High Performance Dummy Fill Insertion with Coupling and Uniformity Constraints

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Outline

- Introduction
- Problem Formulation
- Algorithms
- Experimental Results
- Conclusion
Chemical Mechanical Polishing (CMP)

Example of CMP [source: www.ntu.edu.sg]
Uniformity

Layout uniformity for CMP

Uneven density distribution

Even distribution

Coupling capacitance
Related Works

- Minimize density variation and number of fills
  - Linear Programming (LP)
    - [Kahng+, TCAD’99]
    - [Tian+, TCAD’01]
    - [Xiang+, TCAD’08]
  - Monte Carlo and heuristic approaches
    - [Chen+, ASPDAC’00]
    - [Chen+, DAC’00]
    - [Wong+, ISQED’05]

- Minimize density variation with coupling capacitance constraints
  - ILP
    - [Chen+, DAC’03], [Xiang+, ISPD’07]
Holistic Metrics for Uniformity

- Holistic metrics for layout uniformity from IBM (ICCAD 2014 Contest)
  - Variation (standard deviation)
  - Line hotspots
  - Outlier hotspots
Holistic Metrics for Uniformity

- Holistic metrics for layout uniformity from IBM (ICCAD 2014 Contest)
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Metrics for Coupling Capacitance

- Coupling capacitance
  - Minimize overlay between layers
Problem Formulation

Based on the ICCAD 2014 contest

- **Input**
  - Layout with fill insertion regions
  - Signal wire density information across each window

- **Quality score**
  - Overlay area (20%)
  - Variation/std. dev. (20%)
  - Line hotspot (20%)
  - Outlier hotspot (15%)
  - File size for dummy fill insertion (5%)

- **Normalization function**
  \[ f(x) = \max\left(0, 1 - \frac{x}{\beta}\right) \]

- **Overall score**
  - Quality score (80%)
  - Runtime (15%)
  - Memory usage (5%)

- **Output**
  - Dummy fill positions and dimensions with maximum quality score

The higher score, the better
Outline

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- **Algorithms**
- Experimental Results
- Conclusion
Step 1: Density Planning

- Given density ranges of each window
- Find target density $t_d$ for each window
- Maximize density scores

<table>
<thead>
<tr>
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<th>$d_3$</th>
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Linear scan with a small step to find best target density
## Step 1: Density Planning

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<tr>
<th>Window ID</th>
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<tr>
<td>3</td>
<td>0.6</td>
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<td>0.5</td>
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<tr>
<td>7</td>
<td>0.4</td>
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</table>

- Given density ranges of each window
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Linear scan with a small step to find best target density

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**Searching for Best Target Density $t_d$ (Case II)**

- Density Range
- $t_d$ Lower Bound
- $t_d$ Upper Bound

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12
Step 1: Density Planning

- Given density ranges of each window
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<table>
<thead>
<tr>
<th>Density Range</th>
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Linear scan with a small step to find best target density
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Linear scan with a small step to find best target density

![Graph showing searching for best target density $t_d$ (Case II)]
Step 2: Candidate Fill Generation

- Generate candidate fills with minimum overlay
- With the guidance of target density
- A final fill is a rectangle within a candidate fill
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- Generate candidate fills with minimum overlay
- With the guidance of target density
- A final fill is a rectangle within a candidate fill

\[ q = -\frac{\text{fill overlay}}{\text{fill area}} + \gamma \cdot \frac{\text{fill area}}{\text{window area}} \]
Step 3: Dummy Fill Insertion (1)

- Given a set of candidate fills
- Determine dimension of fills
- Under DRC constraints
- Minimize overlay area and density variation

Solve in an iterative manner
Step 3: Dummy Fill Insertion (2)

