Homework # 5
Due: April 11, 2011, 4:30 PM

Instructor: John C.S. Lui

Note: Deduction Policy for Late Submissions.

Late submissions must be handed to TA’s office (Room 120, SHB) by person. We will not handle any late submissions in the homework collection box in 10/F, SHB.

- Deduct 30% for one day late submission (within 24 hours).
- Deduct 60% for two days late submission (within 48 hours).
- Deduct 100% for more than two days late submission (after 48 hours).

1. Let $X$ be a random variable with probability density function

$$f(x) = \begin{cases} 
    c(1-x^2) & -1 < x < 1 \\
    0 & \text{otherwise}
\end{cases}$$

(a) What is the value of $c$?
(b) What is the cumulative distribution function of $X$?

2. A system consisting of one original unit plus a spare can function for a random amount of time $X$. If the density of $X$ is given (in units of months) by

$$f(x) = \begin{cases} 
    Cxe^{-x/2} & x > 0 \\
    0 & x \leq 0
\end{cases}$$

What is the probability that the system functions for at least 5 months?

3. A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = \begin{cases} 
    5(1-x)^4 & 0 < x < 1 \\
    0 & \text{otherwise}
\end{cases}$$

What must the capacity of the tank be so that the probability of the supply’s being exhausted in a given week is .01?

4. A bus travels between the two cities $A$ and $B$, which are 100 miles apart. If the bus has a breakdown, the distance from the breakdown to city $A$ has a uniform distribution over $(0,100)$. There is a bus service station in city $A$, in $B$, and in the center of the route between $A$ and $B$. It is suggested that it would be more efficient to have the three stations located 25, 50, and 75 miles, respectively, from $A$. Do you agree? Why?

5. Let $X$ be a normal random variable with mean 12 and variance 4. Find the value of $c$ such that $P\{X > c\} = .10.$
6. Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters \( \mu = 71 \) and \( \sigma^2 = 6.25 \). What percentage of 25-year-old men are over 6 feet, 2 inches tall? What percentage of men in the 6-footer club are over 6 feet, 5 inches?

7. A model for the movement of a stock supposes that if the present price of the stock is \( s \), then, after one period, it will be either \( us \) with probability \( p \) or \( ds \) with probability \( 1 - p \). Assuming that successive movements are independent, approximate the probability that the stock’s price will be up at least 30 percent after the next 1000 periods if \( u = 1.012 \), \( d = 0.990 \), and \( p = 0.52 \).