



CENG 4480

Embedded System Development & Applications

Lec 08: Binary/Ternary Network

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(Latest update: October 13, 2025)

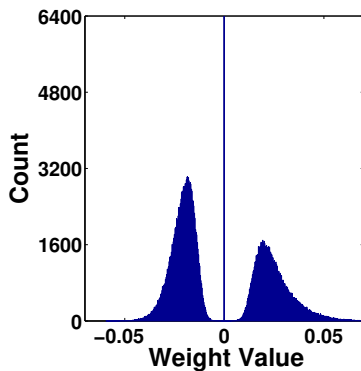
2025 Fall



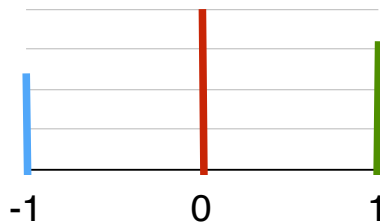
These slides contain/adapt materials developed by

- Ritchie Zhao et al. (2017). “Accelerating binarized convolutional neural networks with software-programmable FPGAs”. In: *Proc. FPGA*, pp. 15–24
- Mohammad Rastegari et al. (2016). “XNOR-NET: Imagenet classification using binary convolutional neural networks”. In: *Proc. ECCV*, pp. 525–542

Binary / Ternary Net: Motivation

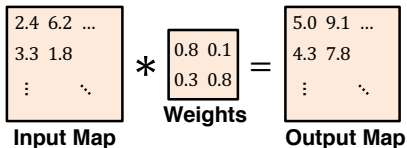


$\Rightarrow$



Binarized Neural Networks (BNN)

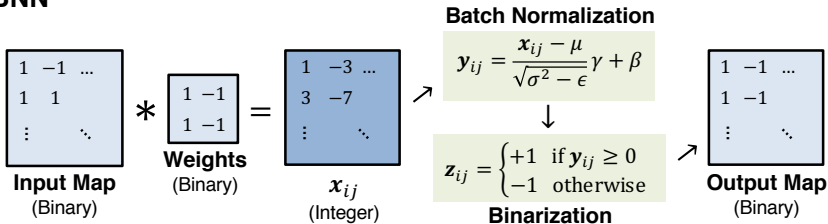
CNN



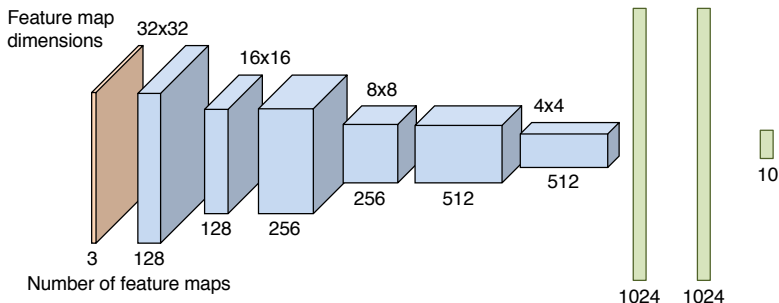
Key Differences

1. Inputs are binarized (-1 or +1)
2. Weights are binarized (-1 or +1)
3. Results are binarized after **batch normalization**

BNN



BNN CIFAR-10 Architecture [2]



- ▶ 6 conv layers, 3 dense layers, 3 max pooling layers
- ▶ All conv filters are 3x3
- ▶ First conv layer takes in floating-point input
- ▶ **13.4 Mbits total model size** (after hardware optimizations)

Advantages of BNN

1. Floating point ops replaced with binary logic ops

b_1	b_2	$b_1 \times b_2$
+1	+1	+1
+1	-1	-1
-1	+1	-1
-1	-1	+1

b_1	b_2	$b_1 \text{ XOR } b_2$
0	0	0
0	1	1
1	0	1
1	1	0

- Encode $\{+1, -1\}$ as $\{0, 1\}$ $\rightarrow$ multiplies become XORs
- Conv/dense layers do dot products $\rightarrow$ XOR and popcount
- Operations can map to LUT fabric as opposed to DSPs

