



CENG 5030

Energy Efficient Computing

Mo04: Binary/Ternary Network

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2023 Fall

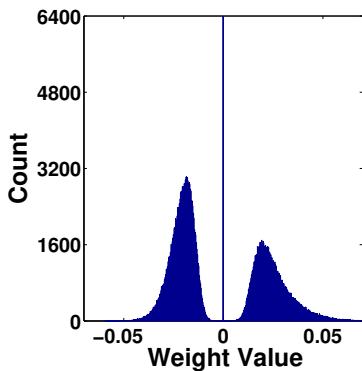


These slides contain/adapt materials developed by

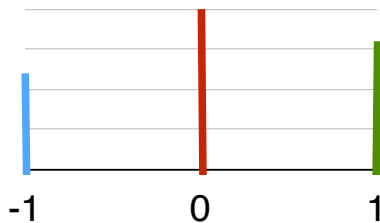
- Ritchie Zhao et al. (2017). “Accelerating binarized convolutional neural networks with software-programmable FPGAs”. In: *Proc. FPGA*, pp. 15–24
- Mohammad Rastegari et al. (2016). “XNOR-NET: Imagenet classification using binary convolutional neural networks”. In: *Proc. ECCV*, pp. 525–542



Binary / Ternary Net: Motivation



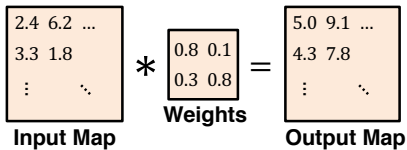
\Rightarrow





Binarized Neural Networks (BNN)

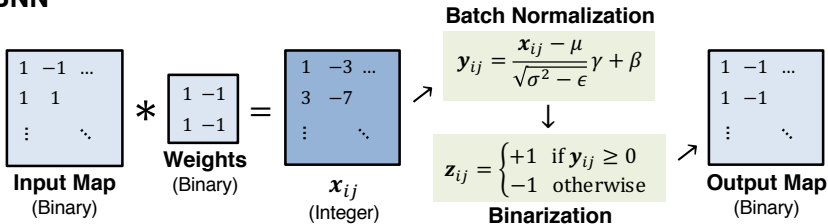
CNN



Key Differences

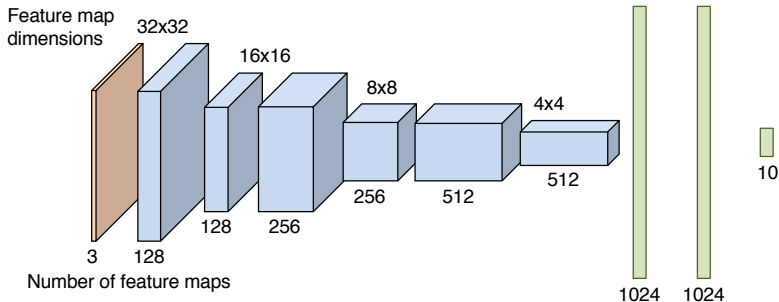
1. Inputs are binarized (-1 or +1)
2. Weights are binarized (-1 or +1)
3. Results are binarized after **batch normalization**

BNN





BNN CIFAR-10 Architecture [2]



- ▶ 6 conv layers, 3 dense layers, 3 max pooling layers
- ▶ All conv filters are 3x3
- ▶ First conv layer takes in floating-point input
- ▶ **13.4 Mbits total model size** (after hardware optimizations)

[2] M. Courbariaux et al. **Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1**. *arXiv:1602.02830*, Feb 2016.



Advantages of BNN

1. Floating point ops replaced with binary logic ops

b_1	b_2	$b_1 \times b_2$
+1	+1	+1
+1	-1	-1
-1	+1	-1
-1	-1	+1

b_1	b_2	$b_1 \text{ XOR } b_2$
0	0	0
0	1	1
1	0	1
1	1	0

- Encode $\{+1, -1\}$ as $\{0, 1\}$ \rightarrow multiplies become XORs
- Conv/dense layers do dot products \rightarrow XOR and popcount
- Operations can map to LUT fabric as opposed to DSPs

2. Binarized weights may reduce total model size

- Fewer bits per weight may be offset by having more weights



BNN vs CNN Parameter Efficiency

Architecture	Depth	Param Bits (Float)	Param Bits (Fixed-Point)	Error Rate (%)
ResNet [3] (CIFAR-10)	164	51.9M	13.0M*	11.26
BNN [2]	9	-	13.4M	11.40

* Assuming each float param can be quantized to 8-bit fixed-point

► Comparison:

- Conservative assumption: ResNet can use 8-bit weights
- BNN is based on VGG (less advanced architecture)
- BNN seems to hold promise!

