CENG 4480

Embedded System Development & Applications

Lecture 03: Operational Amplifier – 2

Bei Yu
CSE Department, CUHK
byu@cse.cuhk.edu.hk

(Latest update: October 27, 2021)

Fall 2021

Overview

Overview

Euler's Identity

$$e^{j\theta} = cos\theta + jsin\theta$$

- real component
- imaginary component
- magnitude

$$|e^{j\theta}| = \sqrt{cos^2\theta + sin^2\theta} = 1$$

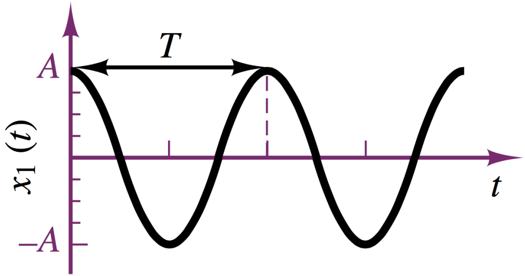
Prove:

$$|\frac{1}{1+ja}| = \frac{1}{\sqrt{1+a^2}}$$

Sinusoidal Signal

$$x(t) = A\cos(\omega t + \phi)$$

- Periodic signals
- A: amplitude
- ω: radian frequency
- ϕ: phase

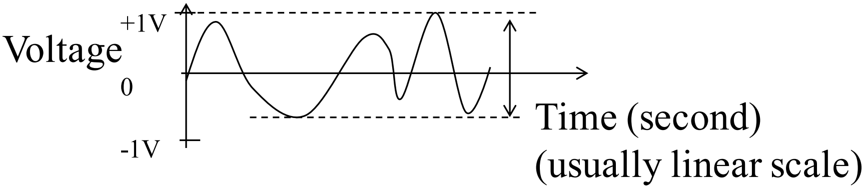


Time Domain

- Voltage gain against time

For sinusoidal signal:

$$v(t) = Acos(\omega t + \phi)$$

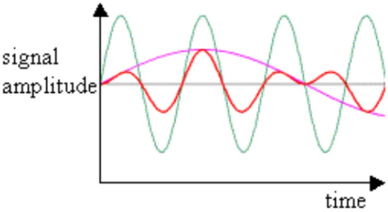


Frequency Domain

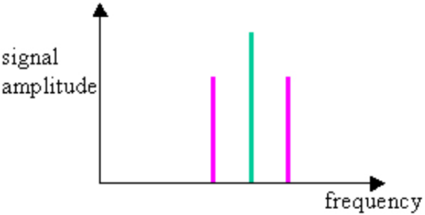
- Voltage gain against frequency

For sinusoidal signal:

$$\mathbf{V}(j\omega) = Ae^{j\phi} = A\angle\phi = Acos\phi + jAsin\phi$$



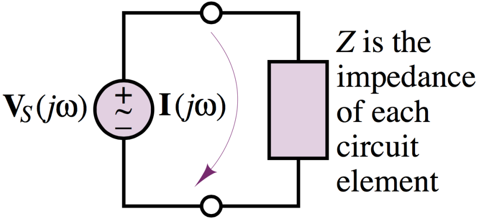
Time domain



Frequency domain

Impedance

A complex resistance or *frequency-dependent resistance*. That is, as resistors whose resistance is a function of the frequency of the sinusoidal excitation.

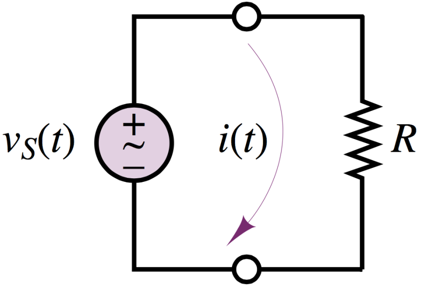


AC circuits in phasor/impedance form

Resistor Impedance

Assume source voltage $v = A\cos(\omega t)$, then

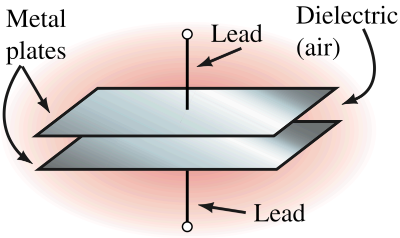
- $\mathbf{V}(j\omega) = A\angle 0$
- $\mathbf{I}(j\omega) = \frac{A}{R}\angle 0$