2. Binarized weights may reduce total model size

- Fewer bits per weight may be offset by having more weights

BNN vs CNN Parameter Efficiency

Architecture	Depth	Param Bits (Float)	Param Bits (Fixed-Point)	Error Rate (%)
ResNet [3] (CIFAR-10)	164	51.9M	13.0M*	11.26
BNN [2]	9	-	13.4M	11.40

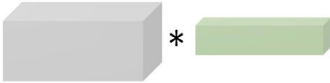
* Assuming each float param can be quantized to 8-bit fixed-point

► Comparison:

- Conservative assumption: ResNet can use 8-bit weights
- BNN is based on VGG (less advanced architecture)
- BNN seems to hold promise!

[2] M. Courbariaux et al. **Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1**. *arXiv:1602.02830*, Feb 2016.

[3] K. He, X. Zhang, S. Ren, and J. Sun. **Identity Mappings in Deep Residual Networks**. *ECCV 2016*.

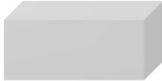
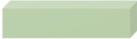
	Operations	Memory	Computation
$\mathbb{R} * \mathbb{R}$	$+ - \times$	$1x$	$1x$

Binary Weight Networks

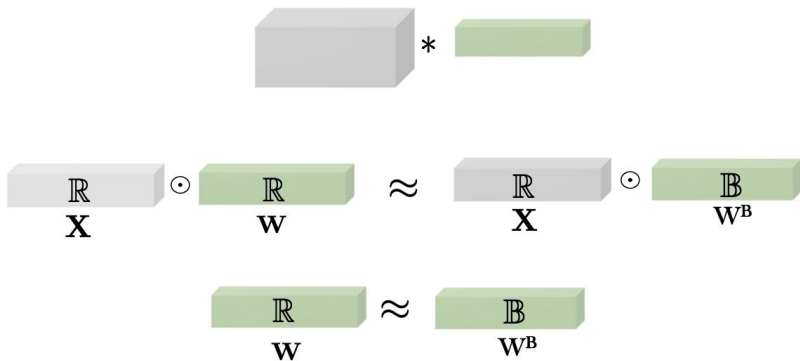
XNOR-Networks

¹Mohammad Rastegari et al. (2016). “XNOR-NET: Imagenet classification using binary convolutional neural networks”. In: *Proc. ECCV*, pp. 525–542.



 * 	Operations	Memory	Computation
$\mathbb{R} * \mathbb{R}$	+ - ×	1x	1x
$\mathbb{R} * \mathbb{B}$	+ -	~32x	~2x
$\mathbb{B} * \mathbb{B}$	XNOR Bit-count	~32x	~58x

¹Mohammad Rastegari et al. (2016). “XNOR-NET: Imagenet classification using binary convolutional neural networks”. In: *Proc. ECCV*, pp. 525–542.



$$\mathbf{W}^B = \text{sign}(\mathbf{W})$$

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Quantization Error

$$W^B = \text{sign}(W)$$

$$\left\| \begin{array}{c} W \\ \mathbb{R} \end{array} - \begin{array}{c} W^B \\ \mathbb{B} \end{array} \right\| \approx 0.75$$



Optimal Scaling Factor

$$\frac{\mathbb{R}}{\mathbf{W}} \approx \alpha \frac{\mathbb{B}}{\mathbf{W}^{\mathbf{B}}}$$

$$\alpha^*, \mathbf{W}^{\mathbf{B}*} = \arg \min_{\mathbf{W}^{\mathbf{B}}, \alpha} \{ \|\mathbf{W} - \alpha \mathbf{W}^{\mathbf{B}}\|^2 \}$$

$$\begin{aligned} \mathbf{W}^{\mathbf{B}*} &= \text{sign}(\mathbf{W}) \\ \alpha^* &= \frac{1}{n} \|\mathbf{W}\|_{\ell_1} \end{aligned}$$



How to train a CNN with binary filters?

