Ten people take off their left and right socks and put them in a common bag. Each person then takes out two random socks from the bag (without replacement). What is the expected number of people who recover both their socks?

Solution: Let \( X_i \) be an indicator random variable for the event that the \( i \)-th person recovers both his/her socks (\( X_i = 1 \) if this happens, \( X_i = 0 \) if it doesn’t.) Then the number of people \( X \) who recover both their socks is

\[
X = X_1 + X_2 + \cdots + X_{10}.
\]

Even though the events \( X_1 = 1, \ldots, X_{10} = 1 \) are not independent, we can apply linearity of expectation to express \( E[X] \) as

\[
E[X] = E[X_1] + \cdots + E[X_{10}].
\]

Since \( X_i \) is an indicator random variable, \( E[X_i] = P(X_i = 1) \). Thus, the probability that person \( i \) recovers both his/her socks is

\[
P(X_i = 1) = 1/\binom{20}{2}
\]

and so \( E[X] = 10/\binom{20}{2} = 1/19 \approx 0.053 \).