An example of the mathematical formulation in one iteration

\[
\text{min } |(x_2 - x_1) \cdot h_A + (x_4 - x_3) \cdot h_B - t_{d1} \cdot A_{\text{win}}| + |(x_6 - x_5) \cdot h_C - t_{d2} \cdot A_{\text{win}}| \\
+ (x_2 - x_5) \cdot h_{AC} + (x_6 - x_3) \cdot h_{BC}
\]

s.t. 
\[
\begin{align*}
  x_2 - x_1 & \geq W_{\text{min}} \\
  x_4 - x_3 & \geq W_{\text{min}} \\
  x_6 - x_5 & \geq W_{\text{min}} \\
  x_3 - x_2 & \geq S_{\text{min}} \\
  (x_2 - x_1) \cdot h_A & \geq A_{\text{min}} \\
  (x_4 - x_3) \cdot h_B & \geq A_{\text{min}} \\
  (x_6 - x_5) \cdot h_C & \geq A_{\text{min}} \\
  x_2 - x_5 & \geq 0 \\
  x_6 - x_3 & \geq 0 \\
  l_i & \leq x_i \leq u_i, \quad i = 1, 2, \ldots, 6
\end{align*}
\]

Overview constraints

- Density variation
- Overlay area
- DRC rules

\(A_{\text{win}}\): area of a window
\(W_{\text{min}}\): minimum width
\(S_{\text{min}}\): minimum spacing
\(A_{\text{min}}\): minimum area
Step 3: Dummy Fill Insertion (3)

\[
\min \left| (x_2 - x_1) \cdot h_A + (x_4 - x_3) \cdot h_B - t_{d1} \cdot A_{\text{win}} \right| + \left| (x_6 - x_5) \cdot h_C - t_{d2} \cdot A_{\text{win}} \right| \\
+ (x_2 - x_5) \cdot h_{AC} + (x_6 - x_3) \cdot h_{BC}
\]

- Further relax to remove absolute operation
- Add tighter bound constraints to variables
Step 3: Dual to Min-Cost Flow

Prime

\[
\min_{x_i} \sum_{i=1}^{N} c_i x_i \\
\text{s.t. } x_i - x_j \geq b_{ij}, \ (i, j) \in E, \\
l_i \leq x_i \leq u_i, \ i = 1, 2, \ldots, N, \\
x_i \in Z
\]

- Convert bound constraints to differential constraints
- Dual to min-cost flow

Dual

\[
c'_i = \begin{cases} 
  c_i & i = 1, 2, \ldots, N \\
  - \sum_{i=1}^{N} c_i & i = 0 
\end{cases}
\]

\[
b'_{ij} = \begin{cases} 
  b_{ij} & (i, j) \in E \\
  l_i & i = 1, 2, \ldots, N, j = 0 \\
  -u_i & i = 0, j = 1, 2, \ldots, N 
\end{cases}
\]
Overall Flow

Initial Fill Regions → Density Planning

Density Planning → Candidate Fill Generation

Candidate Fill Generation → Density Planning

Density Planning → Dummy Fill Insertion

Dummy Fill Insertion → Output Fills
Experimental Environment

- Implemented in C++
- 8-Core 3.4GHz Linux server
- 32GB RAM
- ICCAD 2014 contest benchmarks
Experimental Results

- Compared with contest winners
  - Quality Scores (13% better than the 1st place winner)
  - Overall Scores (10% better than the 1st place winner)
### Experimental Results

#### Detailed results

<table>
<thead>
<tr>
<th>Design</th>
<th>Team</th>
<th>Overlay*</th>
<th>Variation*</th>
<th>Line*</th>
<th>Outlier*</th>
<th>Size*</th>
<th>Run-time*</th>
<th>Memory*</th>
<th>Testcase Quality</th>
<th>Testcase Score</th>
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<tr>
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<td>0.743</td>
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<td>0.948</td>
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<td><strong>0.439</strong></td>
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Conclusion

- Methodology for fill optimization with holistic and multiple objectives
- Validated on industry benchmarks
  - ICCAD 2014 contest benchmark
- Future work
  - Lithography related impacts
Thank you!