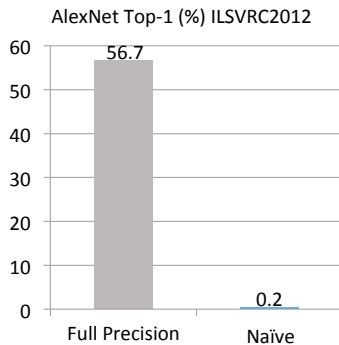
$$\text{R} * \text{R} \approx \left(\text{R} * \text{B} \right) \alpha$$



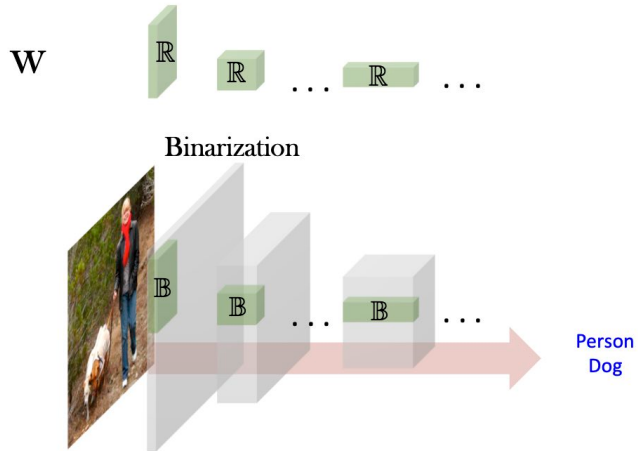
Training Binary Weight Networks

Naive Solution:

1. Train a network with real value parameters
2. Binarize the weight filters







¹Mohammad Rastegari et al. (2016). "XNOR-NET: Imagenet classification using binary convolutional neural networks". In: *Proc. ECCV*, pp. 525–542.



Binary Weight Network

Train for binary weights:

1. Randomly initialize $\mathbf{W}$
2. For $iter = 1$ to N
3. Load a random input image $\mathbf{X}$
4. $\mathbf{W}^B = \text{sign}(\mathbf{W})$
5. $\alpha = \frac{\|\mathbf{W}\|_{\ell_1}}{n}$
6. Forward pass with $\alpha, \mathbf{W}^B$
7. Compute loss function $\mathbf{C}$
8. $\frac{\partial \mathbf{C}}{\partial \mathbf{W}} = \text{Backward pass with } \alpha, \mathbf{W}^B$
9. Update $\mathbf{W}$ ($\mathbf{W} = \mathbf{W} - \frac{\partial \mathbf{C}}{\partial \mathbf{W}}$)

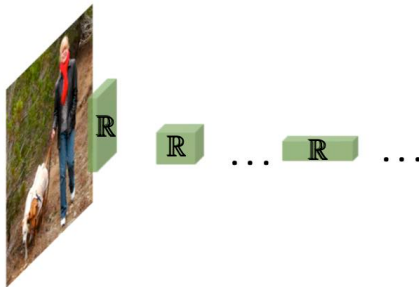


Binary Weight Network

W

Train for binary weights:

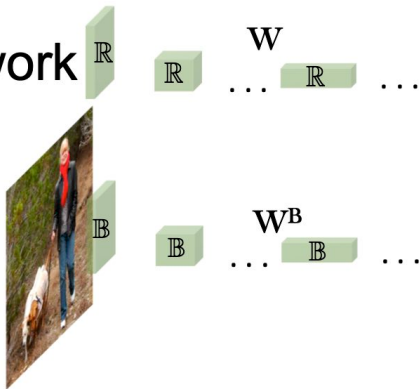
1. Randomly initialize W
2. For $iter = 1$ to N
3. Load a random input image X
4. $W^B = \text{sign}(W)$
5. $\alpha = \frac{\|W\|_{l1}}{n}$
6. Forward pass with α, W^B
7. Compute loss function C
8. $\frac{\partial C}{\partial W} = \text{Backward pass with } \alpha, W^B$
9. Update W ($W = W - \frac{\partial C}{\partial W}$)



