublink as a *vertex*. If a sublink is not interfered by some other sublink, they are assigned by different *colors*. From the well-known result of vertex coloring in graph theory, we know that each sublink can be scheduled at least once in every  $c_0 + 1$  slots to finish the traffic of  $S_{KP}^*(t)$ .

Finally, consider the links in  $S_{KP}^*(t)$  with its length smaller than  $r_c$ , we have  $c = c_0 + 2$ , where  $c$  is not depending on  $K$  or traffic pattern. ■

**Remarks:** First, the assumption of "the transmission power is sufficiently large" is necessary for  $G_K^*$  to be upper-bounded. We illustrate it by an example. Consider there is one flow transmitting from  $A$  to  $B$  in a linear topology. Suppose there is a direct communication between  $A$  and  $B$  when  $P_t = KP$ . So  $C_{KP}^* = W$ . Suppose there are  $m$  hops from  $A$  to  $B$  and each hop distance is exactly  $r$  when  $P_t = P$ , where  $r$  is the transmission range and  $r = \left(\frac{P}{N_0\beta}\right)^{1/\alpha}$  (theoretically, we can assume  $H_r$  is arbitrarily small). Obviously, only one hop can transmit successfully at a time to satisfy the SNR requirement. So  $C_P^* = \frac{W}{m}$ . Therefore  $G_K^* = m$  which is unbounded when  $m \rightarrow \infty$ . Second, the assumption of "uniform node distribution" is not necessary for  $G_K^*$  to be upper-bounded. Actually, we can derive the same result in Theorem 3 if Lemma 3 holds for some other random node distribution, or more generally, if the route between any two nodes can "approximate" the straight line segment of them *whp*.

#### IV. PRACTICAL NETWORK CAPACITY GAIN OF POWER CONTROL

In the previous section we see that network capacity is maximized under the settings of maximal transmission power, optimal routing and link scheduling. However, the latter two are NP-hard problems [11] [12]. In this section, we examine  $G_K$  by using carrier sensing and the minimum hop-count routing, which are the key features commonly used in the link scheduling and routing algorithms nowadays.

First, we discuss carrier sensing. To avoid collisions during transmissions, many current solutions require transmitters to sense channel before transmissions. A transmitter can transmit only when

$$P_s \leq H_s, \quad (15)$$

where  $P_s$  is the noise strength sensed at transmitter side and  $H_s$  is *carrier sensing threshold*. Assume the network is *symmetric*, that is,  $P_s$  at transmitter side is equal to  $\sum I_i + N_0$  at receiver side (Note that the assumption is often invalid in practice). By setting  $H_s = \frac{P_r}{\beta}$ , one can guarantee that  $SNR \geq \beta$  [13]. However, it is difficult in practice for a transmitter to know its  $P_r$  at receiver side. To circumvent this problem, we can conservatively estimate  $P_r$  by  $H_r$ . So we have

$$H_s = \frac{H_r}{\beta}. \quad (16)$$

In practice, exposed terminal problem and hidden terminal problem often occur for Eq.(16). The radical reason is that carrier sensing uses a fixed  $H_s$  and operates at transmitter side, which can not estimate interference accurately. For a detailed discussion, interested readers can refer to Section IV of our technical report [7]. Therefore, even under the optimal routing, network capacity can degrade with higher transmission power by using carrier sensing. For example, consider a network with all one-hop flows, higher transmission power increases  $P_s$ , which can reduce spatial reuse and thus decrease network capacity (see Experiment 1 in Section V).

Next, we discuss the minimum hop-count routing. The authors of [4] argued that even under optimal link scheduling *network capacity under the minimum hop-count routing* is proportional to

$$\frac{1}{r}. \quad (17)$$

So  $G_K = \left(\frac{1}{K}\right)^{1/\alpha}$  by Eq. (6). Their interpretation is as follows. The network capacity consumption of a flow is proportional to the number of hops the flow traverses, i.e.  $\frac{1}{r}$ . Spatial reuse is proportional to  $\frac{1}{r^2}$ . Network capacity is proportional to spatial reuse and *inversely* proportional to the network capacity consumption per flow, i.e.  $\frac{1}{r}$ .

