CENG 4480

Embedded System Development & Applications

Lec 06: Quantization

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These slides contain/adapt materials developed by

- Hardware for Machine Learning, Shao Spring 2020 @ UCB
- 8-bit Inference with TensorRT
- Amir Gholami et al. (2021). "A survey of quantization methods for efficient neural network inference". In: *arXiv preprint*

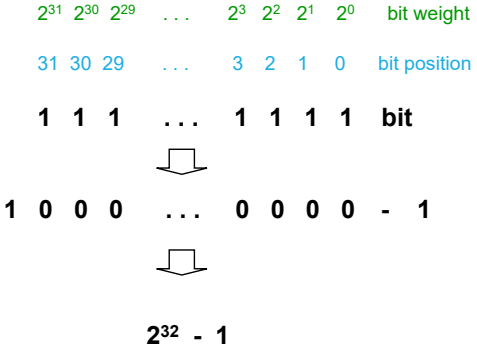
Overview

1. Integer & Fixed-Point Number
2. Quantization Overview
3. Quantization – First Example
4. Post Training Quantization (PTQ)
5. Quantization Aware Training (QAT)

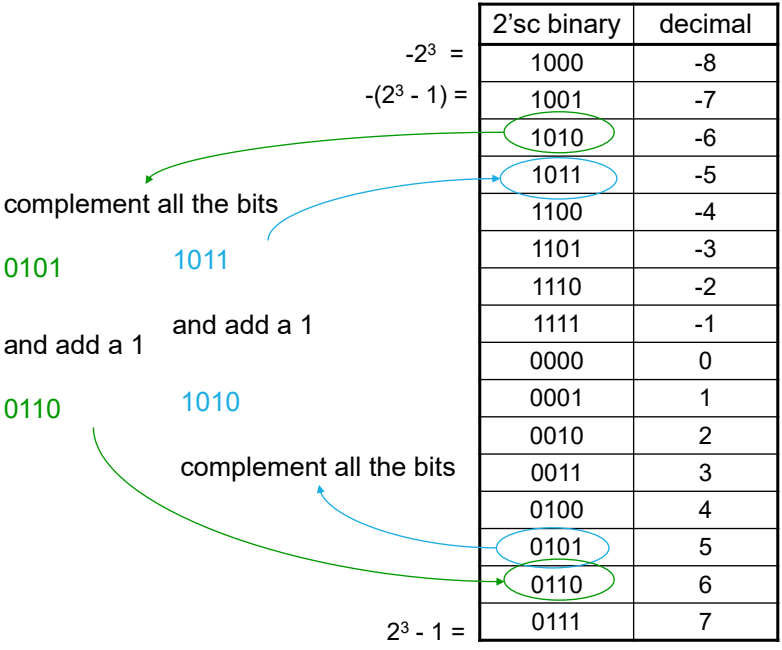
Integer & Fixed-Point Number

Unsigned Binary Representation

Hex	Binary	Decimal
0x00000000	0…0000	0
0x00000001	0…0001	1
0x00000002	0…0010	2
0x00000003	0…0011	3
0x00000004	0…0100	4
0x00000005	0…0101	5
0x00000006	0…0110	6
0x00000007	0…0111	7
0x00000008	0…1000	8
0x00000009	0…1001	9
	…	
0xFFFFFFFC	1…1100	$2^{32} - 4$
0xFFFFFFFD	1…1101	$2^{32} - 3$
0xFFFFFFFE	1…1110	$2^{32} - 2$
0xFFFFFFFF	1…1111	$2^{32} - 1$



Signed Binary Representation



- Integers with a binary point and a bias
  - “slope and bias”: y = s*x + z
  - Qm.n: m (# of integer bits) n (# of fractional bits)

s = 1, z = 0

2^2	2^1	2^0	Val
0	0	0	**0**
0	0	1	**1**
0	1	0	**2**
0	1	1	**3**
1	0	0	**4**
1	0	1	**5**
1	1	0	**6**
1	1	1	**7**

s = 1/4, z = 0

2^0	2^-1	2^-2	Val
0	0	0	**0**
0	0	1	**1/4**
0	1	0	**2/4**
0	1	1	**3/4**
1	0	0	**1**
1	0	1	**5/4**
1	1	0	**6/4**
1	1	1	**7/4**

