Alice, Bob, and Charlie hold a lucky draw for two tickets to a concert with the following odds:

- The probability that Alice gets one of the tickets is 50%.
- The probability that Bob gets one of the tickets is 70%.

What is the probability that Alice and Bob both get tickets?

**Solution:** The sample space consists of the three outcomes \( \{ab, ac, bc\} \), where \( ab \) represents Alice and Bob getting the tickets, and so on. Denote their probabilities by \( p_{ab} \), \( p_{ac} \), and \( p_{bc} \). The event “Alice gets one of the tickets” is \( \{ab, ac\} \) so \( p_{ab} + p_{ac} = 0.5 \). Similarly \( p_{ab} + p_{bc} = 0.7 \). Since the probabilities must add up to one,

\[
p_{ab} = (p_{ab} + p_{ac}) + (p_{ab} + p_{bc}) - 1 = 0.5 + 0.7 - 1 = 0.2.
\]

**Alternative solution:** Let \( A \), \( B \), and \( C \) be the events “Alice gets a ticket” and so on. By the axioms the complementary events have probabilities \( P(A^c) = 0.5 \) and \( P(B^c) = 0.3 \). The events \( A^c \), \( B^c \), and \( C^c \) partition the sample space (they are disjoint and exactly one of the three must occur) so their probabilities must add up to one. Therefore \( P(C^c) = 1 - 0.5 - 0.3 = 0.2 \). The event \( C^c \) happens exactly when Alice and Bob both get tickets, so the desired probability is 20%.