Let $P$ be a random variable that is equally likely to take the values 1/3 and 2/3. Let $X$ be the number of heads when a coin with probability of heads $P$ is tossed 10 times. Find $\text{Var}[E[X|P]]$.

**Solution:** The variance of $P$ is $\frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{2}\right)^2 + \frac{1}{2} \cdot \left(\frac{2}{3} - \frac{1}{2}\right)^2 = \frac{1}{36}$, so

$$\text{Var}[E[X|P]] = \text{Var}[10P] = 100 \cdot \text{Var}[P] = \frac{100}{36} = 2\frac{7}{9}.$$