s = 4, z = 0

2^4	2^3	2^2	Val
0	0	0	**0**
0	0	1	**4**
0	1	0	**8**
0	1	1	**12**
1	0	0	**16**
1	0	1	**20**
1	1	0	**24**
1	1	1	**28**

s = 1.5, z =10

2^2	2^1	2^0	Val
0	0	0	**1.5*0 +10**
0	0	1	**1.5*1 +10**
0	1	0	**1.5*2 +10**
0	1	1	**1.5*3 +10**
1	0	0	**1.5*4 +10**
1	0	1	**1.5*5 +10**
1	1	0	**1.5*6 +10**
1	1	1	**1.5*7 +10**

Catastrophic Cancellation

$(a - b)$ is inaccurate when $a >> b$ or $a << b$

Decimal Example 1:

- Using 2 significant digits
- Compute mean of 5.1 and 5.2 using the formula $(a + b)/2$:
- $a + b = 10$ (with 2 significant digits, 10.3 can only be stored as 10)
- $10/2 = 5.0$ (the computed mean is less than both numbers!!!)

Decimal Example 2:

- Using 8 significant digits to compute sum of three numbers:
- $(11111113 + (-11111111)) + 7.5111111 = 9.5111111$
- $11111113 + ((-11111111) + 7.5111111) = 10.000000$

Catastrophic Cancellation

Catastrophic cancellation occurs when

$$\left| \frac{[\text{round}(x) \times \text{round}(y)] - \text{round}(x \times y)}{\text{round}(x \times y)} \right| >> \epsilon$$

Quantization Overview

Quantization in DNN

Quantization:

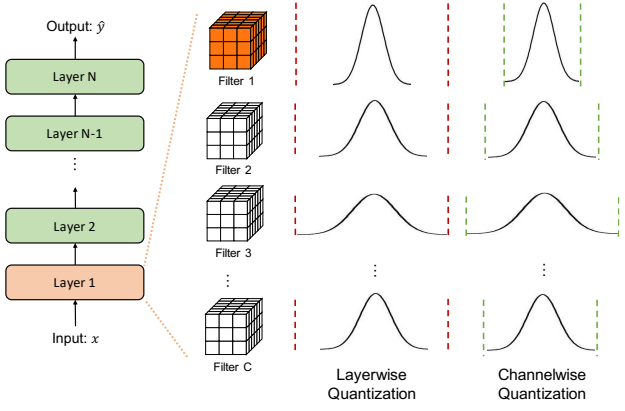
$$Q(r) = \text{Int}(r/S) - Z$$

Dequantization:

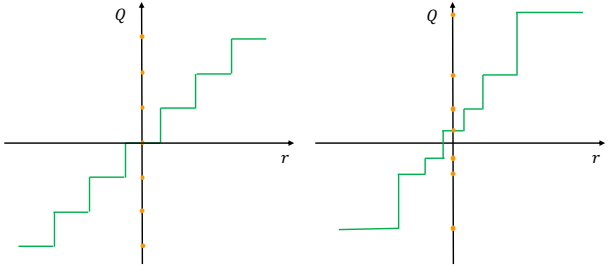
$$\hat{r} = S(Q(r) + Z)$$

Granularity:

- Layerwise
- Groupwise
- Channelwise



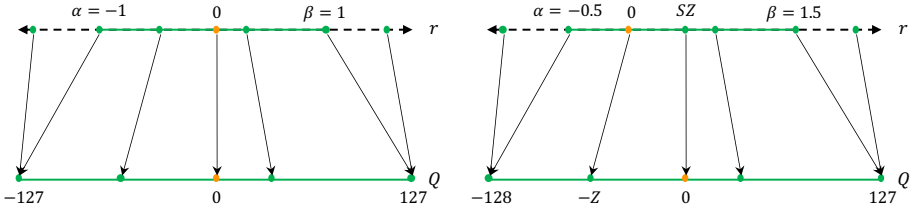
Uniform vs. Non-Uniform



(a) uniform quantization (b) non-uniform quantization

- Real values in the continuous domain r are mapped into discrete
- Lower precision values in the quantized domain Q.
- **Uniform** quantization: distances between quantized values are the same
- **Non-uniform** quantization: distances between quantized values can vary