Binary Weight Network

Train for binary weights:

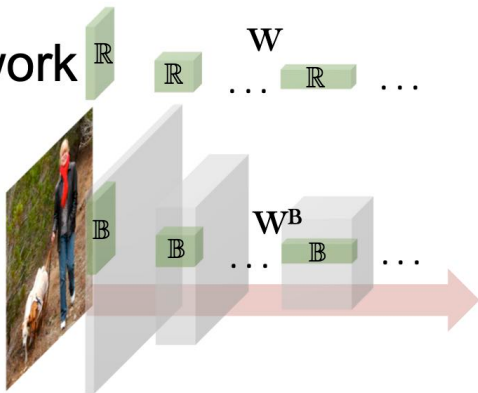
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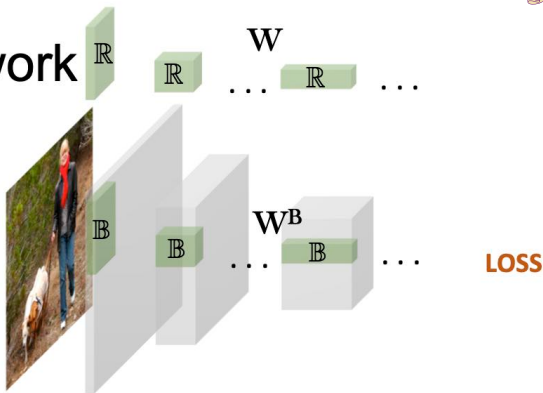
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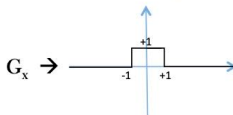
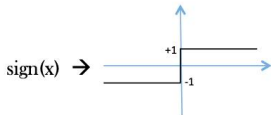
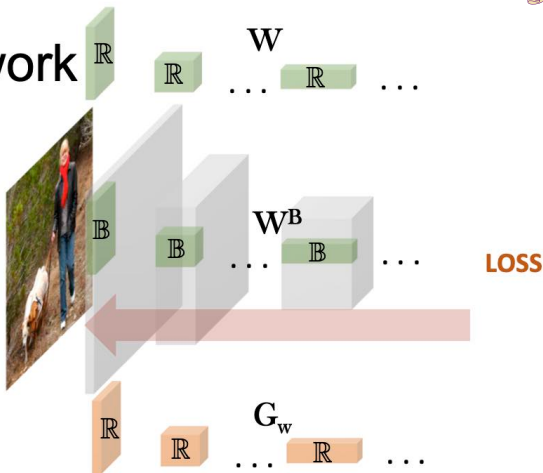




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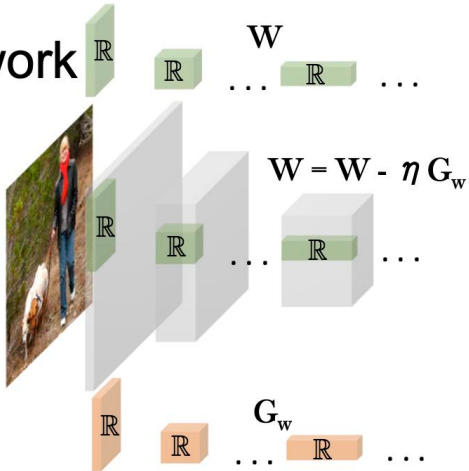
[Hinton et al. 2012]

