Homework # 4

Due: Mar. 24, 2011, 4:30 PM

Instructor: John C.S. Lui

Note: Deduction Policy for Late Submissions.
Late submissions must be handed to the office of TA (Room 120, SHB) by person. We will not handle any late submissions in the homework collection box in 10/F, SHB.

- Deduct 30% for one day late submission (within 24 hours).
- Deduct 60% for two days late submission (within 48 hours).
- Deduct 100% for more than two days late submission (after 48 hours).

1. Three dice are rolled. By assuming that each of the $6^3 = 216$ possible outcomes is equally likely, find the probabilities attached to the possible values that X can take on, where X is the sum of the 3 dice.
2. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X?
3. Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find $P\{X = i\}, i = 0, 1, 2, 3, 4$
4. Suppose that the distribution function of X is given by

$$y = \begin{cases} 0 & b < 0 \\ b/4 & 0 \le b < 1 \\ 1/2 + (b-1)/4 & 1 \le b < 2 \\ 11/12 & 2 \le b < 3 \\ 1 & 3 \le b \end{cases}$$

   (a) Find $P\{X = i\}, i = 1, 2, 3$
   (b) Find $P\{\frac{1}{2} < X < \frac{3}{2}\}$
5. Four independent flips of a fair coin are made. Let X denote the number of heads obtained. Plot the probability mass function of the random variable $X - 2$.
6. Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25 and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.

   (a) Which of $E[X]$ or $E[Y]$ do you think is larger? Why?
   (b) Compute $E[X]$ and $E[Y]$.
7. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.7. Assume that the results of the flips are independent, and let X equal the total number of heads that result.

   (a) Find $P\{X = 1\}$.
   (b) Determine $E[X]$.