

Each of the questions is worth 10 points. Please turn in solutions to *four* questions of your choice. Write your name, your student ID, and your TA's name on the solution sheet.

Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you must write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Questions

- Which of the following pairs of statements are logically equivalent?
 - If it is raining then the ground is wet.
If the ground is not wet then it isn't raining.
 - If it is raining then the ground is wet.
It is not raining or the ground is wet.
 - It is not true that it raining and the ground is wet.
It is not raining or the ground is not wet.
 - If you are rich then you are famous.
You are not famous if you are not rich.
- "Jason, I heard that you are learning logic", says Bob. "Please translate this strange note into English for me. You can use the this dictionary:"
 - $SPY(x)$ means " x is a spy."
 - $FRIEND(x, y)$ means " x and y are friends."
 - $EXPOSE(x, y)$ means "Operation x is exposed by y ."
 - $FAIL(x)$ means "Operation x fails."

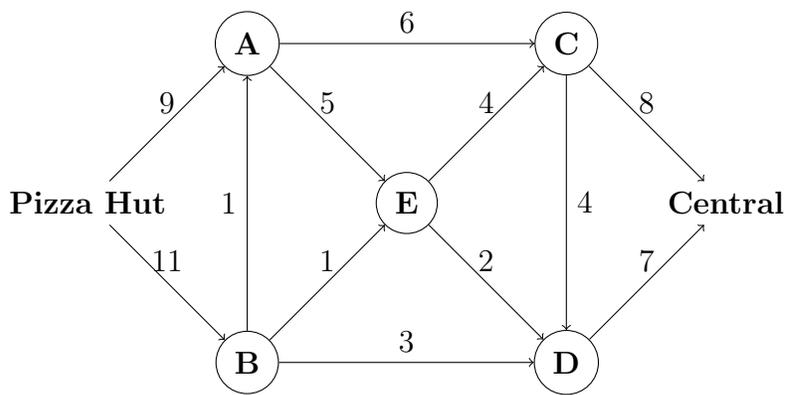
The note says:

- $\forall x: (x \neq \text{Alice}) \longrightarrow SPY(x)$
 - $\forall x, y: (SPY(x) \text{ AND } FRIEND(x, y)) \longrightarrow SPY(y)$
 - $\forall x \exists y: EXPOSE(x, y) \longrightarrow FAIL(x)$
 - $\forall x \forall y: EXPOSE(x, y) \longrightarrow SPY(y)$
- Express the following statements about positive integers using quantifiers and the predicates $P(x)$ which stands for " x is a prime number" and $E(x)$ which stands for " x is an even number".
 - Every even integer greater than 2 is the sum of two primes.
 - There is exactly one even prime number.

- (c) If you raise 2 to the power of a prime number and subtract one, you always get a prime number.
- (d) Two prime numbers are called *twin primes* if they differ by exactly two. There are infinitely many twin primes.
4. This is about the adventures of Alex, Bob, and Chris, three students of logic:
- (a) Alex, Bob and Chris go a village to collect rotten tomatoes and they get lost. They ask a villager for help. “Don’t worry”, the villager says, “I will help you if and only if I tell the truth.” Will the villager help them or not?
- (b) Alex, Bob, and Chris go to a cafe. The waiter asks, ”Does everyone want an espresso?” Alex says, “I am not sure if everyone wants an espresso.” Then Bob says, “I am not sure if everyone wants an espresso.” Finally, Chris says, “Yes, please bring everyone an espresso.” Can you explain what happened?
- (c) (**Extra credit**) Alex, Bob, and Chris stand in line, with Chris in front. Jason takes three white hats and two black hats, puts a hat on each head, and tosses out the remaining two hats. Alex observes Bob’s and Chris’s hats and announces “I don’t know the colour of my hat.” Bob sees Chris’s hat and says “I don’t know the colour of my hat.” What is the colour of Chris’s hat?
5. Jason is asking his friends whether they will join Occupy Central or not. He has collected the following information from four of his friends, Alex, Bob, Chris and David.
- Alex and Bob will not join together.
 - If Chris joins then Bob will join.
 - If David doesn’t join then Chris will join.
 - If David joins then Bob will join.

He wants to know who will join and he asks you to help him. Show your deduction clearly and carefully.

6. Jason is organising the distribution of pizza lunches for Occupy Central. Here is the network of roads that can be used to transport food:



An arrow pointing from X to Y with label t indicates that at most t pizza trucks can pass from X to Y before the officials figure out something suspicious is going on. For example, Jason can route 3 but not 4 trucks from Pizza Hut through B and then D to Central.

- (a) How should Jason route the pizza trucks in order to maximise the number of them that reach Central (without alerting the authorities)?
- (b) Explain convincingly why it is not possible for Jason to route any additional trucks.