[2] M. Courbariaux et al. **Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1**. *arXiv:1602.02830*, Feb 2016.

[3] K. He, X. Zhang, S. Ren, and J. Sun. **Identity Mappings in Deep Residual Networks**. *ECCV 2016*.





- 1 Minimize the Quantization Error
- 2 Reduce the Gradient Error



1 Minimize the Quantization Error

2 Reduce the Gradient Error




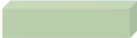
 * 	Operations	Memory	Computation
\mathbb{R} * \mathbb{R}	+ - ×	1x	1x

Binary Weight Networks

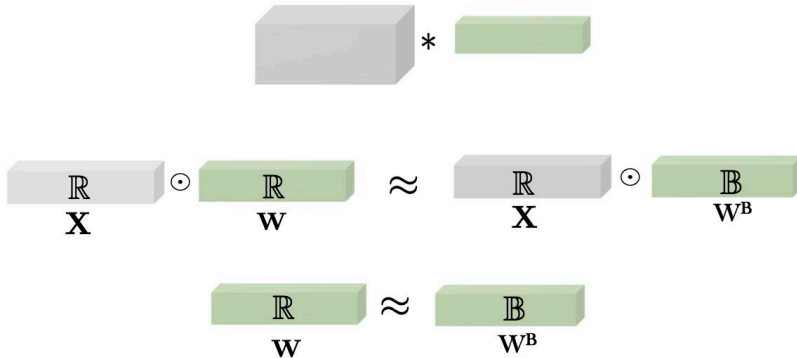
XNOR-Networks

¹Mohammad Rastegari et al. (2016). "XNOR-NET: Imagenet classification using binary convolutional neural networks". In: *Proc. ECCV*, pp. 525–542.



 * 	Operations	Memory	Computation
$\mathbb{R} * \mathbb{R}$	+ - x	1x	1x
$\mathbb{R} * \mathbb{B}$	+ -	~32x	~2x
$\mathbb{B} * \mathbb{B}$	XNOR Bit-count	~32x	~58x

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$$\text{W}^{\text{B}} = \text{sign}(\text{W})$$

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Quantization Error

$$W^B = \text{sign}(W)$$

$$\left\| \begin{array}{c} W \\ \mathbb{R} \end{array} - \begin{array}{c} W^B \\ \mathbb{B} \end{array} \right\| \approx 0.75$$



Optimal Scaling Factor

$$\begin{array}{c} \mathbb{R} \\ \mathbf{W} \end{array} \approx \alpha \begin{array}{c} \mathbb{B} \\ \mathbf{W}^{\mathbb{B}} \end{array}$$

$$\alpha^*, \mathbf{W}^{\mathbb{B}*} = \arg \min_{\mathbf{W}^{\mathbb{B}}, \alpha} \{ \|\mathbf{W} - \alpha \mathbf{W}^{\mathbb{B}}\|^2 \}$$

$$\begin{array}{l} \mathbf{W}^{\mathbb{B}*} = \text{sign}(\mathbf{W}) \\ \alpha^* = \frac{1}{n} \|\mathbf{W}\|_{\ell_1} \end{array}$$



How to train a CNN with binary filters?

$$\mathbb{R} * \mathbb{R} \approx (\mathbb{R} * \mathbb{B}) \alpha$$

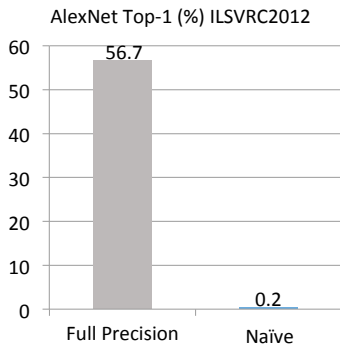
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Training Binary Weight Networks

Naive Solution:

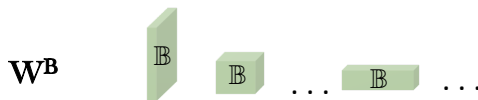
1. Train a network with real value parameters
2. Binarize the weight filters