Symmetric vs. Asymmetric

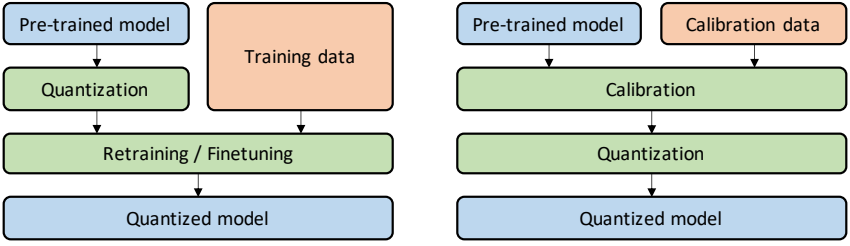


(a) Symmetric quantization

(b) Asymmetric quantization

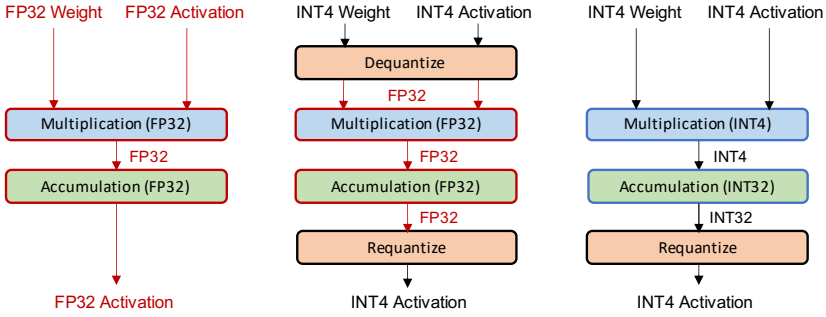
- Symmetric vs. Asymmetric: $Z = 0$?
- Fig. (a) Symmetric w. restricted range maps [-127, 127],
- Fig. (b) Asymmetric w. full range maps to [-128, 127]
- Both for 8-bit quantization case.

QAT and PTQ



- quantization-aware training (QAT): model is quantized using training data to adjust parameters and recover accuracy degradation.
- post-training quantization (PTQ): a pre-trained model is calibrated using finetuning data (e.g., a small subset of training data) to compute the clipping ranges and the scaling factors.
- Key difference: Model parameters fixed/unfixed.

Simulated quantization vs Integer-Only quantization



Left : Full-precision

Middle : Simulated quantization

Right : Integer-only quantization

Backend Support for Quantization Deployment

Hardware Support

- Nvidia GPU: Tensor Core support FP16, Int8 and Int4
- Arm: Neon 128-bit SIMD instruction: 4×32bit or 8×16bit up to 16×8bit
- Intel: SSE intrinsics, same as above
- DSP, AI Chip

Some common architectures:

- For CPU: Tensorflow Lite, QNNPACK, NCNN
- For GPU: TensorRT
- Versatile Compiler such TVM.qnn

Quantization – First Example

Linear quantization

Representation:

Tensor Values = FP32 scale factor * int8 array + FP32 bias

Do we really need bias?

Two matrices:

```
A = scale_A * QA + bias_A
B = scale_B * QB + bias_B
```

Let's multiply those 2 matrices:

```
A * B = scale_A * scale_B * QA * QB +
        scale_A * QA * bias_B       +
        scale_B * QB * bias_A       +
        bias_A  * bias_B
```

Do we really need bias?

Two matrices:

```
A = scale_A * QA + bias_A
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Let's multiply those 2 matrices:

```
A * B = scale_A * scale_B * QA * QB +
        scale_A * QA * bias_B       +
        scale_B * QB * bias_A       +
        bias_A  * bias_B
```

Do we really need bias? No!

Two matrices:

```
A = scale_A * QA
B = scale_B * QB
```

Let's multiply those 2 matrices:

