## CENG4480 Homework 2

Due: Nov. 13, 2018
Q1 The circuit shown in Figure 1 represents a simple 4-bit digital-to-analog converter. Each switch is controlled by the corresponding bit of the digital number if the bit is 1 the switch is up; if the bit is 0 the switch is down. Let the digital number be represented by $b_{3} b_{2} b_{1} b_{0}$. Please answer the following two questions:
(1) Determine an expression relating $v_{o}$ to the binary input bits.
(2) Use this converter, design another 4-bit digital-to-analog converter whose output is given by

$$
\begin{equation*}
v_{o}=-\frac{1}{10}\left(8 b_{3}+4 b_{2}+2 b_{1}+b_{0}\right) V \tag{1}
\end{equation*}
$$




Figure 2: RF DAC.

Figure 1: 4-bit DAC.

Q2 For the DAC circuit shown in Figure 2 (using an ideal op-amp), what value of $R_{F}$ will givean output range of $-10 \leq V_{0} \leq 0 \mathrm{~V}$ ? Assume that logic $0=0 \mathrm{~V}$ and $\operatorname{logic} 1=5 \mathrm{~V}$.

Q3 A simple Infra-Red Sensor system to detect passing human is presented as in Figure 3. A and B are IR Sensors which will generate different output voltages for different infra-red intensity, and higher voltage level corresponds to high light intensity.
(1) Explain how this system works for counting passing pedestrians.
(2) To increase counting accuracy, usually $B$ is covered with materials that can reflect infra-red light. Explain why.


Figure 3: IR-System.

Q4 Exemplify the working principles of sensors that measure: (1) Flow; (2) Temperature; (3) Pressure; (4) Motion; (5) Liquid Level.

Q5 Briefly describe how PID affects motor control.
Q6 Given a linear system

$$
\left\{\begin{align*}
\boldsymbol{x}_{t} & =\boldsymbol{A}_{t-1} \boldsymbol{x}_{t-1}+\boldsymbol{\omega}_{t-1},  \tag{2}\\
\boldsymbol{z}_{t} & =\boldsymbol{B}_{t} \boldsymbol{x}_{t}+\boldsymbol{v}_{t} \\
\boldsymbol{v}_{t} & =\boldsymbol{C}_{t-1} \boldsymbol{v}_{t-1}+\boldsymbol{n}_{t-1}
\end{align*}\right.
$$

where $\boldsymbol{\omega}_{t}$ and $\boldsymbol{n}_{t}$ are independent and obey Gaussian distribution zero-mean and covariance $Q_{t}$ and $\boldsymbol{R}_{t}$, respectively. Please give the estimate equation and measurement equation of the system.

Q7 Given two Gaussian distributions $N\left(x_{0} ; \mu_{0}, \sigma_{0}\right)$ and $N\left(x_{1} ; \mu_{1}, \sigma_{1}\right)$, try to give the expectation and variance of a new distribution which is the product of these two Gaussian distributions.

Q8 For the 4-bit R-2R DAC, calculate $V_{0}$ in terms of $V_{b, 0}-V_{b, 4}$ if $V_{r e f}$ is grounded (Figure4).


Figure 4: R-2R DAC.

Q9 [UPDATED] Assume the liner estimate system equation is $\mathbf{x}_{t+1}=\mathbf{A} \mathbf{x}_{t}+\mathbf{w}_{t}$. Given a second-autoregression random series:

$$
\begin{equation*}
x(t)=2.32 x(t-1)-0.76 x(t-2)+\omega_{t} \tag{3}
\end{equation*}
$$

Kalman Filter is used to estimate $x(t)$ (Here $x(t)$ is a scalar). Try to give the formulations of state transition matrix $\mathbf{A}$ and noise vector $\mathbf{w}_{t}$.