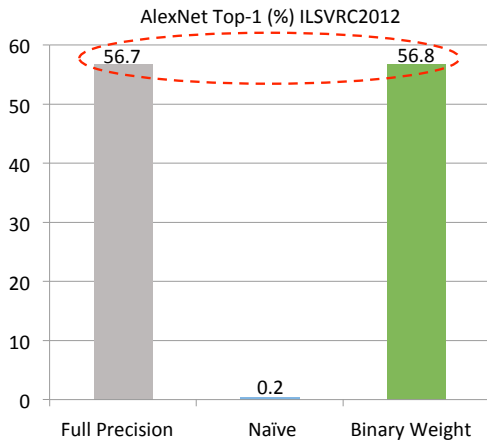


Binary Weight Network

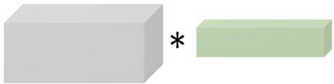
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¹Mohammad Rastegari et al. (2016). “XNOR-NET: Imagenet classification using binary convolutional neural networks”. In: *Proc. ECCV*, pp. 525–542.

		Operations	Memory	Computation
$\mathbb{R}$	$*$ $\mathbb{R}$	+ - $\times$	1x	1x
$\mathbb{R}$	$*$ $\mathbb{B}$	+ -	$\sim 32x$	$\sim 2x$
$\mathbb{B}$ $*$ $\mathbb{B}$ XNOR-Networks		XNOR Bit-count	$\sim 32x$	$\sim 58x$

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Binary Input and Binary Weight (XNOR-Net)

$$\begin{array}{c} \mathbb{R} \\ \mathbf{X} \end{array} \odot \begin{array}{c} \mathbb{R} \\ \mathbf{W} \end{array} \approx \beta \begin{array}{c} \mathbb{B} \\ \mathbf{X}^{\mathbf{B}} \end{array} \odot \alpha \begin{array}{c} \mathbb{B} \\ \mathbf{W}^{\mathbf{B}} \end{array}$$

1

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Binary Input and Binary Weight (XNOR-Net)

$$\underbrace{\begin{matrix} \text{R} \\ \text{X} \end{matrix}}_{\text{Y}} \odot \underbrace{\begin{matrix} \text{R} \\ \text{W} \end{matrix}}_{\text{Y}} \approx \underbrace{\beta \alpha}_{\gamma} \underbrace{\begin{matrix} \text{B} \\ \text{X}^{\text{B}} \end{matrix}}_{\text{Y}^{\text{B}}} \odot \underbrace{\begin{matrix} \text{B} \\ \text{W}^{\text{B}} \end{matrix}}_{\text{Y}^{\text{B}}}$$

$$\mathbf{Y} \approx \gamma \mathbf{Y}^{\text{B}}$$

$$\mathbf{Y}^{\text{B}*}, \gamma^* = \arg \min_{\mathbf{Y}^{\text{B}}, \gamma} \|\mathbf{Y} - \gamma \mathbf{Y}^{\text{B}}\|^2$$

$$\mathbf{Y}^{\text{B}*} = \text{sign}(\mathbf{Y}) \quad \gamma^* = \frac{1}{n} \|\mathbf{Y}\|_{\ell_1}$$

$$\mathbf{X}^{\text{B}*} = \text{sign}(\mathbf{X}) \quad \mathbf{W}^{\text{B}*} = \text{sign}(\mathbf{W})$$

$$\alpha^* = \frac{1}{n} \|\mathbf{W}\|_{\ell_1} \quad \beta^* = \frac{1}{n} \|\mathbf{X}\|_{\ell_1}$$

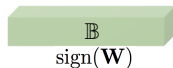
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(1) Binarizing Weights

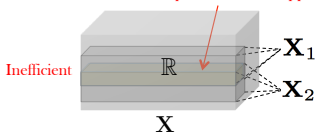


$$\frac{1}{n} \|\mathbf{W}\|_{\ell_1} = \alpha$$



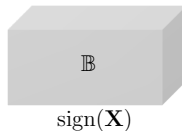
(2) Binarizing Input

Redundant computation in overlapping areas

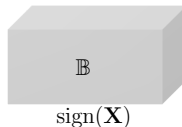
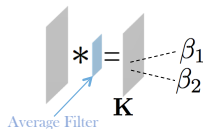
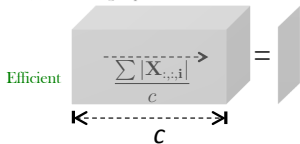


$$\begin{aligned} \frac{1}{n} \|\mathbf{X}_1\|_{\ell_1} &= \beta_1 \\ \frac{1}{n} \|\mathbf{X}_2\|_{\ell_1} &= \beta_2 \end{aligned}$$