```
A * B = scale_A * scale_B * QA * QB
```

Symmetric linear quantization

Representation:

Tensor Values = FP32 scale factor * int8 array

One FP32 scale factor for the entire int8 tensor

Q: How do we set scale factor?

MINIMUM QUANTIZED VALUE

- Integer range is not completely symmetric. E.g. in 8bit, [-128, 127]
  - If use [-127, 127], $s = \frac{127}{\alpha}$
    - Range is symmetric
    - 1/256 of int8 range is not used. 1/16 of int4 range is not used
  - If use full range [-128, 127], $s = \frac{128}{\alpha}$
    - Values should be quantized to 128 will be clipped to 127
    - Asymmetric range may introduce bias

EXAMPLE OF QUANTIZATION BIAS

Bias introduced when int values are in [-128, 127]

$$A = [-2.2 \quad -1.1 \quad 1.1 \quad 2.2] \text{ , } B = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.3 \\ 0.5 \end{bmatrix} \text{ , } AB = 0$$

8bit scale quantization, use [-128, 127]. s_A=128/2.2, s_B=128/0.5

$$[-128 \quad -64 \quad 64 \quad 127] * \begin{bmatrix} 127 \\ 77 \\ 77 \\ 127 \end{bmatrix} = -127$$

Dequantize -127 will get -0.00853. A small bias is introduced towards -∞

EXAMPLE OF QUANTIZATION BIAS

No bias when int values are in [-127, 127]

$$A = [-2.2 \quad -1.1 \quad 1.1 \quad 2.2] \text{ , } B = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.3 \\ 0.5 \end{bmatrix} \text{ , } AB = 0$$

8-bit scale quantization, use [-127, 127]. s_A=127/2.2, s_B=127/0.5

$$[-127 \quad -64 \quad 64 \quad 127] * \begin{bmatrix} 127 \\ 76 \\ 76 \\ 127 \end{bmatrix} = 0$$

Dequantize 0 will get 0

MATRIX MULTIPLY EXAMPLE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

MATRIX MULTIPLY EXAMPLE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

8bit quantization

choose [-2, 2] fp range (scale 127/2=63.5) for first matrix and [-1, 1] fp range (scale = 127/1=127) for the second

$$\begin{pmatrix} -98 & 14 \\ -17 & 41 \end{pmatrix} * \begin{pmatrix} 44 \\ -65 \end{pmatrix} = \begin{pmatrix} -5222 \\ -3413 \end{pmatrix}$$

MATRIX MULTIPLY EXAMPLE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

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$$\begin{pmatrix} -98 & 14 \\ -17 & 41 \end{pmatrix} * \begin{pmatrix} 44 \\ -65 \end{pmatrix} = \begin{pmatrix} -5222 \\ -3413 \end{pmatrix}$$

The result has an overall scale of 63.5*127 . We can *dequantize* back to float

$$\begin{pmatrix} -5222 \\ -3413 \end{pmatrix} * \frac{1}{63.5 * 127} = \begin{pmatrix} -0.648 \\ -0.423 \end{pmatrix}$$

REQUANTIZE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

8bit quantization

choose [-2, 2] fp range for first matrix and [-1, 1] fp range for the second

$$\begin{pmatrix} -98 & 14 \\ -17 & 41 \end{pmatrix} * \begin{pmatrix} 44 \\ -65 \end{pmatrix} = \begin{pmatrix} -5222 \\ -3413 \end{pmatrix}$$

