CSC6480 Homework 1: Static Games

Submission at class of March 6, 2008

Question 1

Consider an individual whose utility function of wealth, w, is given by $u(w) = 1 - e^{kw}$ where $k > 0$. Assume that wealth is distributed by a normal distribution $N(\mu, \sigma^2)$. Show

(a)Individual's expected utility can be represented as a trade-off between μ and σ^2.

(b)Classify the utility function.

Question 2 (Book Exercise 2.2)

Exercise 2.2

Show that the following behavioural and mixed strategies for the "nickel or dime" game of Example 2.1 all have the same payoff.

$$\beta = \left(\left(\frac{1}{2}, \frac{1}{2} \right), (1,0) \right)$$

$$\sigma = \frac{1}{2} ND + \left(\frac{1}{2} - x \right) DD + xDN \qquad \text{with} \qquad x \in \left[0, \frac{1}{2} \right]$$

Example 2.1

Suppose that the adult will offer the "nickel or dime" choice at most twice: if the girl takes the dime on the first occasion, then the choice will be offered only once. The nickel and dime problem can then be represented by the tree shown in Figure 2.1. If she chooses a dime (action D) at the first opportunity, then she receives ten cents and no further offer is made. On the other hand, if she chooses the nickel (action N), she gets five cents and a second choice. It is clear what the girl should do. If she chooses the nickel the first time and then the dime, she gets a payoff of fifteen cents; if she follows any other course of action, she gets only ten cents. Therefore, she should choose the nickel first and then the dime.

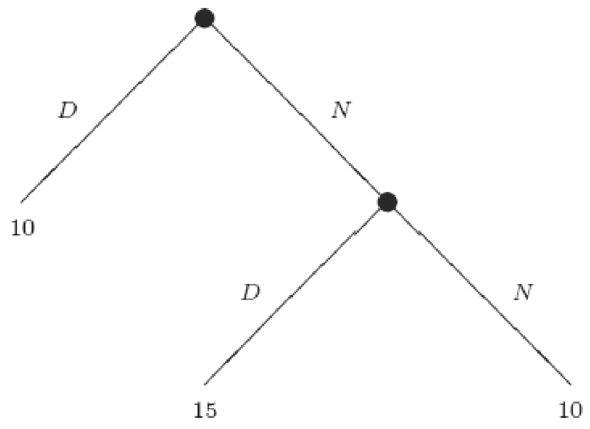

Figure 2.1 Choosing a nickel (N) or a dime (D) on (at most) two occasions. The payoff in cents is given at the end of each branch of the tree.

Question 3 (Book Exercise 2.3)

Exercise 2.3

Consider the decision tree shown in Figure 2.3. Find the all behavioural strategy equivalents for the mixed strategies (a) $\sigma = \frac{1}{2}a_1b_1c_1 + \frac{1}{2}a_2b_2c_2$ and (b) $\sigma = \frac{1}{3}a_1b_1c_1 + \frac{1}{3}a_1b_2c_1 + \frac{1}{3}a_1b_1c_2$.

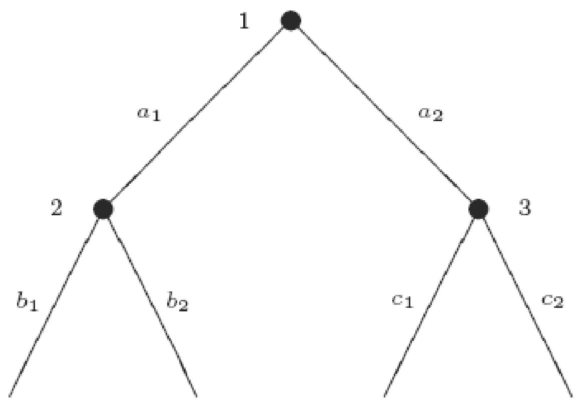

Figure 2.3 Decision tree for Exercise 2.3.

Question 4 (Book Exercise 2.4)

Exercise 2.4

Consider a female bird choosing a mate from three displaying males. The attributes of the males are summarised by the following table.

Male	Genetic quality	Cares for chicks?
1	High	No
2	Medium	Yes
3	Low	Yes

Suppose that the value of offspring depends on the genetic quality of the father. The value of offspring is v_H, v_M, and v_L for the males of high, medium, and low quality, respectively, with $v_H > v_M > v_L$. Once she has mated, the female can choose to care for the chicks or desert them. Chicks that are cared for by both parents will certainly survive; those cared for by only one parent (of either sex) have a 50% chance of survival; and those deserted by both parents will certainly die. Draw the decision tree and find the female's optimal strategy.

Question 5 (Book Exercise 4.1)

Exercise 4.1

In Puccini's opera *Tosca*, Tosca's lover has been condemned to death. The police chief, Scarpia, offers to fake the execution if Tosca will sleep with him. The bargain is struck. However, in order to keep her honour, Tosca stabs and kills Scarpia. Unfortunately, Scarpia has also reneged on the deal and Tosca's lover has been executed. Construct a game theoretic representation of this operatic plot.

Question 6 (Book Exercise 4.2)

Exercise 4.2

Solve the following abstract games using the (iterated) elimination of dominated strategies. For the second game, does the solution depend on the order of elimination?