$\mathbf{K}$



(2) Binarizing Input



(3) Convolution with XNOR-Bitcount

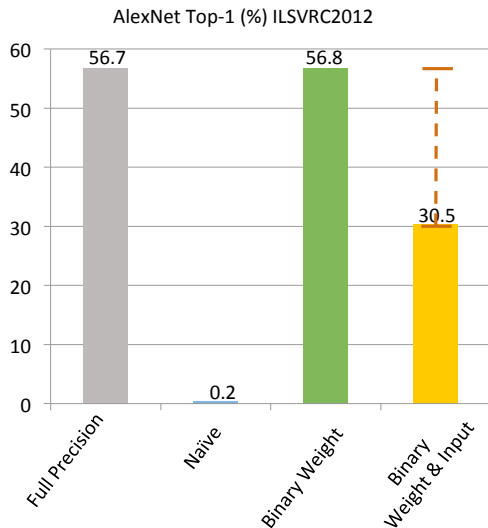
$$\mathbf{R} * \mathbf{W} \approx \left[\text{sign}(\mathbf{X}) \circledast \text{sign}(\mathbf{W}) \right] \odot \mathbf{K} \odot \alpha$$



$$R * R \approx \left[B_{\text{sign}(X)} * B_{\text{sign}(W)} \right] \odot \beta \odot \alpha$$

1. Randomly initialize W
2. For $iter = 1$ to N
3. Load a random input image X
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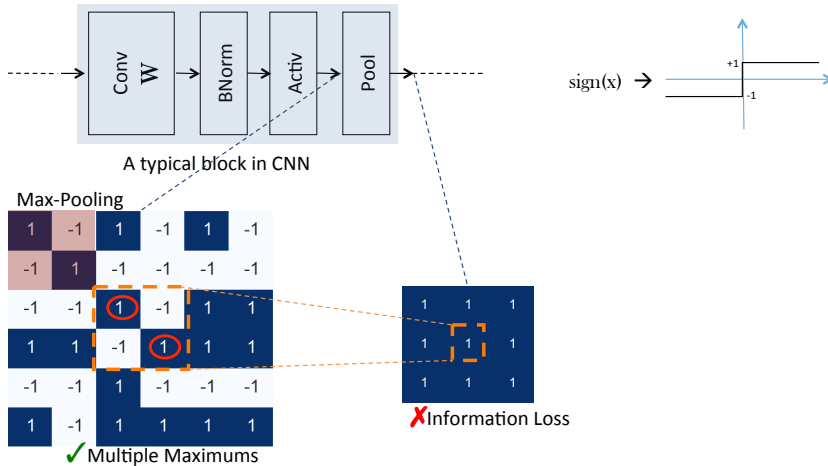
1



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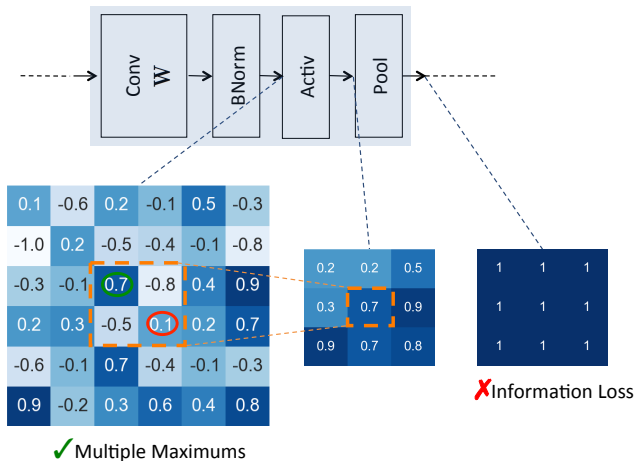