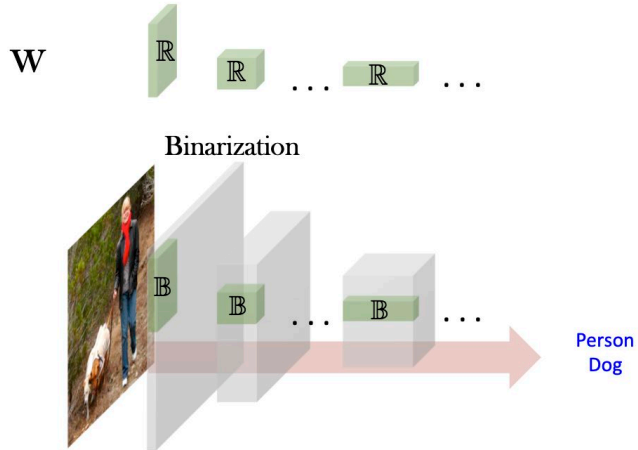
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Binarization



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Binary Weight Network

Train for binary weights:

1. Randomly initialize W
2. For $iter = 1$ to N
3. Load a random input image X
4. $W^B = \text{sign}(W)$
5. $\alpha = \frac{\|W\|_{l1}}{n}$
6. Forward pass with α, W^B
7. Compute loss function C
8. $\frac{\partial C}{\partial W} =$ Backward pass with α, W^B
9. Update W ($W = W - \frac{\partial C}{\partial W}$)



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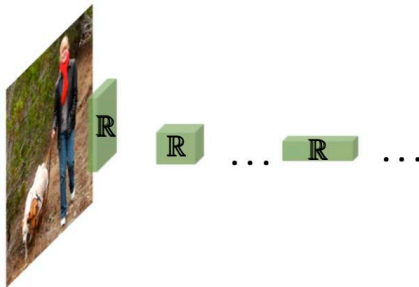


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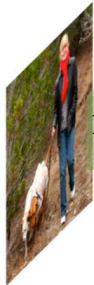




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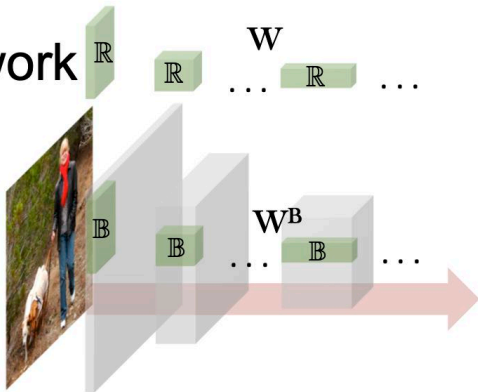




Binary Weight Network

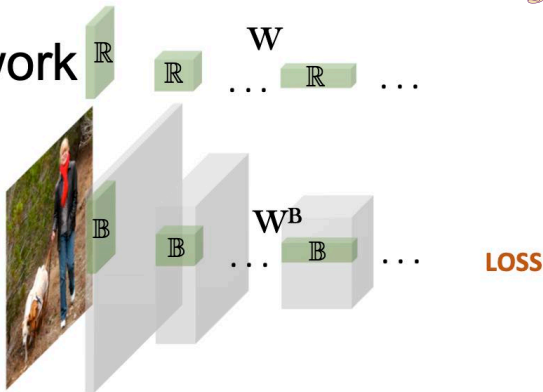
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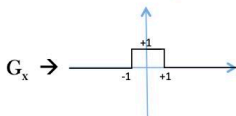
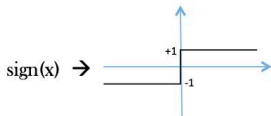
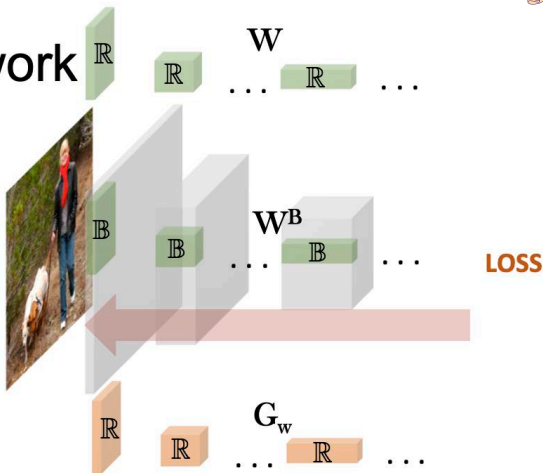
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[Hinton et al. 2012]

