Optimal pricing for a peer-to-peer sharing platform under network externalities *

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ABSTRACT

In this paper, we analyse how a peer-to-peer sharing platform should price its service (when imagined as an excludable public good) to maximize profit, when each user's participation adds value to the platform service by creating a positive externality to other participants. To characterize network externalities as a function of the number of participants, we consider different bounded and unbounded user utility models. The bounded utility model fits many infrastructure sharing applications with bounded network value, in which complete coverage has a finite user valuation (e.g., WiFi or hotspot). The unbounded utility model fits the large scale data sharing and explosion in social media, where it is expected that the network value follows Metcalfe's or Zipf's law. For both models, we analyze the optimal pricing schemes to select heterogeneous users in the platform under complete and incomplete information of users' service valuations. We propose the concept of price of information (PoI) to characterize the profit loss due to lack of information, and present provable PoI bounds for different utility models. We show that the PoI = 2 for the bounded utility model, meaning that just half of profit is lost, whereas the PoI ≥ 2 for the unbounded utility model and increases as for a less concave utility function. We also show that the complicated differentiated pricing scheme which is optimal under incomplete user information, can be replaced by a single uniform price scheme that is asymptotic optimal. Finally, we extend our pricing schemes to a two-sided market by including a new group of 'pure' service users contributing no externalities, and show that the platform may charge zero price to the original group of users in order to attract the pure user group.

CCS CONCEPTS

• **Networks → Network economics**;

1 INTRODUCTION

Due to advances in wireless technology and more powerful mobile devices (e.g., smartphones), it is common today that when users join a peer-to-peer sharing platform they not only enjoy the provided service but also contribute to the service's value. There are roughly two types of peer-to-peer sharing platforms: infrastructure and content sharing [12]. The former type of platforms allows users to cooperate and contribute physical resources to create networking or computing services. For example, FON is a WiFi sharing platform whose user opens its home WiFi connection to the community and can access the others' WiFi access points [10]. The latter type of platforms includes online social media (e.g., WeChat, WhatsApp), where platform users create and share massive content with each other and their number has reached 1.6 billion in 2014. The global revenue of such peer-to-peer sharing platforms is fast growing and is expected to increase to US$40 billions by 2022 [11]. How to price their services for selected users under network externalities is a key question for such profit-maximizing platforms.

Peer-to-peer sharing economy of such excludable public goods has been widely studied in the recent literature. [1] and [5] study how to address the incentive issues for efficient sharing in peer-to-peer networks via mechanism design. [8] sudies effects of user diversity on incentives in peer-to-peer information sharing platforms. Courcourbetis and Weber in [4] study pricing of an infrastructure-sharing platform (e.g., peer-to-peer file sharing) and find the network value (profit) in an asymptotic sense and find that network value/profit is bounded when each user randomly caches and shares a subset of distinct files. Metcalfe and Zipf's laws study the network value for the social media platforms, showing the service value to an individual increases super-linearly with the total user number and is thus unbounded [2]. In [10], [12] and [7], users' dual modes (i.e., contributors and consumers) are considered and optimal pricing schemes for network externalities is designed under complete information. Assuming full information of users' private utilities, [3] investigates the optimal pricing according to the network structure, and proposes a simplified approximation using uniform pricing, i.e. every users sees the same price. [6] further considers that the network externalities can be positive or negative, affecting the final pricing design. Different from these works, we consider the challenging scenario of incomplete information for optimal pricing design of excludable public goods, and study the feasibility to employ a simple pricing approach for profit maximization (without users' reporting of private information as in VCG auction). The newly proposed concept, price of information is unique to characterize the profit loss due to lack of information.

Our main contributions and key novelty are summarised as follows.

- We study the optimal pricing for a peer-to-peer sharing platform under incomplete information, by considering both

the infrastructure and content sharing applications (with bounded and unbounded network externalities, see Section 2). The platform is profit-maximizing and designs pricing to include target users to contribute to the excludable public goods.

- For both bounded and unbounded user utility models, we analyze the optimal pricing schemes to select heterogeneous users in the platform under complete and incomplete information of users' service valuations. We propose the concept of price of information, which is defined as the ratio of profits under complete and incomplete information, to characterize the profit loss due to lack of information, and present provable PoI bounds for different utility models. We prove that the PoI = 2 for the bounded utility model, meaning that just half of profit is lost. For a general unbounded utility model, we prove the PoI is in the interval $[2, 27/8]$, that is, PoI is at least 2 and is greater for a less concave utility function.

