1. Solution:
   (a) $B$ is a subset of $C$. Hence $B \cup C = C$.
   (b) Hence $B \cap C = B$.
   (c) $A \cup C = S$, the universal event.
   (d) $A \cap C = \text{null event}$.

2. Solution:
The sample space consists of 64 6-tuples of 0’s and 1’s. Enumerating these 6-tuples, we determine that the cardinality of event and corresponding probability as follows:
   (a) $|A| = 56$, $P(A) = \frac{56}{64} = \frac{7}{8}$,
   (b) $|B| = 41$, $P(B) = \frac{41}{64}$,
   (c) $|C| = 32$, $P(C) = \frac{1}{2}$,
   (d) $|A \cup B \cup C| = 58$, $P(A \cup B \cup C) = \frac{29}{32}$.

3. Solution:
   (a) If we can use the same digit again, we can construct $5 \times 2 = 10$ even digits.
   (b) If we cannot use the same digit, we can construct $3 \times 2 + 2 \times 1 = 8$ even digits.

4. Solution:
   When $k$ is different, we have different probability.
   (a) When $k = 0$, $P(1) = \frac{3^3}{10^4} = 0.125$.
   (b) When $k = 1$, $P(1) = \frac{3 \times 5 \times 5^2}{10^4} = 0.375$.
   (c) When $k = 2$, $P(1) = \frac{3 \times 5 \times 5^2}{10^4} = 0.375$.
   (d) When $k = 3$, $P(1) = \frac{5 \times 5^2}{10^4} = 0.125$.
   (BTW: here it is assumed that three-digit numbers include 0-999. You can also assume that they only include 100-999.)

5. Solution:
   $|S| = \binom{15}{5} = 455$, $|E| = \binom{9}{3} = 10$
   $P(E) = \frac{|E|}{|S|} = \frac{10}{455} = 2.1978 \times 10^{-2}$.

6. Solution:
The sample space is $S = \{(x_1, x_2, x_3, x_4, x_5) : x_i \in \{1, ..., 356\}\}$, $|S| = 356^5$
   Define event $E = \text{at least two have same birthday}$, $\bar{E} = \text{none have the same birthday}$.
   $|E| = P(365, 5) = \frac{365!}{360!}$
   $P(E) = 1 - P(\bar{E}) = 1 - \frac{365!}{360!365^5} = 0.0272$. 

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7. Solution:

The sample space is \( S = \{(i_1, i_2, ..., i_n) : i_j \in \{1, 2, ..., n\}\} \). Hence \(|S| = n^n\).

Let \( n^n \) possible arrangements be equally likely. Compute the probability that only one processor is empty. First we compute the probability that processor labelled 1 is idle. Let \( A_1 \) be the event of interest. Then \( n \) jobs are distributed such that none of the \( n - 1 \) processors are empty and hence all except one processor hold 1 job each.

Let \( B_j \) be the event that processor \( j \) has two jobs, \( B_j = \{(i_1, i_2, ..., i_n) : i_k \in \{2, ..., n\}, i_{k_1} = i_{k_2} = j\} \)

Then, \( A_1 = \bigcap_{j=2}^{n} B_j \), \( P(B_j) = \binom{n}{2} \left(\frac{1}{n^n}\right)^2 \).

\[ P(A_1) = \frac{(n-1)(n-2)!}{n^n} = \frac{n!}{n^2} \frac{n-1}{2} \]

Now the probability of exactly 1 processor idle \( = \bigcap_{i=1}^{n} A_i = \frac{n-1}{n^2} \left(\frac{n-1}{n}\right)^n \).

8. Solution:

The shortest path must of be length \(m + n\). In each step, we should choose go up or go left. So the number of shortest path is \( \binom{m+n}{n} \).