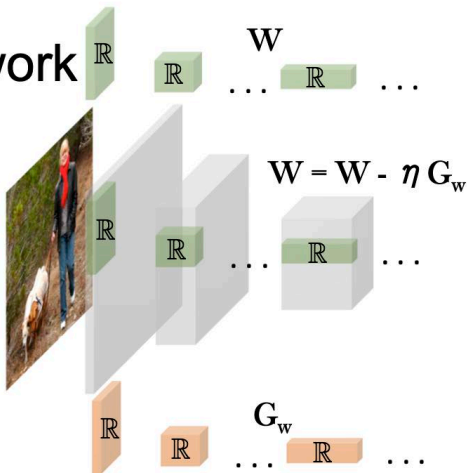
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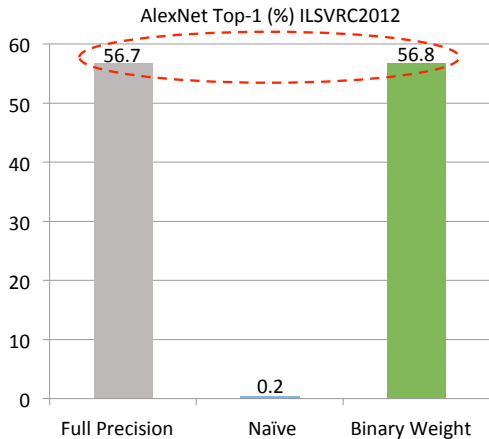


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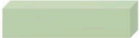
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$\mathbb{B} * \mathbb{B}$ XNOR-Networks	XNOR Bit-count	$\sim 32x$	$\sim 58x$

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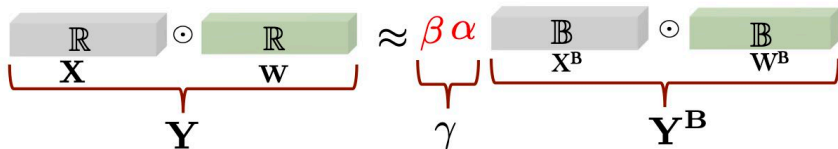


Binary Input and Binary Weight (XNOR-Net)

$$\begin{array}{c} \mathbb{R} \\ \mathbf{X} \end{array} \odot \begin{array}{c} \mathbb{R} \\ \mathbf{W} \end{array} \approx \beta \begin{array}{c} \mathbb{B} \\ \mathbf{X}^{\mathbb{B}} \end{array} \odot \alpha \begin{array}{c} \mathbb{B} \\ \mathbf{W}^{\mathbb{B}} \end{array}$$



Binary Input and Binary Weight (XNOR-Net)



$$\mathbf{Y} \approx \gamma \mathbf{Y}^B$$

$$\mathbf{Y}^{B^*}, \gamma^* = \arg \min_{\mathbf{Y}^B, \gamma} \|\mathbf{Y} - \gamma \mathbf{Y}^B\|^2$$

$$\mathbf{Y}^{B^*} = \text{sign}(\mathbf{Y}) \quad \gamma^* = \frac{1}{n} \|\mathbf{Y}\|_{\ell_1}$$

$$\mathbf{X}^{B^*} = \text{sign}(\mathbf{X}) \quad \mathbf{W}^{B^*} = \text{sign}(\mathbf{W})$$

$$\alpha^* = \frac{1}{n} \|\mathbf{W}\|_{\ell_1} \quad \beta^* = \frac{1}{n} \|\mathbf{X}\|_{\ell_1}$$

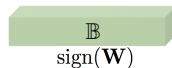
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(1) Binarizing Weights

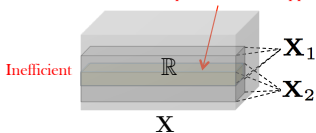


$$\frac{1}{n} \|\mathbf{W}\|_{\ell_1} = \alpha$$

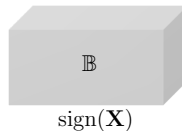


(2) Binarizing Input

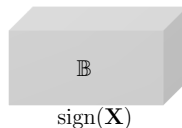
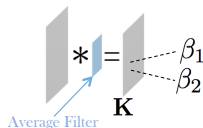
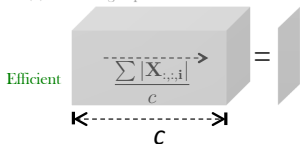
Redundant computation in overlapping areas



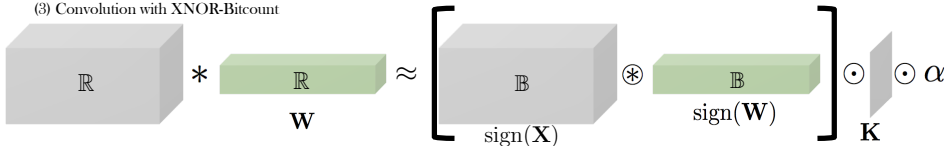
$$\frac{1}{n} \|\mathbf{X}_1\|_{\ell_1} = \beta_1$$
$$\frac{1}{n} \|\mathbf{X}_2\|_{\ell_1} = \beta_2$$