- We simplify the complicated differentiated pricing scheme under incomplete information, by replacing it by a single uniform price. The uniform price mechanism does not need users to report their private information of service valuations and achieves asymptotical optimality as user number goes to infinity for both bounded and unbounded user utility models.

- We extend our pricing schemes to a two-sided market by including a new group of 'pure' service users contributing no externalities. We show that the platform needs to decide different pricing to different groups of users and may charge zero price to the original group of users in order to attract the pure user group. We prove that the uniform pricing scheme is still asymptotically optimal as user number goes to infinity and that PoI increases as the fraction of original group of users decreases.

2 SYSTEM MODEL

We consider a peer-to-peer platform who wants to maximize its profit. It faces a set of potential users $N = \{1, \ldots, n\}$ who choose to participate in the subscribing to the platform service or not. Define binary variable $\pi_i = 1$ or 0, telling that user i will or will not participate. The vector $\pi = (\pi_1, \ldots, \pi_n)$ summarizes all users' participation decisions. The total service value is denoted by $\phi(\pi)$, which is a function of π to tell the network externalities. Consider that each user contributes equally to the service as a public good, then $\phi(\pi)$ can be rewritten as a function of the number of platform users denoted by $m = \sum_{i=1}^{n} \pi$, that is, $\phi(\pi) = \phi(m)$. We will introduce the detailed formulation of bounded and unbounded $\phi(\cdot)$ in Sections 2.1 and 2.2, respectively.

Users have heterogeneous service valuations towards the platform service. Let θ_i be the user i's service valuation and this is his private information. Without loss of generality, we assume $\theta_1 > \theta_2 > \cdots > \theta_n$ and denote valuation vector $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$. The utility of a participant i is proportional to his valuation and the total service value, that is, $\theta_i \phi(\pi)$. The platform can charge differently for different users' subscriptions. Let p_i be the membership fee charged to i. The payoff of user i is his utility of the total service value minus the membership fee, that is,

$$u_i = \pi_i(\theta_i \phi(\pi) - p_i). \tag{1}$$

The platform's goal is to maximize its total profit and it may not include all users. Let c be the platform cost (e.g., equipment fee for installing an access point in WiFi sharing) for adding a user to access shared service with the exsiting others. The total profit,

denoted by Π, is a function of π and c as follows

$$\Pi = \sum_{i \in N} \pi_i(p_i - c). \tag{2}$$

2.1 Bounded User Utility Model

In an infrastructure sharing platform, the service coverage or value is bounded (e.g., by 100% citywide), no matter how many users participate. Thus, user utility function is bounded in this model. For modelling bounded $\phi(m)$, take WiFi sharing in a finite region of a normalized unit square surface for example. n users are randomly distributed in the square and each user can cover a circle of radius r ($0 < r << 1$) or an area πr^2. The total coverage depends on the total user number m. For an arbitrary point in the square surface, the probability that it is not covered by a single user is $\rho = 1 - \pi r^2$ and the probability that it is not covered by the m users is ρ^m. That is,

$$\phi(m) = 1 - \rho^m,$$

which is bounded by 1 and is concavely increasing in m. We can rewrite user i's payoff (1) as follows,

$$u_i = \pi_i(\theta_i(1 - \rho^{\sum_{j \in N} \pi_j}) - p_i). \tag{3}$$

In Section 3, we will focus on this bounded utility model and analyse the optimal pricing schemes under complete and incomplete information.

2.2 Unbounded User Utility Model

In an online social media, user utility increases super-linearly with the number of users, following Metcalfe's or Zipf's laws. Metcalfe's law suggests that a user will get equal benefits from the other $m - 1$ participants. The user's utility is proportional to m and when m is sufficiently large, $\phi(m) \approx m$ [2]. Zipf's law suggests that a user will benefit from the others differently, in inverse proportion to the frequency with which he interacts with(i.e., frequency $1/i$ with the i-th closest user among m users). Then $\phi(m) = \sum_{i=1}^{m-1}(1/m) \approx \log m$ [2]. As a result, user i's payoff (1) becomes,

$$u_i = \begin{cases} \pi_i(\theta_i \log(\sum_j \pi_j) - p_i), & \text{if Zipf's law;} \tag{4} \\ \\ \pi_i(\theta_i(\sum_j \pi_j) - p_i), & \text{if Metcalfe's law.} \tag{5} \end{cases}$$

Due to page limit, we skip most proofs in the 6-page version. The full version including all the proofs can be found in our online technical report in [9].