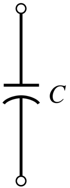
Impedance of A Resistor

$$Z_R(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} = R\angle 0 = R$$

Capacitor ABC



(a) Basic construction



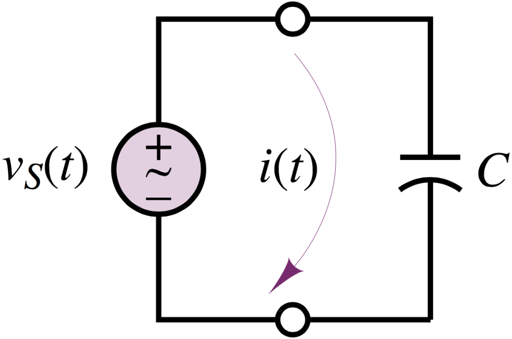
(b) Symbol

Capacitance C

A measure of how much charge a capacitor can hold.

- Amount of charge $Q = C \cdot V$
- **current** is the rate of movement of charge: $I = \frac{dQ}{dt} = C \cdot \frac{dV}{dt}$

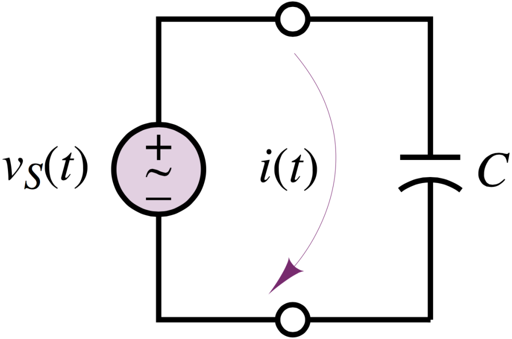
$$\mathbf{V}(j\omega) = A\angle 0$$
$$\mathbf{I}(j\omega) = \omega C A\angle \pi/2$$



Impedance of A Capacitor

$$Z_C(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)}$$

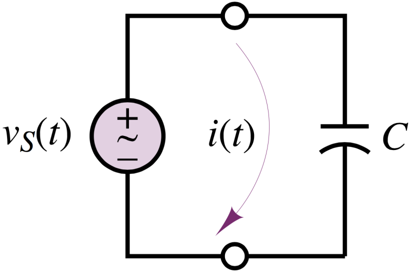
$$\mathbf{V}(j\omega) = A\angle 0$$
$$\mathbf{I}(j\omega) = \omega C A\angle \pi/2$$