We make some comments on Eq. (17). First, although it properly characterizes the *order* of network capacity as a function of  $r$ , it has some deviations from practice. For example, the network diameter (in term of the number of hops) may be so small that the spatial reuse may not decrease as much as  $\frac{1}{r^2}$  due to edge effect<sup>3</sup>. As a result, the network

<sup>3</sup>In here, the edge effect means that the network diameter is so small that most links are near the periphery of the network

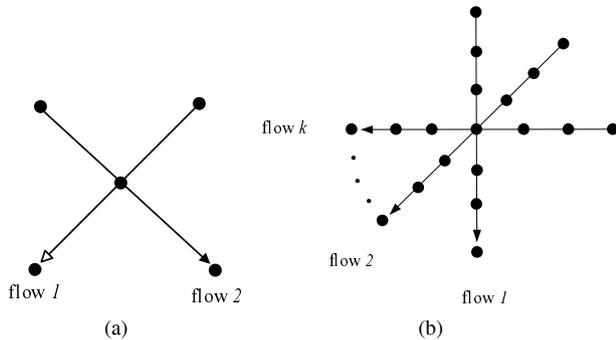


Fig. 3. Examples of network capacity increasing with higher power.

capacity may increase with larger  $r$ . Fig. 3(a) shows such an example where there are five nodes and two flows of equal rate in the network. When the transmission power is low, both flows need to traverse the centered node to reach their respective destinations. Since there are four links contending the channel, the network capacity is  $\frac{1}{4}W \cdot 2 = \frac{1}{2}W$ . When we increase the transmission power so that packets can be transmitted directly from sources to destinations, there are two links contending the channel, and network capacity is  $\frac{1}{2}W \cdot 2 = W$ . Actually, the spatial reuse here is always one transmitting link per time slot for any power level due to edge effect. The network capacity increases with higher transmission power due to a less number of hops per flow. Second, it *may not* hold for the networks with *non-uniform link load distribution*. Fig. 3(b) shows such an example where there are  $k$  flows of equal rate traversing through the centered node. The link load distribution is non-uniform here as the centered node is the biggest bottleneck. It is easy to see that the spatial reuse decreases as  $\frac{1}{r^2}$  here. However, the network capacity does not decrease as  $\frac{1}{r}$ . To see this, we consider two specific cases. In the first case of using the minimal transmission power, each flow is  $m$ -hop ( $m \gg 2$ ). So there are at least  $2k$  links neighboring the centered node, resulting in the network capacity of at most  $\frac{W}{2k} \cdot k = \frac{1}{2}W$ . In the second case of using the maximal transmission power, each flow is 1-hop. So there are  $k$  links contending the channel, resulting in the network capacity of  $\frac{W}{k} \cdot k = W$ .

Based on the above observations, one can explain why network capacity sometimes increases with higher transmission power under the minimum hop-count routing [5].

In summary, current carrier sensing and the minimum hop-count routing do not guarantee  $G_K \geq 1$  and may lead to significant capacity degradation with higher transmission power. But network capacity may increase significantly with higher transmission power in some scenarios. Therefore, there is a paradox in practice on whether to use higher transmission power to increase network capacity.

## V. SIMULATION RESULTS

In this section, we examine the impact of power control on network capacity via simulation. We use carrier sensing and the minimum hop-count routing as the link scheduling and routing algorithms in our simulations.

We set  $\alpha = 4$  for simulating the two-ray ground path loss model [10]. We set  $\beta = 10$  and  $H_r = -81\text{dBm}$ . Therefore,

$H_s = \frac{1}{10}H_r$  by Eq. (16). We ignore  $N_0$  which is often much smaller than the interference strength. For better illustrations, we use the transmission range  $r$  to represent the transmission power. We increase the transmission power so that  $r = 250\text{m}$ ,  $500\text{m}$ ,  $750\text{m}$  and  $1000\text{m}$ . Actually, one can change  $r$  proportionally and scale network topologies at the same time to obtain similar results.

We implemented a TDMA simulator for performance evaluation. That is, nodes transmit in synchronous time-slotted mode and each DATA transmission and its ACK occupies one time slot. Transmitters sense the channel one by one at the beginning of each time slot. A transmitter will transmit a DATA packet when  $P_s \leq H_s$  and its backoff timer expires. The receiver returns an ACK to the transmitter when it receives the packet successfully. If the transmitter does not receive an ACK due to packet collision, it will carry out the exponential backoff. The backoff mechanism is similar to that of 802.11 except that we backoff the time slot here.

We calculate network capacity according to Eq. (8). In our simulations, each flow is assigned a traffic workload of 500 equal-sized packets. We generate CBR traffic for each flow until completing its traffic workload. The CBR rate is set large enough to saturate the network. Besides, the packet buffer in each node is set sufficiently large since we do not consider queue management at the moment.