3 OPTIMAL PRICING FOR BOUNDED UTILITY MODEL

In this section, we will analyse the platform's pricing strategy for bounded user utility $\phi(m) = 1 - \rho^m$.

3.1 Pricing under Complete Information

Under complete information about all users' valuations θ_i's, the platform's optimization problem is to choose prices p_i's and control admission π_i's to maximize its profit. The payoff of a participant in (3) cannot be negative, otherwise he will choose not to participate. Formally, the problem is

$$\max_{\{(\pi_i, p_i), i \in N\}} \sum_{i \in N} \pi_i(p_i - c)$$

$$\text{s. t. } \pi_i\left(\theta_i(1 - \rho^{\sum_j \pi_j}) - p_i\right) \geq 0, \quad \text{for all } i \in N. \tag{6}$$

At optimality, the constraints in problem (6) are tight. For any user with $\pi_i = 1$ or 0, it is optimal to leave a zero payoff to him by setting the price to be

$$p_i^*(\pi) = \theta_i(1 - \rho^{\sum_j \pi_j}).$$

This result helps simplify problem (6) to

$$\max_{\{\pi_i, \ i \in N\}} \sum_{i \in N} \pi_i\left(\theta_i(1 - \rho^{\sum_j \pi_j}) - c\right). \tag{7}$$

To help solve this problem, we start with a lemma about the platform's preference among users.

LEMMA 1. *At the optimality of problem* (7), *for any two users* $i, j \in N$ *with* $\theta_i > \theta_j$, *if user j is included in the platform (i.e., $\pi_j = 1$), then user i should also be included* ($\pi_i = 1$).

It follows from Lemma 1 that the platform will select m users with the largest service valuations and problem (7) reduces to

$$\max_{m \in N} \left((1 - \rho^m)\sum_{i=1}^{m} \theta_i - mc\right). \tag{8}$$

This problem's objective function is not a monotonic function of m and it is not possible to derive closed-form solution of m. Yet we can use the efficient one-dimensional search method to find the optimal m numerically.

3.2 Pricings under Incomplete Information

Under incomplete information, the platform does not know θ_i's exactly but their distributions. We assume θ_i's are independent and identically distributed on $[0, 1]$ with cumulative distribution function F. The cost is comparable and we have $c \in (0, 1)$. We will derive a optimal (differentiated) pricing scheme and then propose a uniform pricing scheme as approximation. We will compare these two different pricing schemes asymptotically.

3.2.1 Optimal/Differentiated Pricing Scheme. Under incomplete information, the platform will require each user i to declare his θ_i. Given the θ_i's (may or may not be truthful) declared by the users, the platform should choose p_i's and π_i's as functions of the θ_i's distribution to maximize its profit, i.e.,

$$\max_{\pi_i(\cdot), \ p_i(\cdot)} E_\theta\left(\sum_{i=1}^{n} \pi_i(\theta)(p_i(\theta) - c)\right) \tag{9}$$

subject to

$$E_{\theta_{-i}}\left(\pi_i(\theta_i, \theta_{-i})\left(\theta_i(1 - \rho^{\sum_j \pi_j(\theta_i, \theta_{-i})}) - p_i(\theta_i, \theta_{-i})\right)\right) \geq 0, \tag{10}$$

$$E_{\theta_{-i}}\left(\pi_i(\theta_i, \theta_{-i})\left(\theta_i(1 - \rho^{\sum_j \pi_j(\theta_i, \theta_{-i})}) - p_i(\theta_i, \theta_{-i})\right)\right)$$

$$\geq E_{\theta_{-i}}\left(\pi_i(\theta_i', \theta_{-i})\left(\theta_i(1 - \rho^{\sum_j \pi_j(\theta_i, \theta_{-i})}) - p_i(\theta_i', \theta_{-i})\right)\right), \tag{11}$$

for all i and θ_i',

where $\theta_{-i} = (\theta_1, \cdots, \theta_{i-1}, \theta_{i+1}, \cdots, \theta_n)$ is a vector consists of all the users' valuations except θ_i. Constraint (10) is to ensure individual rationality or participation, i.e., user i's expected payoff conditional on θ_{-i} is nonnegative, and constraint (11) is to ensure incentive compatibility, i.e., user i must declare his valuation truthfully.