Impedance of A Capacitor

$$Z_C(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)}$$

$$Z_C(j\omega) = \frac{1}{j\omega C}$$



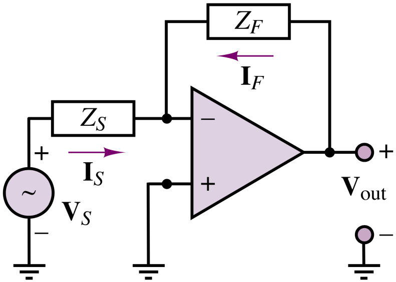
Capacitor Rule 1

Low Frequency $\Rightarrow$ Open circuit

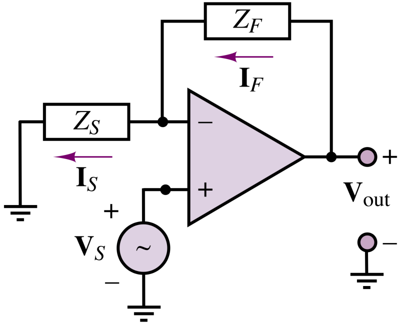
Capacitor Rule 2

High Frequency $\Rightarrow$ Short circuit

Overview

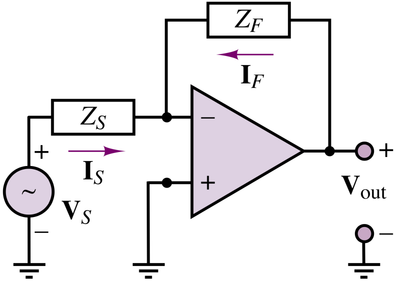


Inverting

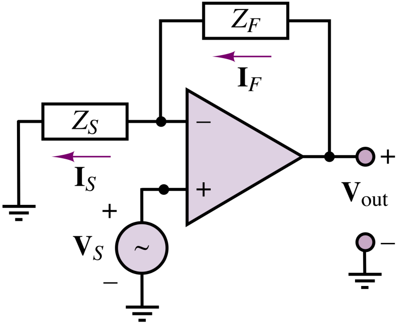


Noninverting

Frequency Response of An Op-Amp



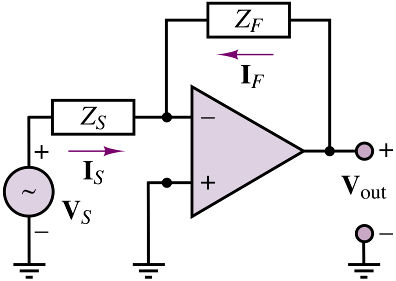
Inverting



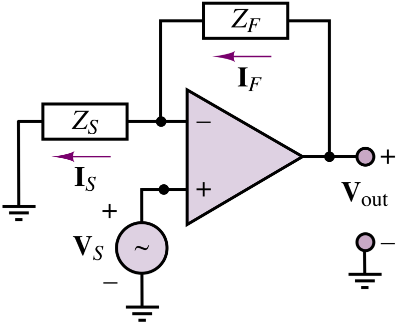
Noninverting

- Inverting amplifier: $\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$

Frequency Response of An Op-Amp

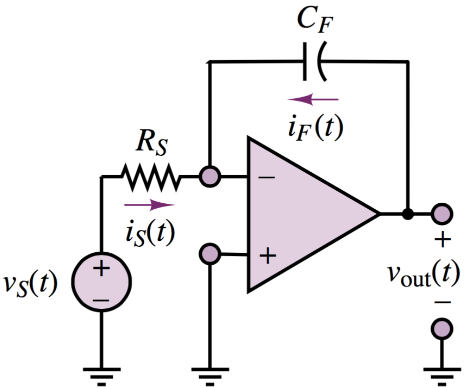


Inverting



Noninverting

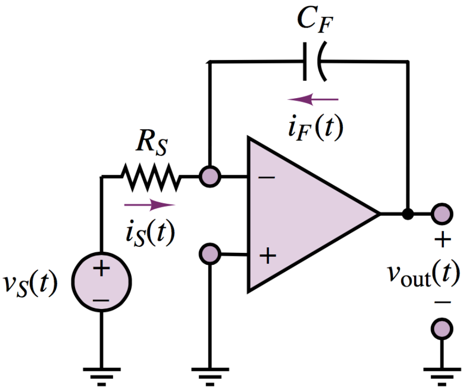
- Inverting amplifier: $\dfrac{V_{out}}{V_S}(j\omega) = -\dfrac{Z_F}{Z_S}$
- Non-Inverting amplifier: $\dfrac{V_{out}}{V_S}(j\omega) = 1 + \dfrac{Z_F}{Z_S}$



$$i_S(t) = -i_F(t)$$

$$i_S(t) = \frac{v_S(t)}{R_S}$$

$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$



$$i_S(t) = -i_F(t)$$

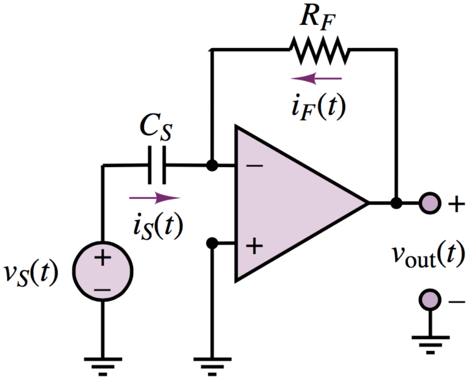
$$i_S(t) = \frac{v_S(t)}{R_S}$$

$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$

Therefore:

$$v_{out}(t) = -\frac{1}{R_S C_F}\int_{-\infty}^{t} v_S(t')dt'$$

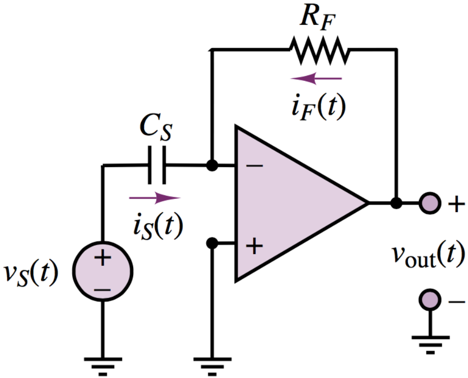
Differentiator



$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$

Differentiator



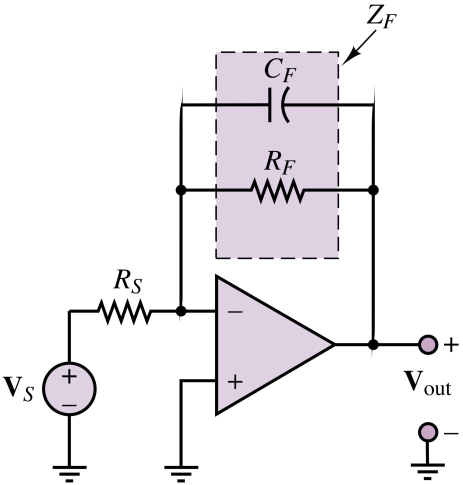
$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$