(2) Binarizing Input

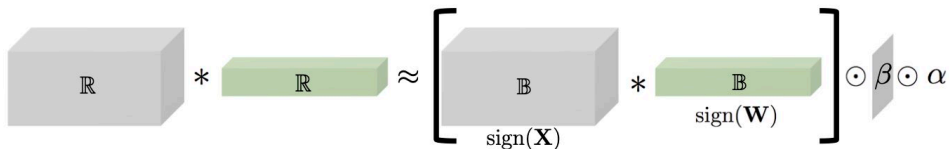


(3) Convolution with XNOR-Bitcount



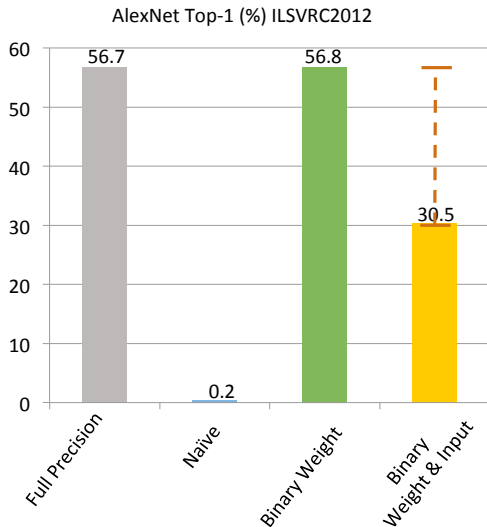
1

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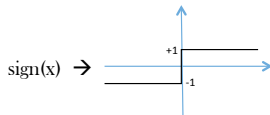
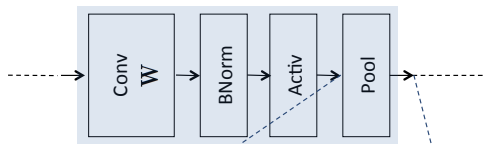


1

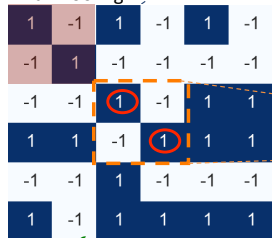
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Network Structure in XNOR-Networks