For simplicity, in the following experiments, we use CS to denote carrier sensing and use HOP to denote the minimum hop-count routing. We implemented a centralized link scheduling, named Cen, as a benchmark, which schedules links one by one in a centralized and collision-free way and thus ensures maximal spatial reuse. In each experiment, we take the average of all simulation results for ten networks.

In the first experiment, we study the interaction of power control and carrier sensing by considering one-hop flows so as to isolate the interaction of routing.

**Experiment 1** Network capacity vs Power in a random network with *one-hop flows*. There are  $n = 200$  nodes uniformly placed in a square of  $3000\text{m} \times 3000\text{m}$ , which form a connected network when  $r = 250\text{m}$ . Each node randomly communicates with one of its nearest neighbors.

Fig. 4 shows the network capacity as a function of  $r$ . Obviously, the network capacity by using Cen is almost a constant in this scenario. However, when we use CS, higher transmission power causes more exposed terminals and decrease network capacity, since the carrier sensing threshold is fixed.

In the following experiments, we study the interaction of power control, carrier sensing and the minimum hop-count routing by considering multi-hop flows.

**Experiment 2** Network capacity vs Power in a random network with *multi-hop flows* and *small network diameter* (in terms of the number of hops). There are  $n = 20$  nodes uniformly placed in a square of  $1000\text{m} \times 1000\text{m}$ , which form a connected network when  $r = 250\text{m}$ . Each node randomly communicates with any other node in the network.

Fig. 5(a) shows the network capacity as a function of  $r$ . First, in a sharp contrast to Eq. (17), the network capacity by using HOP significantly increases with  $r$ . The reason is that the network diameter is so small (4-6 hops) that the spatial reuse

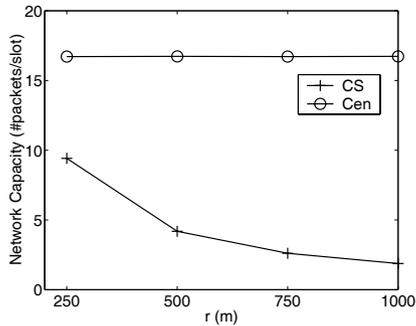
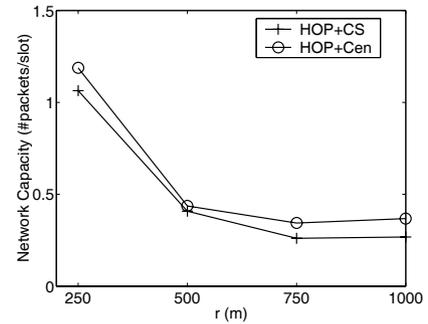
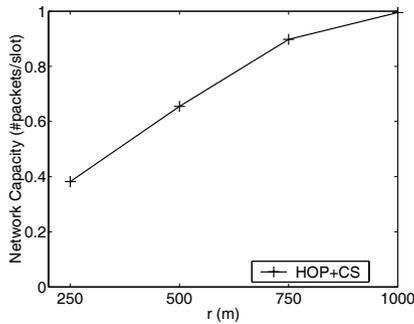
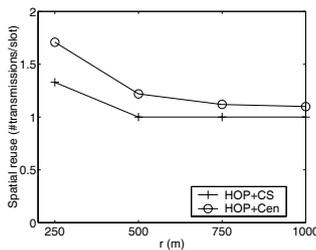
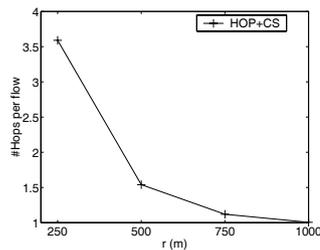
Fig. 4. Experiment 1: Network capacity vs  $r$ .Fig. 6. Experiment 3: Network capacity vs  $r$ .(a) Network capacity vs  $r$ (b) Avg. spatial reuse vs  $r$ (c) Avg. # of hops per flow vs  $r$ 

Fig. 5. Experiment 2.

only decreases slightly with larger  $r$ , as shown in Fig.5(b). Actually, only a few links can transmit simultaneously in this scenario due to edge effect. HOP minimizes the number of hops that flows traverse, as shown in Fig.5(c), which is the dominant factor for the significant increase of network capacity. Second, CS works reasonably well in this experiment, as compared with Cen (see Fig.5(b)). The reason is that HOP prefers longest forwarding links for multi-hop flows, which is close to the case that we derive  $H_s$  in Eq. (16).