Therefore:

$$v_{out}(t) = -R_F C_S \cdot \frac{dv_S(t)}{dt}$$

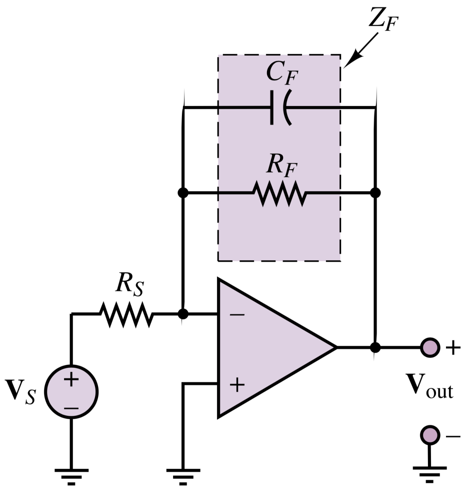
Overview



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F || \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F || \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$

⇒

$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$

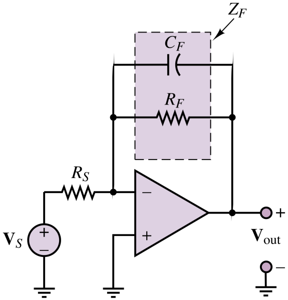
Given:

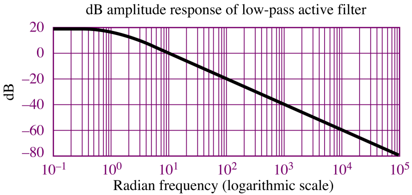
$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$

$$w_c = \frac{1}{R_F C_F}$$

Prove:

$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2/w_c^2}}$$



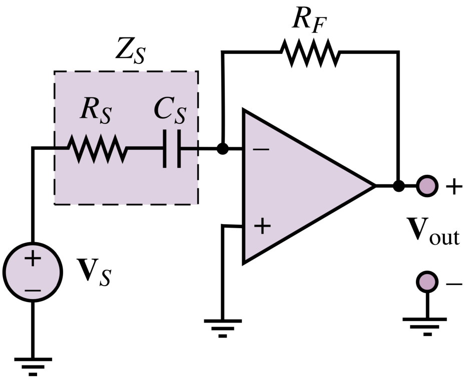


$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2 / w_c^2}}$$

- $w_c = \dfrac{1}{R_F C_F}$
- **3-dB** frequency
- or **cutoff** frequency

BTW, $\lim_{\omega \to 0} |A| = \frac{R_F}{R_S}$, $\lim_{\omega \to \infty} |A| = 0$

High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

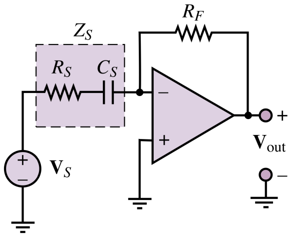
$$Z_S = R_S + \frac{1}{j\omega C_S}$$

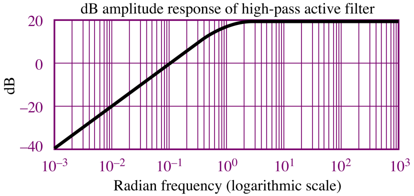
$$Z_F = R_F$$

⇒

$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$

High-Pass Filter



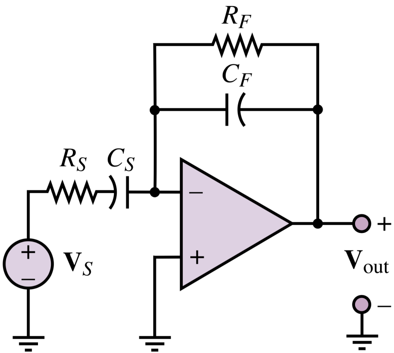


$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$

$$\lim_{\omega \to 0} |A| = 0$$

$$\lim_{\omega \to \infty} |A| = \frac{R_F}{R_C}$$

High freq. cutoff unintentionally created by Op-amp


R_F
C_F
R_S
C_S
+
−
$\mathbf{V}_S$
−
+
+
$\mathbf{V}_{\text{out}}$
−