Max-Pooling



✓ Multiple Maximums

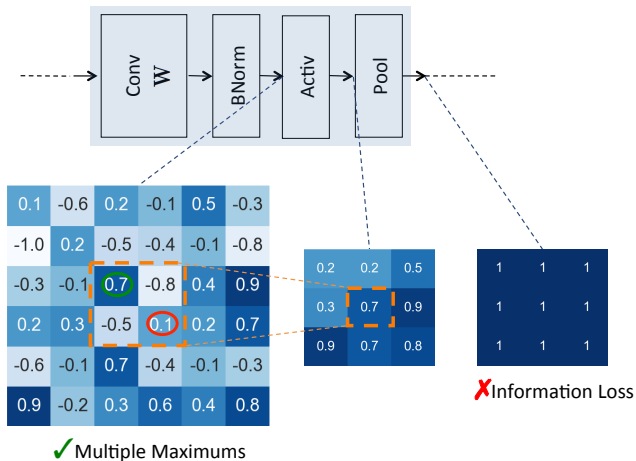


✗ Information Loss

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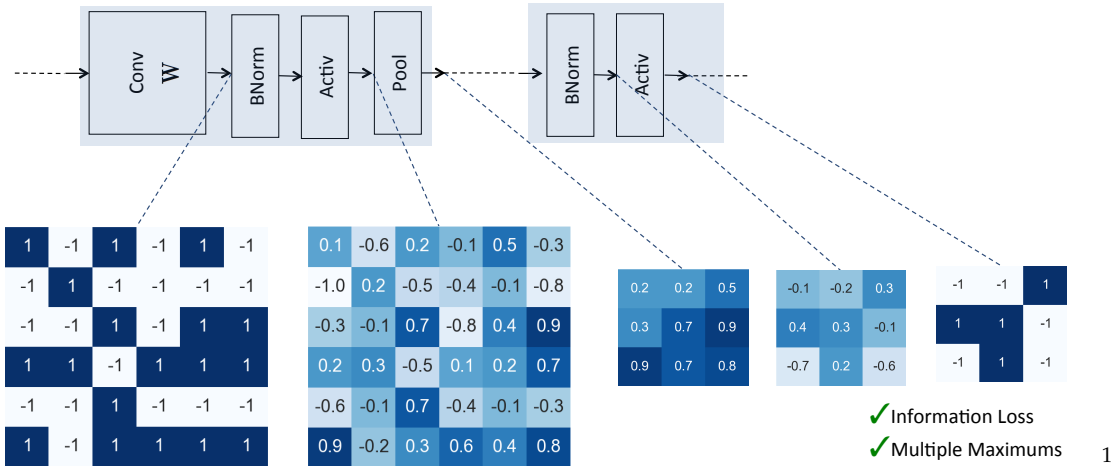
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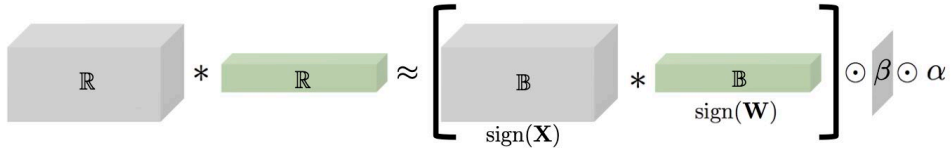
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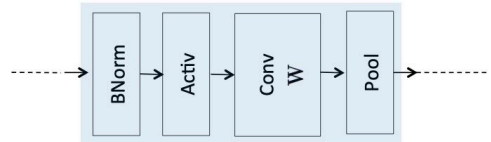
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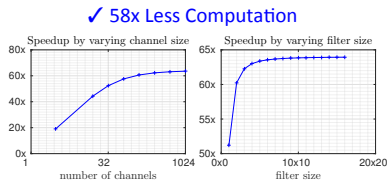
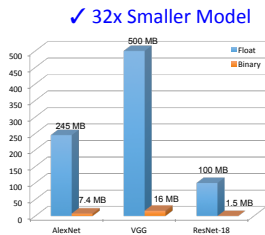
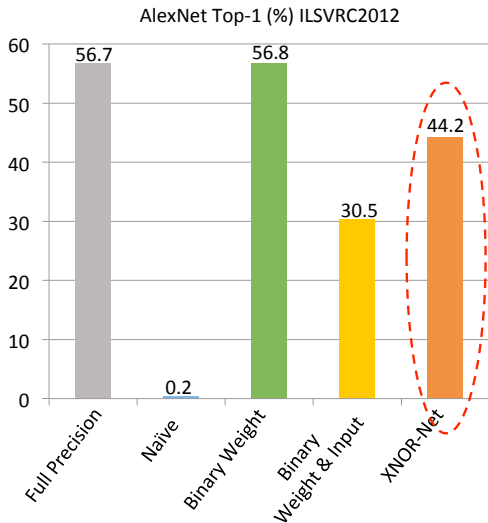


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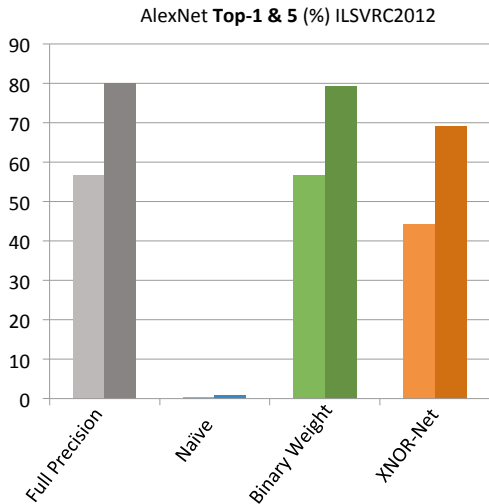


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Motivation

- Naive methods (Matthieu Courbariaux, Yoshua Bengio, and Jean-Pierre David (2015). “Binaryconnect: Training deep neural networks with binary weights during propagations”. In: *Advances in neural information processing systems*, pp. 3123–3131, Matthieu Courbariaux, Itay Hubara, et al. (2016). “Binarized neural networks: Training deep neural networks with weights and activations constrained to +1 or -1”. In: *arXiv preprint arXiv:1602.02830*) suffer the accuracy loss

Intuition

- Quantized parameter should approximate the full precision parameter as closely as possible



Towards Accurate Binary Convolutional Neural Network



Contribution

- Approximate full-precision weights with the linear combination of multiple binary weight bases
- Introduce multiple binary activations