Let us define three functions:

$$g(\theta_i) = \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}, \tag{12}$$

$$V_i(\theta_i) = \int \pi_i(\theta_i, \theta_{-i})(1 - \rho^{\sum_j \pi_j(\theta_i, \theta_{-i})})dF^{n-1}(\theta_{-i}), \tag{13}$$

$$P_i(\theta_i) = \int \pi_i(\theta_i, \theta_{-i})p_i(\theta_i, \theta_{-i})dF^{n-1}(\theta_{-i}). \tag{14}$$

Note that $\theta_i V_i(\theta_i)$ and $P_i(\theta_i)$ are the expected utility and expected payment of user i given his valuation report θ_i, respectively. Assume that g is a nondecreasing function as in the literature of mechanism design. Intuitively, $g(\theta_i)$ is less than θ_i to give users incentives to truthfully report their θ_i's in the incomplete information scenario. We let $g(\theta_{(i)})$ be the ith greatest among $g(\theta_1), \ldots, g(\theta_n)$, then we have $g(\theta_{(1)}) \geq \cdots \geq g(\theta_{(n)})$. The following lemma helps simplify the constraints in problem (9).

PROPOSITION 2 (NECESSARY AND SUFFICIENT FOR INCENTIVE COMPATIBILITY). $V_i(\theta_i)$ *is non-decreasing in θ_i, and the differentiated pricing $P_i(\theta_i)$ is given by,*

$$P_i(\theta_i) = \theta_i V_i(\theta_i) - \int_0^{\theta_i} V_i(\eta)d\eta. \tag{15}$$

As a result, the platform's maximal profit, denoted by Π_D, in (9) can be written as

$$\int \max_{m \in N} \left((1 - \rho^m)\sum_{i=1}^{m} g(\theta_{(i)}) - mc\right)dF^n(\theta). \tag{16}$$

The proof is given in [9]. Proposition 2 indicates that at the optimum, the platform will include m users whose $g(\theta_i)$'s are the greatest.

3.2.2 Uniform Pricing Scheme As Approximation. Although the differentiated pricing mechanism in (15) is optimal, it is complicated to compute and implement in practice. While it guarantees that truthful reporting is the best response for users , it is difficult for a user to check (11) for any θ_i' and θ_{-i}. Next, we propose a uniform pricing scheme which does not even require users to declare their θ's.

In this simple scheme, the platform announces a single price P to users without any admission control. As users are i.i.d. distributed, there is a common valuation threshold $\bar{\theta}$ for subscription decision-making and $\bar{\theta}$ depends on P. User i will decide subscription by comparing his θ_i to $\bar{\theta}$ and participates if $\theta_i \geq \bar{\theta}$. Approximately $m = n(1 - F(\bar{\theta}))$ users will finally subscribe and contribute to the network externalities.. User i's payoff in (3) becomes

$$u_i = \theta_i\left(1 - \rho^{n(1 - F(\bar{\theta}))}\right) - P \geq 0, \quad \text{for all } \theta_i \geq \bar{\theta}.$$

This should be zero for an indifferent user with $\theta_i = \bar{\theta}$. Thus,

$$P = \bar{\theta}\left(1 - \rho^{n(1 - F(\bar{\theta}))}\right), \tag{17}$$

which is a function of $\bar{\theta}$, or we can equivalently express $\bar{\theta}$ as a function of P. The platform's optimization problem is

$$\max_{\bar{\theta}} n(1 - F(\bar{\theta}))\bar{\theta}\left(1 - \rho^{n(1 - F(\bar{\theta}))}\right) - n(1 - F(\bar{\theta}))c. \tag{18}$$

Since each θ_i follows the uniform distribution on $[0, 1]$, problem (18) becomes

$$\max_{\bar{\theta}} n(1 - \bar{\theta})\bar{\theta}\left(1 - \rho^{n(1 - \bar{\theta})}\right) - n(1 - \bar{\theta})c \tag{19}$$

The uniform pricing problem (though non-convex) can be solved efficiently via an one-dimensional search. We next present the analytical results as $n \to \infty$ and characterize the network value/profit.