**Experiment 3** Network capacity vs Power in a grid network with *multi-hop flows* and *large network diameter* (in terms of the number of hops). There are  $n = 625$  nodes placed in a  $25 \times 25$  grid. There is a distance of 200m between any two horizontally or vertically neighboring nodes. There are 25 flows from the leftmost nodes to the rightmost nodes horizontally and 25 flows from the topmost nodes to the bottommost nodes vertically. This configuration ensures a large network diameter and uniform link load distribution.

We observe that the network capacity decreases significantly with larger  $r$ , as shown in Fig. 6, because of the significant decreasing of spatial reuse under the minimum hop-count routing. We also plot the network capacity by using HOP and

Cen, which confirms our explanation.

We also test the random networks with multi-hop flows and a large network diameter. We observe that the network capacity significantly decreases with larger  $r$  in this scenario when  $n$  is sufficiently large.

In summary, the following conclusions can be made from our analysis (Section IV) and simulations. When we use carrier sensing and the minimum hop-count routing,

- In the networks with *one-hop flows*, the network capacity significantly decreases with higher transmission power due to exposed terminal problem.
- In the networks with *multi-hop flows* and a *small network diameter* of a few hops, the network capacity can increase significantly with higher transmission power because the edge effect makes spatial reuse only decrease slightly with larger  $r$ . This can find applications in small WMNs. Currently, many WMNs tend to have a small network diameter (in term of the number of hops), because the end-to-end throughput of a flow drops significantly with an increasing number of hops [8] [14].
- In the networks of *multi-hop flows* and a *large network diameter*, there are two subcases. Under *uniform link load distribution*, the network capacity decreases significantly with higher transmission power as shown in Eq. (17); Under *non-uniform link load distribution*, it is hard to make a conclusion. The network capacity *may* increase with higher transmission power as illustrated by Fig. 3(b).

## VI. RELATED WORK

Research on power control can be classified into two classes: energy oriented and capacity oriented. The first class of works focus on energy-efficient power control [15] [16] [17]. The application is in mobile ad hoc networks (MANETs) or wireless sensor networks (WSNs), where nodes have limited battery life. Low transmission power is preferred here to maximize the throughput per unit of energy consumption, while maximizing overall network capacity is the secondary consideration. As a result, their solutions often achieve moderate network capacity. The second class of works focus on capacity-oriented power control. The application is in WMNs where mesh routers have sufficient power supply and maximizing network capacity is the first consideration.

Authors in [4] indicated that network capacity decreases significantly with higher transmission power under the minimum hop-count routing and they suggested using the lowest

transmission power to maximize network capacity. There are a lot of works following this suggestion, e.g. [18] [19], and they observed capacity improvement by using lower transmission power. However, there is an opposite argument recently. Park *et al* showed via simulation that network capacity sometimes increase with higher transmission power [5]. Behzad *et al.* formulated the problem of power control as an optimization problem and proved that network capacity is maximized by properly increasing transmission power [6].

We also proved that the optimal network capacity is a non-decreasing function of common transmission power in a simpler way. Furthermore, we characterized the theoretical network capacity gain of power control. Besides, we studied the interactions of power control, carrier sensing and the minimum hop-count routing. As a result, we explained the above paradox successfully from both theoretical and practical perspective. Our work provides a deep understanding on the *structure* of the power control problem and is an extension to [4]- [6].

## VII. CONCLUSION

This work thoroughly studies the impact of power control on network capacity from both theoretic and practical perspective. In the first part, we provided a formal proof that the optimal network capacity is a non-decreasing function of common transmission power. Then we characterize the theoretical capacity gain of power control in the case of the optimal network capacity. We proved that the optimal network capacity can be increased unlimitedly with higher transmission power in some network configurations. However, the increase of network capacity is bounded by a constant with higher transmission power *whp* for the networks with uniform node distribution. In the second part, we analyzed why network capacity increases or decreases with higher transmission power in different scenarios, by using carrier sensing and the minimum hop-count routing in practice. We also conduct simulations to study this problem under different scenarios such as a small network diameter vs a large network diameter and one-hop flows vs multi-hop flows. The simulation results verify our analysis. In particular, we observe that network capacity can be significantly improved with higher transmission power in the networks with a small network diameter, which can find applications in small WMNs.

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