Weights Binarization

- Weights tensors in one layer: $W \in \mathbb{R}^{w \times h \times c_{in} \times c_{out}}$

$$B_1, B_2, \dots, B_M \in \{-1, +1\}^{w \times h \times c_{in} \times c_{out}}$$

$$W \approx \alpha_1 B_1 + \alpha_2 B_2 + \dots + \alpha_M B_M$$

$$B_i = F_{u_i}(W) = \text{sign}(\bar{W} + u_i \text{std}(W)), i = 1, 2, \dots, M$$

where $\bar{W} = W - \text{mean}(W)$, u_i is a shift parameter (e.g. $u_i = -1 + (i-1)\frac{2}{M-1}$)
 α can be calculated via $\min_{\alpha} J(\alpha) = \|W - B\alpha\|^2$



Forward and Backward

- Forward

$$B_1, B_2, \dots, B_M = F_{u_1}(W), F_{w_2}(W), \dots, F_{u,u}(W)$$

$$\text{solve } \min_{\alpha} J(\alpha) = \|W - B\alpha\|^2 \text{ for } \alpha$$

$$O = \sum_{m=1}^M \alpha_m \text{Conv}(B_m, A)$$

- Backward

$$\frac{\partial c}{\partial W} = \frac{\partial c}{\partial O} \left(\sum_{m=1}^M \alpha_m \frac{\partial O}{\partial B_m} \frac{\partial B_m}{\partial W} \right) \stackrel{STE}{=} \frac{\partial c}{\partial O} \left(\sum_{m=1}^M \alpha_m \frac{\partial O}{\partial B_m} \right) = \sum_{m=1}^M \alpha_m \frac{\partial c}{\partial B_m}$$



Multiple Binary Activations

- Bounded Activation Function

$$h(x) \in [0, 1]$$

$$h_r(x) = \text{clip}(x + v, 0, 1)$$

where v is a shift parameter

- Binarization Function

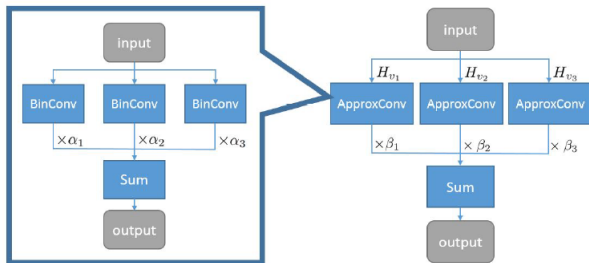
$$H_v(\mathbf{R}) := 2\mathbb{I}_{h_v(\mathbf{R}) \geq 0.5} - 1$$

$$A_1, A_2, \dots, A_N = H_{v_1}(R), H_{v_2}(R), \dots, H_{v_N}(R)$$

$$R \approx \beta_1 A_1 + \beta_2 A_2 + \dots + \beta_N A_N$$

where R is the real-value activation

- A_1, A_2, \dots, A_N is the base to represent the real-valued activations



- ApproxConv is expected to approximate the conventional full-precision convolution with linear combination of binary convolutions
- The right part is the overall block structure of the convolution in ABC-Net. The input is binarized using different functions H_{v1}, H_{v2}, H_{v3}

$$\text{Conv}(\mathbf{W}, \mathbf{R}) \approx \text{Conv} \left(\sum_{m=1}^M \alpha_m \mathbf{B}_m, \sum_{n=1}^N \beta_n \mathbf{A}_n \right) = \sum_{m=1}^M \sum_{n=1}^N \alpha_m \beta_n \text{Conv}(\mathbf{B}_m, \mathbf{A}_n)$$



Read the paper² if you want to learn the specific details of the algorithm

Towards Accurate Binary Convolutional Neural Network

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²Xiaofan Lin, Cong Zhao, and Wei Pan (2017). “Towards accurate binary convolutional neural network”. In: *Advances in Neural Information Processing Systems*, pp. 345–353.



1 Minimize the Quantization Error

2 Reduce the Gradient Error



Motivation

- Although STE is often adopted to estimate the gradients in BP, there exists obvious gradient mismatch between the gradient of the binarization function
- With the restriction of STE, the parameters outside the range of $[-1 : +1]$ will not be updated.