THEOREM 3. *Given users' bounded utility model in* (3), *as $n \to \infty$, the optimal uniform price under incomplete information is $P^* \to \frac{1+c}{2}$, the optimal user threshold is $\bar{\theta}^* \to \frac{1+c}{2}$ and the maximal profit is $\Pi_U \sim (\frac{1-c}{2})^2 n$. As $n \to \infty$, the maximum profit achieved by the differentiated pricing scheme in* (15) *is $\Pi_D \sim (\frac{1-c}{2})^2 n$. Thus, uniform pricing is asymptotically optimal, i.e., $\lim_{n\to\infty} \frac{\Pi_U}{\Pi_D} = 1$.*

The proof is given in [9]. Theorem 3 shows that uniform pricing scheme's profit grows at the same rate with n as the differentiated pricing scheme.

3.2.3 Price of Information. Now we are ready to compare the expected maximal profits under complete and incomplete information. We define price of information (PoI) as the ratio of the expected maximal profit under complete and incomplete information as $n \to \infty$, i.e.,

$$\text{PoI} = \lim_{n\to\infty} \frac{E_\theta(\Pi)}{\Pi_U}. \tag{20}$$

PoI is of course similar in concept to the well-known idea of Price of Anarchy. However, the second refers to the social welfare that is lost when users act self-interestedly vis-a-vis for the community. Note that price discounts are given as incentives under incomplete information, and the profit is greater under complete information. Thus, PoI ≥ 1. One can also replace Π_U by Π_D in (20) without changing the PoI value, according to the uniform pricing's asymptotic optimality in Theorem 3.

PROPOSITION 4. *Given users' bounded utility model in* (3), *the price of information is PoI = 2.*

The proof is given in [9]. We note that PoI does not depend on parameter ρ. Recall that ρ tells the service coverage contributed by an individual user. As n goes to infinity, the total bounded coverage is fixed to 100% , and hence ρ has no impact on PoI .

4   OPTIMAL PRICING FOR UNBOUNDED UTILITY MODEL

In this section, we will analyse the platform's pricing strategy for unbounded user utility $\phi(m) = \log m$.

4.1 Pricing under Complete Information

Assume user's utility is given by (4), which follows from Zipf's law. Similar to Section 3.1, it is optimal to leave a zero payoff to user i by setting the price to be

$$p_i^*(\pi) = \theta_i \log(\sum_i \pi_i).$$

The platform's optimization problem is

$$\max_{\{\pi_i,\ i\in N\}} \sum_{i\in N} \pi_i \left(\theta_i \log(\sum_{j\in N} \pi_j) - c \right). \tag{21}$$

Lemma 1 still holds here, the problem reduces to

$$\max_{m\in N} \left(\log m \sum_{i=1}^m \theta_i - mc \right). \tag{22}$$

Thus, similarly, the platform will select m users with the largest service valuations and we can use one-dimensional search to find the optimal m.

4.2 Pricings under Incomplete Information

Inherit the same logic from Section 3.2, We will derive a optimal (differentiated) pricing scheme and then propose a uniform pricing scheme as approximation. We will compare these two different pricing schemes asymptotically.

4.2.1 Optimal/Differentiated Pricing Scheme. The platform's optimization problem in differentiated pricing scheme is the same as (9)-(11) except service value $(1 - \rho^{\sum_j \pi_j(\theta_i,\theta_{-i})})$ is replaced by $\log(\sum_{j=1}^n \pi_j(\theta_i, \theta_{-i}))$. We can similarly define $g(\theta_i)$, $V_i(\theta_i)$, and $P(\theta_i)$ as in (12), (13), and (14). Then Proposition 2 still holds here, and similar to (16), the platform's maximal profit can be written as

$$\int \max_{m\in N} \left(\log m \sum_{i=1}^m g(\theta_{(i)}) - mc \right) dF^n(\theta). \tag{23}$$

This indicates that at the optimum, the platform will include m users whose $g(\theta_i)$'s are the greatest.

4.2.2 Uniform Pricing Scheme as Approximation. Now we analyse the uniform pricing mechanism. Similar to Section 3.2.2, the payoff should be zero for an indifferent user with $\theta_i = \bar{\theta}$. Thus, similar to (17), we have

$$P = \bar{\theta} \log \left(n(1 - F(\bar{\theta})), \right.$$

and the platform's optimization problem is

$$\max_{\bar{\theta}} \bar{\theta}(1-\bar{\theta})n \log(n(1-\bar{\theta})) - n(1-\bar{\theta})c. \tag{24}$$

The uniform pricing problem (though non-convex) can be solved efficiently via an one-dimensional search. We next present the analytical results as $n \to \infty$ and characterize the network value/profit.