Network Structure in XNOR-Networks



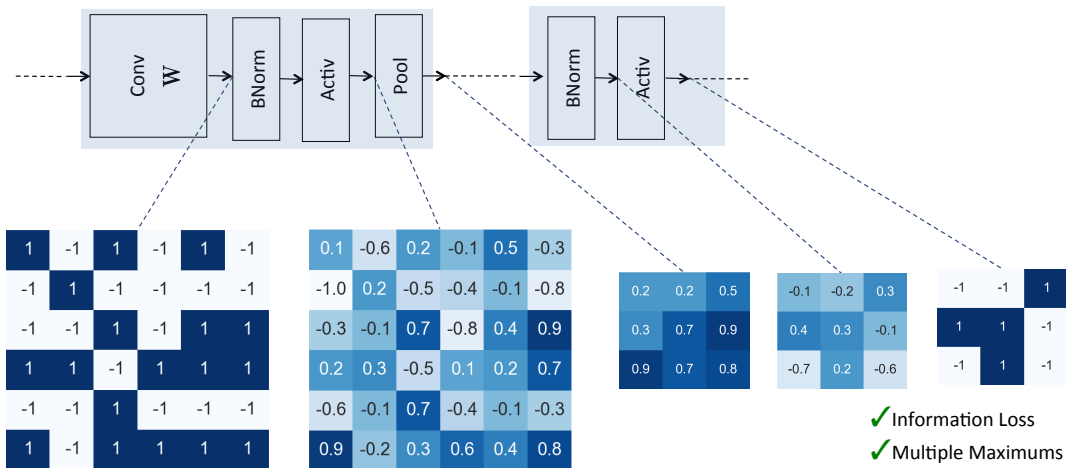
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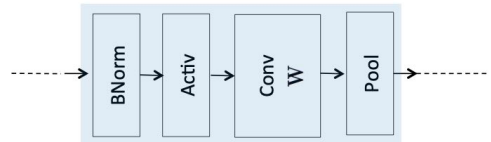


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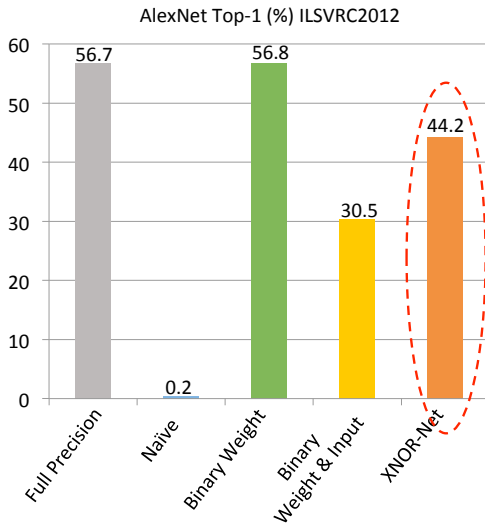


$$\mathbb{R} * \mathbb{R} \approx \left[\underset{\text{sign}(\mathbf{X})}{\mathbb{B}} * \underset{\text{sign}(\mathbf{W})}{\mathbb{B}} \right] \odot \beta \odot \alpha$$

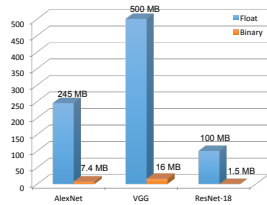
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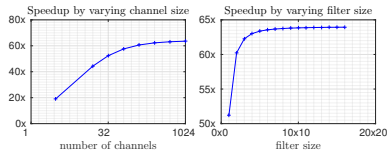
1



✓ 32x Smaller Model



✓ 58x Less Computation



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