Diffusion in Random Networks: Impact of Degree Distribution

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ABSTRACT

Word of Mouth (WoM) is known as a powerful marketing force, as numerous empirical studies reveal that consumers' purchasing decisions are based on the advice of those in their social networks rather than on direct advertising. A recent international survey by Nielsen reports that 92% of consumers around the world count on recommendations from friends and family more than all other forms of advertising. With technological advances in online communications that enable consumers to easily share their experience with their "friends," the effect and importance of WoM has only grown. In this new era, firms not only harness the power of WoM, but also improve its efficacy by targeting consumers based on the wealth of information available about their online activities. In particular, firms can utilize information on connections among consumers (1) to predict the diffusion trajectory (for both time and cost), and (2) to devise effective seeding strategies to impact the trajectory. In this work, we provide a theoretical framework to study the diffusion process for a general class of network models and drive insights on the impact of heterogeneity in the degree of connections on the cost and speed of diffusion as well as on optimal seeding strategies.

To this end, we study a diffusion process of a new product that spreads through the contacts that adopters make with their neighbors. In particular, we assume that an adopter makes contact with each of her neighbors according to an independent Poisson process with rate γ. We assume that the network underlying the connections is a random network with a given degree distribution. This general class of network models has been extensively used in the study of social networks, and it serves as the network model when the firm's knowledge about the pattern of connections is limited to the degree of each agent, rather that having access to the identity of every neighbor of an agent.

In our setting, the firm incurs a fixed cost c for each contact by an adopter. Further, we assume that the firm has a budget for *seeding*. In particular, prior to the adoption process, he can directly contact a fraction $q > 0$ of agents, who become adopters. The firm decides who to seed with the goal of minimizing the total cost or the total time to reach his target proportion of adopters. Targeting agents based on social network information is a common practice. It has been studied in network economics literature in monopolistic settings as well as in competitive settings, mainly under the assumption that the firm has complete information about the network structure. However, in our setting the firm can only target based on the degree of an agent, since he does not have access to more detailed information. Seeding agents based on their degrees seems to be more practical (as it requires the firm to acquire much less information), and empirically it has been shown to be effective.

Summary of Our Contributions: In the above setting,

(1) We compute the cost and time to reach any adoption proportion $q < s < 1$, for any general bounded degree distribution in the limit as the number of agents grows. To the best of our knowledge, this is the first exact characterization of the diffusion process for such a general class of degree distributions. The other exact characterizations are for the special cases of a complete network, which is equivalent to the Bass model, and a one-dimensional grid (i.e., a cycle).

(2) Using our exact characterization, we study the impact of degree distribution on the cost and time to reach any adoption proportion $q < s < 1$ and demonstrate a trade-off between contact cost and speed. In particular, we show that lower variability in the degree distribution results in lower cost. Fixing the average degree $k \in \mathbb{N}$, the most cost-efficient network (to reach any adoption fraction s) is the k-regular network. The impact of degree distribution on timing is more involved, as it depends on the target adoption level (i.e., s) as well as higher moments of the degree distribution. However, our numerical analysis suggests that unless the target level is very high (e.g., $s = 0.9$), higher variance improves the speed.

(3) We also study the problem of optimal seeding that the firm faces for a network with a given degree distribution. Somewhat contrary to the general wisdom, we show that the optimal strategy does not necessarily entail seeding high degree agents. In fact, we prove that for the objective of minimizing cost (to achieve any target level of adoption), the optimal strategy is to maximally seed low degree agents. For the objective of minimizing time, the optimal strategy depends on the target level s and on the seeding budget q. We present examples illustrating that the optimal strategy can be to seed a mixture of high and low degree agents.

(4) In the absence of seeding, diffusion has a very slow start simply because there are not enough adopters to make contacts. We also study diffusion in such a setting by assuming that the diffusion starts with a single (randomly selected) agent. We characterize the cost and time it takes to reach $\alpha \log(n)$ adopters, where n is the number of agents in the network, and $\alpha > 0$ is a constant. We call this phase of diffusion the *early adoption regime*. We provide comparative statics with respect to the degree distribution in the early adoption regime, and we show that the cost is independent of degree distribution. Further, the time to diffuse to $\alpha \log(n)$ adopters only depends on the first two moments of the degree distribution. Fixing the average degree, the time decreases as variance increases.[1]

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[1]Full version is available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3174391.