THEOREM 5. *Given users' unbounded utility model in* (4), *as $n \to \infty$, the optimal uniform price under incomplete information is $P^* \to \frac{1}{2} \log \frac{n}{2}$, the optimal user threshold is $\bar{\theta}^* \to \frac{1}{2}$ and the maximal profit is $\Pi_U \sim \frac{n}{4} \log(\frac{n}{2})$. As $n \to \infty$, the maximum profit achieved by the differentiated pricing scheme is $\Pi_D \sim \frac{n}{4} \log(\frac{n}{2})$. Therefore, uniform pricing is asymptotically optimal, i.e., $\lim_{n\to\infty} \frac{\Pi_U}{\Pi_D} = 1$.*

The proof is given in [9]. Note that the cost c does not play a role in the optimal price or maximal profit. This is because when utility is unbounded, as $n \to \infty$, the user's perceived network value grows super-linearly with the number of participants, while the cost only grows linearly and is negligible.

We next also consider Metcalfe's law rather than Zipf's law and $\phi(m) = m$ as a less concave function than $log(m)$. Then user i's payoff is now given by (5) and we can prove similar results as Theorem 5 below. The proof is given in [9].

COROLLARY 6. *Given users' unbounded utility model in* (5), *as $n \to \infty$, the optimal uniform price under incomplete information is $P^* \to (2/9)n$, the optimal user threshold $\bar{\theta}^* = 1/3$, and the maximal profit is $\Pi_U \sim (4/27)n^2$. As $n \to \infty$, the maximum profit achieved by the differentiated pricing scheme is $\Pi_D \sim (4/27)n^2$. Therefore, uniform pricing is asymptotically optimal, i.e., $\lim_{n\to\infty} \frac{\Pi_U}{\Pi_D} = 1$.*

4.2.3 Price of Information. We can still define price of information by (20). We more generally consider users' payoff function (not limited to (4) and (5)) as follows,

$$u_i = \theta_i v(m) - p_i, \tag{25}$$

where $v(m)$ is an unbounded, increasing and concave function with $v(0) = 0$. Then we have the following proposition.

Proposition 7. *Given users' general unbounded utility model in* (25), *the price of information is* $PoI \in [2, 27/8]$. *More specifically, if users' utility model follows Zipf's law in* (4), $PoI = 2$. *If users' utility model follows Metcalfe's law in* (5), $PoI = 27/8$.

The proof is given in [9]. As the utility function becomes more concave (from m in Metcalfe's law to $log(m)$ in Zipf's law), the profit loss due to lack of information decreases since the network externality decreases and there is less consumer surplus to be transformed to platform's profit. This holds true for a general cumulative distribution function F.

5 PRICING EXTENSION TO A TWO-SIDED MARKET

In this section, we include another group/type of users to the platform, who are simply consumers and do not contribute to the network externalities . Denote the set of original users (both contributors and consumers) as $N_1 = \{1, 2, \cdots, n_1\}$, and the new user set by $N_2 = \{n_1 + 1, \cdots, n_1 + n_2\}$. Within each set, we reorder users according to their service valuations such that $\theta_1 > \cdots > \theta_{n_1}$ and $\theta_{n_1+1} > \cdots > \theta n_1 + n_2$. Note that to which set a user belongs is public information as it is easy to verify whether a user can contribute or not. However, within each set, users' service valuations are still private information. As the two user sets' subscriptions affect each other, we wonder how the platform should jointly decide pricing schemes to the two sets of users. We also wonder if we can still approximate the two user groups' differentiated pricing via two uniform prices to achieve asymptotic optimality. The pricing schemes considered in Sections 3 and 4 can be similarly applied to the two sets of users. However, the asymptotical analysis becomes challenging as dimension increases.

Without much loss of generality, we apply Metcalfe's law here, where $\phi(m) = m$ and m only counts the original users in N_1 who can contribute.

