Practice questions
Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. A point is chosen uniformly at random inside a triangle with base 1 and height 1. Let $X$ be the distance from the point to the base of the triangle. Find the CDF and the PDF of $X$. *(Textbook problem 3.2.5)*

2. There are 100 students in class. The arrival times of students (in minutes) are exponential random variables with rate $\lambda = 0.2$, starting from 09:20.
   (a) What is the expected number of students that have arrived by 09:30?
   (b) Assuming students’ arrivals are independent, what is the probability that everyone has made it by 09:45?

3. Three points are dropped at random on the perimeter of a circle with 1 unit circumference.
   (a) What is the probability that they all fall within $1/4$ of a unit of one another?
   (b) What is the probability that every pair of them is at least $1/4$ of a unit apart? *(Hint: Fix one of the three points.)*

4. A coin has probability $P$ of being heads, where $P$ itself is a Uniform(0, 1) random variable. Find the PMF of the number of heads after performing two independent coin flips.

5. Here is a way to solve Buffon’s needle problem without calculus. Recall that an $\ell$ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.
   (a) Let $A$ be the number of lines that the needle hits. Let $B$ be the number of times that a polygon of perimeter $\ell$ hits a line. Show that $E[A] = E[B]$. *(Hint: Use linearity of expectation.)*
   (b) Assume that $\ell < \pi$. Calculate the expected number of times that a circle of perimeter $\ell$ hits a line.
   (c) Assume that $\ell < 1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. *(Hint: The number of hits is a 0/1-valued random variable.)*