Requantize output to a different quantized representation with fp range [-3, 3]:

$$\begin{pmatrix} -5222 \\ -3413 \end{pmatrix} * \frac{127/3}{63.5 * 127} = \begin{pmatrix} -27 \\ -18 \end{pmatrix}$$

Post Training Quantization (PTQ)

Greedy Layer-wise Quantization[1]

- For a fixed-point number, it representation is:

$$n = \sum_{i=0}^{bw-1} B_i \cdot 2^{-f_l} \cdot 2^i,$$

where bw is the bit width and f_l is the fractional length which is dynamic for different layers and feature map sets while static in one layer.

- Weight quantization: find the optimal f_l for weights:

$$f_l = \arg\min_{f_l} \sum |W_{float} - W(bw, f_l)|,$$

where W is a weight and $W(bw, f_l)$ represents the fixed-point format of W under the given bw and f_l.

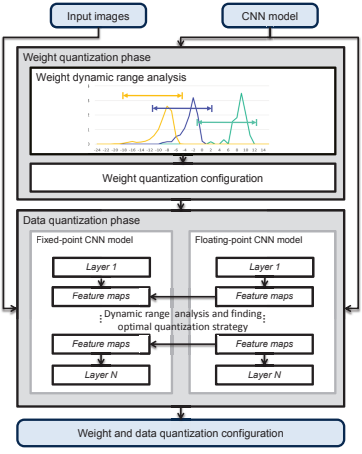
[1] Jiantao Qiu et al. (2016). "Going deeper with embedded fpga platform for convolutional neural network". In: *Proc. FPGA*, pp. 26–35.

Greedy Layer-wise Quantization

- Feature quantization: find the optimal f_l for features:

$$f_l = \arg\min_{f_l} \sum |x^+_{float} - x^+(bw, f_l)|,$$

where x^+ represents the result of a layer when we denote the computation of a layer as $x^+ = A \cdot x$.

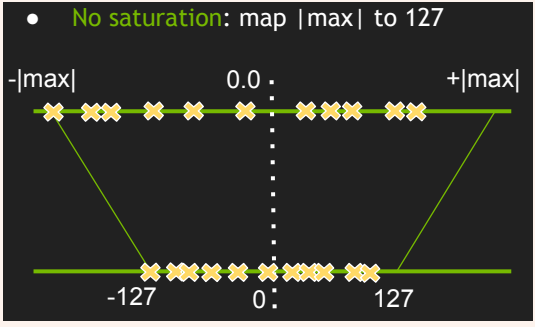


Dynamic-Precision Data Quantization Results

Network	VGG16						
Data Bits	Single-float	16	16	8	8	8	8
Weight Bits	Single-float	16	8	8	8	8	8 or 4
Data Precision	N/A	2^{-2}	2^{-2}	Impossible	$2^{-5}/2^{-1}$	Dynamic	Dynamic
Weight Precision	N/A	2^{-15}	2^{-7}	Impossible	2^{-7}	Dynamic	Dynamic
Top-1 Accuracy	68.1%	68.0%	53.0%	Impossible	28.2%	66.6%	67.0%
Top-5 Accuracy	88.0%	87.9%	76.6%	Impossible	49.7%	87.4%	87.6%

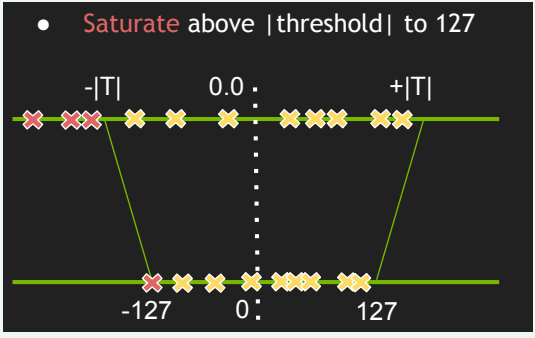
Network	CaffeNet			VGG16-SVD		
Data Bits	Single-float	16	8	Single-float	16	8
Weight Bits	Single-float	16	8	Single-float	16	8 or 4
Data Precision	N/A	Dynamic	Dynamic	N/A	Dynamic	Dynamic
Weight Precision	N/A	Dynamic	Dynamic	N/A	Dynamic	Dynamic
Top-1 Accuracy	53.9%	53.9%	53.0%	68.0%	64.6%	64.1%
Top-5 Accuracy	77.7%	77.1%	76.6%	88.0%	86.7%	86.3%

No Saturation Quantization – INT8 Inference



- Map the maximum value to 127, with unifrom step length.
- Suffer from outliers.

Saturation Quantization – INT8 Inference



- Set a threshold as the maxiumum value.
- Divide the value domain into 2048 groups.
- Traverse all the possible thresholds to find the best one with minimum KL divergence.

Relative Entropy of two encodings

- INT8 model encodes the same information as the original FP32 model.
- Minimize the loss of information.
- Loss of information is measured by Kullback-Leibler divergence (*a.k.a.*, relative entropy or information divergence).
  - P, Q - two discrete probability distributions:

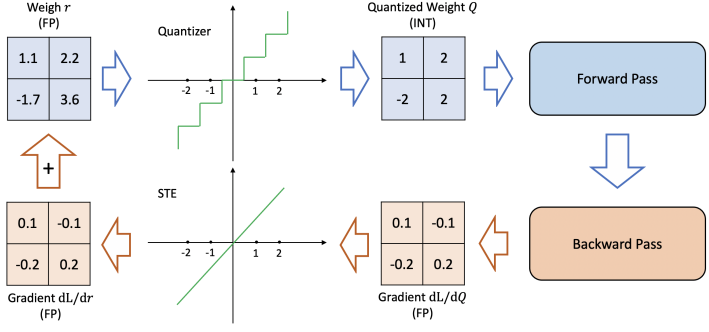