5.1 Pricing under Complete Information

Similar to Section 3.1, it is optimal to leave a zero payoff to user i of any user set by setting the price to be

$$p_i^*(\pi) = \theta_i \sum_{j \in N_1} \pi_j.$$

The platform's optimization problem is

$$\max_{\{\pi_i,\ i \in N\}} \sum_{i \in N} \pi_i \left(\theta_i \sum_{j \in N_1} \pi_j - c \right)$$

Similar to Lemma 1, at the optimality of problem (26), for any two users $i, j \in N_1$ or $i, j \in N_2$ with $\theta_i > \theta_j$, if user j is included (i.e., $\pi_j = 1$), then user i should also be included ($\pi_i = 1$). It follows that the platform will select m_1 users with the largest service valuations in N_1 and m_2 users with the largest service valuations in N_2 and problem (26) reduces to

$$\max_{m_1, m_2} m_1 \left(\sum_{i=1}^{m_1} \theta_i + \sum_{i=n_1+1}^{n_1+m_2} \theta_i \right) - (m_1 + m_2)c, \quad (26)$$

which is an extension of (22) for a single user set. We have the following theorem regarding the optimal solution to (26).

Proposition 8. *Let* $\bar{m}_2$ *be the largest user number* m_2 *such that* $n_1 \theta_{n_1+m_2} \geq c$. *Then if*

$$\left(\sum_{i=1}^{n_1} \theta_i + \sum_{i=n_1+1}^{n_1+\bar{m}_2} \theta_i \right) - \frac{n_1 + \bar{m}_2}{n_1} c > 0,$$

then the optimal solution to (26) *is* $m_1^* = n_1$ *and* $m_2^* = \bar{m}_2$. *Otherwise, the optimal solution to* (26) *is* $m_1^* = 0$ *and* $m_2^* = 0$.

The proof is given in [9]. It is optimal to either include all the potential contributors in the platform for the maximum network externality or include no users due to high cost. Note that if no user of the first set is selected, the network value is zero and the platform cannot attract any pure user from the other set.

5.2 Pricing under Incomplete information

5.2.1 Differentiated Pricing Scheme. Under incomplete information, the platform will require each user i of each type to declare his θ_i and then choose p_i's and π_i's to maximize its profit. We can similarly decide the differentiated pricing in (15) as Proposition 2 still applies, and the platform's optimization problem can be written as

$$\Pi = \int \max_{m_1, m_2} m_1 \left(\sum_{i=1}^{m_1} g(\theta_{(i)}) + \sum_{i=n_1+1}^{n_1+m_2} g(\theta_{(i)}) \right) - (m_1 + m_2)c \, dF^n(\theta).$$

5.2.2 Uniform Pricing Scheme as Approximation. Unlike the single user type case, in the two-sided market, the platform sets different uniform prices for different user types. The price for type-1 users (dual-role) is P_1 and and the price for type-2 user (pure consumers) is P_2. There is a unique threshold for each type of users: $\bar{\theta}_1$ for type-1 and $\bar{\theta}_2$ for type 2 . Assume θ_i is uniformly distributed in $[0, 1]$. Similar to (17), for a type-1 user $i \in N_1$ with $\theta_i = \bar{\theta}_1$, we have

$$\bar{\theta}_1(1 - \bar{\theta}_1)n_1 = P1.$$

and for a type 2 user user $i \in N_2$ with $\theta_i = \bar{\theta}_2$, we have

$$\bar{\theta}_2(1 - \bar{\theta}_1)n_1 = P2.$$

The platform's optimization problem can be written as

$$\max_{\bar{\theta}_1, \bar{\theta}_2 \in [0,1]} n_1(1 - \bar{\theta}_1)(\bar{\theta}_1(1 - \bar{\theta}_1)n_1)n_2 - c)$$
$$+ n_2(1 - \bar{\theta}_2)(\bar{\theta}_2(1 - \bar{\theta}_1)n_1 - c).$$

Assume $n_1/n_2 = k$ where k is a positive constant, when n_1 and n_2 or simply n go to infinity, we can compute the two optimal uniform prices under incomplete information, i.e., P_1^* and P_2^* and we can still prove that uniform pricing is asymptotically optimal, i.e., $\lim_{n \to \infty} \Pi_U/\Pi_D = 1$. The results and proofs are given in [9]. We note that when n_1/n_2 is small (less than $1/4$), the platform platform's profit comes mostly from the type-2 pure users and desires the maximum network externalities contributed by the type-1 users. Thus, it charges zero price to motivate all type-1 users to contribute to the network externalities. As n_1/n_2 increases, the fraction of potential contributors increases, the platform with larger network externalities can charge more from the pure users of type-2, while keeping more contributors of type-1 at a lower price. Thus, P_1^* decreases and P_2^* increases with n_1/n_2.