Bi-real net: Enhancing the performance of 1-bit CNNs with improved representational capability and advanced training algorithm



Naive Binarization Function

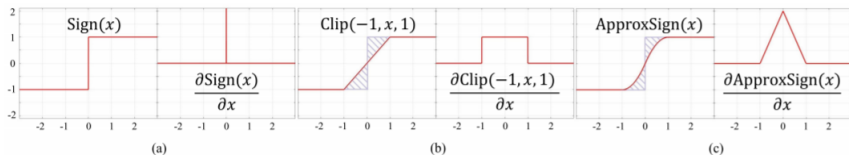
- Recall the partial derivative calculation in back propagation

$$\frac{\partial \mathcal{L}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \mathbf{A}_b^{l,t}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \text{Sign}(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}} \approx \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial F(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}}$$

- Sign* function is a non-differentiable function, so F is an approximation differentiable function of *Sign* function



$$\frac{\partial \mathcal{L}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \mathbf{A}_b^{l,t}}{\partial \mathbf{A}_r^{l,t}} = \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial \text{Sign}(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}} \approx \frac{\partial \mathcal{L}}{\partial \mathbf{A}_b^{l,t}} \frac{\partial F(\mathbf{A}_r^{l,t})}{\partial \mathbf{A}_r^{l,t}}$$



Approximation of Sign function

- Naive Approximation $F(x) = \text{clip}(x, 0, 1)$, see fig(b)
- More Precious Approximation in Bi-Real, see fig(c)

$$\text{Approxsign}(x) = \begin{cases} -1, & \text{if } x < -1 \\ 2x + x^2, & \text{if } -1 \leq x < 0 \\ 2x - x^2, & \text{if } 0 \leq x < 1 \\ 1, & \text{otherwise} \end{cases} \quad \frac{\partial \text{Approxsign}(x)}{\partial x} = \begin{cases} 2 + 2x, & \text{if } -1 \leq x < 0 \\ 2 - 2x, & \text{if } 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$



Read the paper³ if you want to learn the specific details of the algorithm

Bi-Real Net: Enhancing the Performance of 1-bit CNNs With Improved Representational Capability and Advanced Training Algorithm

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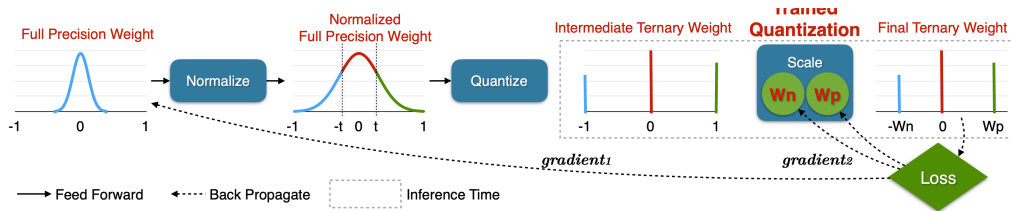
² Tencent AI lab

³ Huazhong University of Science and Technology

³Zechun Liu et al. (2018). “Bi-real net: Enhancing the performance of 1-bit cns with improved representational capability and advanced training algorithm”. In: *Proceedings of the European conference on computer vision (ECCV)*, pp. 722–737.



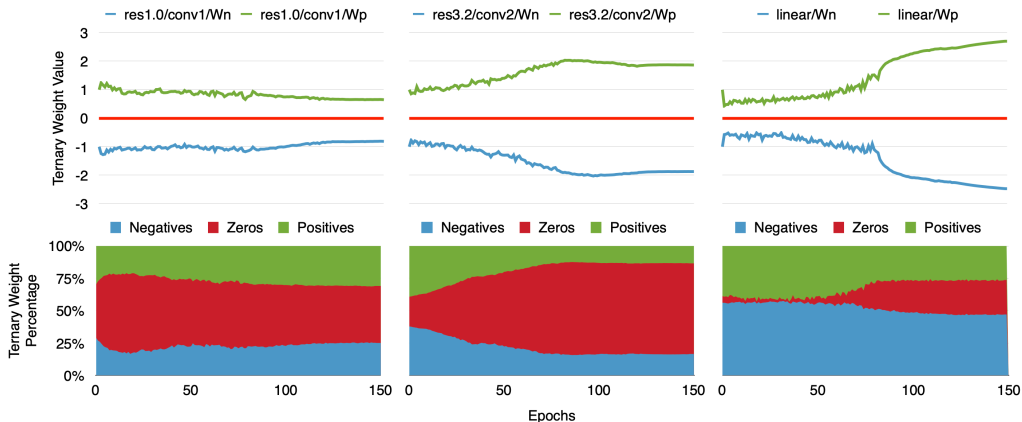
Trained ternary quantization



Overview of the trained ternary quantization procedure.

⁴Chenzhuo Zhu et al. (2017). "Trained ternary quantization". In: *Proc. ICLR*.

Trained Ternary Quantization⁴



Ternary weights value (above) and distribution (below) with iterations for different layers of ResNet-20 on CIFAR-10.

⁴Chenzhuo Zhu et al. (2017). "Trained ternary quantization". In: *Proc. ICLR*.



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