$$D_{KL}(P\|Q) = \sum_{i=1}^{N} P(x_i) \log \frac{P(x_i)}{Q(x_i)}$$

- Intuition: KL divergence measures the amount of information lost when approximating a given encoding.

Quantization Aware Training (QAT)

Straight Through Estimator (STE)[2]

- Forward integer, Backward floating point
- Rounding to nearest



[2]Yoshua Bengio, Nicholas Léonard, and Aaron Courville (2013). "Estimating or propagating gradients through stochastic neurons for conditional computation". In: *arXiv preprint arXiv:1308.3432*.

Is Straight-Through Estimator (STE) the best?

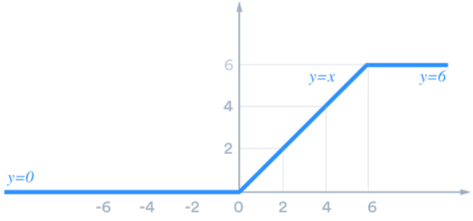
- Gradient mismatch: the gradients of the weights are not generated using the value of weights, but rather its quantized value.
- Poor gradient: STE fails at investigating better gradients for quantization training.

QAT: Activation

PArameterized Clipping acTivation (PACT)[3]

- Relu6 $\rightarrow$ clipping
- threshold $\rightarrow$ clipping range in quantization
- range upper/lower bound trainable

$$y = PACT(x) = 0.5(|x| - |x - \alpha| + \alpha) = \begin{cases} 0, & x \in (-\infty, 0) \\ x, & x \in [0, \alpha) \\ \alpha, & x \in [\alpha, +\infty) \end{cases}$$



[3]Jungwook Choi, Zhuo Wang, et al. (2018). "Pact: Parameterized clipping activation for quantized neural networks". In: *arXiv preprint arXiv:1805.06085*.

PArameterized Clipping acTivation Function (PACT)[4]

- A new activation quantization scheme in which the activation function has a parameterized clipping level α.
- The clipping level is dynamically adjusted vias stochastic gradient descent (SGD)-based training with the goal of minimizing the quantization error.
- In PACT, the convolutional ReLU activation function in CNN is replaced with:

$$f(x) = 0.5\,(|x| - |x-\alpha| + \alpha) = \begin{cases} 0, & x \in (\infty, 0) \\ x, & x \in [0, \alpha) \\ \alpha, & x \in [\alpha, +\infty) \end{cases}$$

where α limits the dynamic range of activation to $[0, \alpha]$.

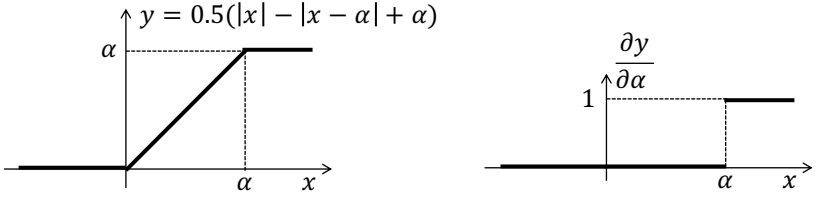
[4] Jungwook Choi, Swagath Venkataramani, et al. (2019). "Accurate and efficient 2-bit quantized neural networks". In: *Proceedings of Machine Learning and Systems* 1.

PArameterized Clipping acTivation Function (PACT)

- The truncated activation output is the linearly quantized to k-bits for the dot-product computations:

$$y_q = \text{round}\,(y \cdot \frac{2^k - 1}{\alpha}) \cdot \frac{\alpha}{2^k - 1}$$

- With this new activation function, α is a variable in the loss function, whose value can be optimized during training.
- For back-propagation, gradient $\frac{\partial y_q}{\partial \alpha}$ can be computed using STE to estimate $\frac{\partial y_q}{\partial y}$ as 1.



PACT activation function and its gradient.