(a) P_1 (rows), P_2 (columns)

	L	R
U	$3, 0$	$2, 1$
D	$2, 1$	$1, 0$

(b) P_1 (rows), P_2 (columns)

	L	R
U	$0, 3$	$10, 2$
C	$10, 4$	$0, 0$
D	$3, 1$	$3, 1$

Question 7 (Book Exercise 4.3)

Exercise 4.3

Consider the following game. Show that (D, L) and (U, M) are Nash equilibria.

P_1 (rows), P_2 (columns)

	L	M	R
U	$10, 0$	$5, 1$	$4, -2$
D	$10, 1$	$5, 0$	$1, -1$

Question 8 (Book Exercise 4.4)

Exercise 4.4

Find all the Nash equilibria of the following games.

(a) P_1 (rows), P_2 (columns)

	L	R
U	$4, 3$	$2, 2$
D	$2, 2$	$1, 1$

(b) P_1 (rows), P_2 (columns)

	R	W
F	$0, 0$	$2, 1$
M	$1, 2$	$0, 0$

We can often simplify the process of finding Nash equilibria by making use of the next two theorems. The first of these theorems makes it easy to find pure-strategy Nash equilibria.

Theorem 4.23

Suppose there exists a pair of pure strategies (s_1^*, s_2^*) such that

$$\pi_1(s_1^*, s_2^*) \geq \pi_1(s_1, s_2^*) \quad \forall s_1 \in \mathbf{S}_1$$
$$\text{and} \quad \pi_2(s_1^*, s_2^*) \geq \pi_2(s_1^*, s_2) \quad \forall s_2 \in \mathbf{S}_2 .$$

Then (s_1^*, s_2^*) is a Nash equilibrium.

Theorem 4.27 (Equality of Payoffs)

Let (σ_1^*, σ_2^*) be a Nash equilibrium, and let $\mathbf{S}_1^*$ be the support of σ_1^*. Then $\pi_1(s, \sigma_2^*) = \pi_1(\sigma_1^*, \sigma_2^*) \quad \forall s \in \mathbf{S}_1^*$.

Question 9 (Book Exercise 4.5)

Exercise 4.5

Find the pure strategy Nash equilibria for the following game.

P_2

P_1		L	M	R
	U	$4, 3$	$2, 7$	$0, 4$
	D	$5, 5$	$5, -1$	$-4, -2$

Question 10 (Book Exercise 4.7)

Exercise 4.7

Consider the children's game "Rock-Scissors-Paper", where 2 children simultaneously make a hand sign corresponding to one of the three items. Playing "Rock" (R) beats "Scissors" (S),"Scissors" beats "Paper" (P), and "Paper" beats "Rock". When both children play the same action (both R, both S, or both P) the game is drawn. (a) Construct a payoff table for this game with a payoff of $+1$ for a win, -1 for losing, and 0 for a draw. (b) Solve this game.

Question 11

Consider the following game with 2 players:

	P_2 (A)	P_2 (B)
P_1 (A)	a, a	b, c
P_1 (B)	c, b	d, d

Show that such a game has at least one symmetric Nash Equilibrium.

Question 12 (Book Exercise 4.10)

Exercise 4.10

Find all the Nash equilibria for the following non-generic games. Draw the best response graphs for the first game.

(a)

P_1 \ P_2	C	D
A	$6, 0$	$5, 3$
B	$6, 1$	$0, 0$

(b)

P_1 \ P_2	B	F	H
G	$5, 0$	$-1, 1$	$2, 0$
J	$5, 3$	$-2, 3$	$2, 3$

Question 13 (Book Exercise 4.14)

Exercise 4.14

Represent the game from Example 4.45 by a pair of payoff tables "chosen" by player 2. Confirm that the game has the same Nash equilibrium when represented in this way. [Hint: show that there are no pure strategy Nash equilibria, then use the Equality of Payoffs theorem to find a Nash equilibrium involving mixed strategies.]

Example 4.45

Consider a static three-player game where the first player chooses between U and D, the second player chooses between L and R, and the third player chooses between A and B. Instead of trying to draw a three-dimensional payoff table, we represent this game by a pair of payoff tables such as the ones shown in Figure 4.2. (We can interpret this as player 3 choosing the game that players 1 and 2 have to play, so long as we remember that players 1 and 2 do not know which of the payoff tables player 3 has chosen.) We can find a Nash equilibrium for the game with the payoffs shown in Figure 4.2 as follows. First, suppose that player 3 chooses A. Then the best responses for players 1 and 2 are the strategies $\hat{\sigma}_1 = \hat{\sigma}_2 = (0, 1)$. However, we do not have a Nash equilibrium because choosing A is not player 3's best response to this pair of strategies. Now suppose that player 3 chooses B. Then the best responses for players 1 and 2 are the strategies $\hat{\sigma}_1 = \hat{\sigma}_2 = (\frac{1}{2}, \frac{1}{2})$. Because player 3 would get a payoff of $\frac{3}{2}$ if he switches to A, we have a Nash equilibrium $(\sigma_1^*, \sigma_2^*, \sigma_3^*)$ with

$$\sigma_1^* = (\frac{1}{2}, \frac{1}{2}) \qquad \sigma_2^* = (\frac{1}{2}, \frac{1}{2}) \qquad \sigma_3^* = (0, 1)$$

		P_2	
	A	L	R
P_1	U	$1, 1, 0$	$2, 2, 3$
	D	$2, 2, 3$	$3, 3, 0$

		P_2	
	B	L	R
P_1	U	$-1, -1, 2$	$2, 0, 2$
	D	$0, 2, 2$	$1, 1, 2$

Figure 4.2 A representation of the three player game from example 4.45.