5.2.3 Price of Information. Similar to Section 3.2.3, we can define price of information by (20) and calculate it straightforwardly. The PoI results and proofs are given in [9]. We note that as n_1/n_2 increases, the fraction of potential contributors increases, the platform under incomplete information still needs to provide price discounts as incentives. As a result, the PoI or profit loss due to lack of information increases.
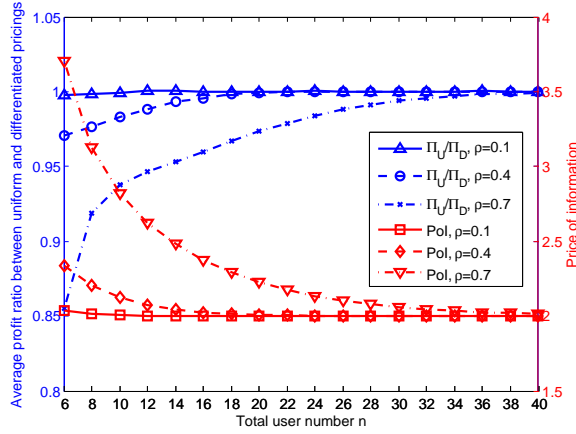
Figure 1: Average profit ratio between uniform and differentiated pricing and price of information for bounded utility model.

Figure 2: Average profit ratio between uniform and differentiated pricing and price of information for unbounded utility model.

6 SIMULATION RESULTS

We plot ratios of average profits under different pricing schemes in Figure 1. for bounded utility model and Figure 2. for unbounded utility model, by averaging 1 million sample data with different θ realizations.

Figure 1. shows that the average profit ratio between the uniform and differentiated pricing schemes increases with user number. This is consistent with Theorem 3, which shows uniform pricing scheme is asymptotically optimal as n goes to infinity. The convergence rate at which Π_U/Π_D approaches 1 decreases with ρ. As service coverage contributed by an individual user increases (ρ decreases), total service converges to 100% faster and hence uniform pricing scheme approaches optimality faster. Figure 1. also shows PoI as an decreasing function of n, approaches to 2 as in Proposition 4. PoI decreases with n since the information of users' valuation distribution helps pricing design of the platform more as n increases. The convergence rate at which PoI approaches 2 increases as ρ decreases is also due to the fact that total service converges to 100% faster as ρ decreases.

Figure 2. shows that the average profit ratio between the uniform and differentiated pricing mechanisms increases with user number. This is consistent with Theorem 5 and Corollary 6, which show uniform pricing scheme is asymptotically optimal as n goes to infinity. Logarithm utility model converges faster than linear utility model. This is because network externalities grow faster in linear utility model than logarithm utility model and hence uniform pricing cause greater profit loss in linear utility model than logarithm utility model. Figure 2. also shows PoI as an decreasing function of n approaches to 2 for logarithm utility model and 27/8 (=3.375) for linear utility model. This is consistent with Proposition 7.

7 CONCLUSION

This paper studies how a peer-to-peer sharing platform should price its service to maximize its profit. We consider both bounded and unbounded user utility models. For both bounded and unbounded user utility models, we analyze the optimal pricing schemes to select heterogeneous users in the platform under complete and incomplete information of users' service valuations. The profit loss due to lack of information becomes greater as the utility function becomes less concave. We show that the complicated differentiated
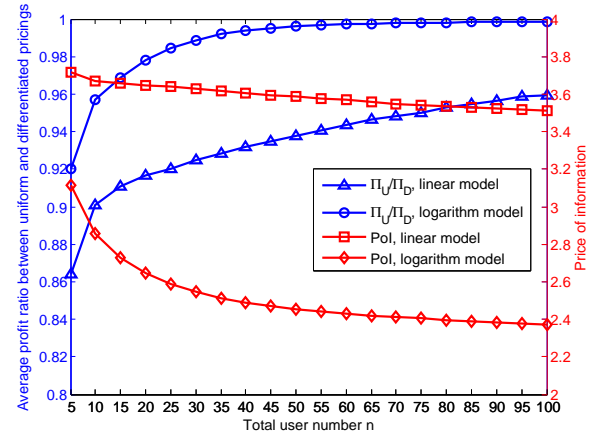
pricing scheme under incomplete information can be replaced by a single uniform price with asymptotic optimality. We also extend our pricing schemes to a two-sided market. Platform may charge zero price to the original group of users in order to attract the pure user group. Uniform pricing scheme is still asymptotically optimal as user number goes to infinity and price of information increases as the fraction of original users decreases.

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