Question 1

Let $n$ be a power of 3. The recursive majority of threes on $n$ bits is defined by the recursive formula

$$RMAJ(x, y, z) = MAJ(RMAJ(x), RMAJ(y), RMAJ(z)),$$

where $x, y, z \in \{0, 1\}^{n/3}$ with $RMAJ(x) = x$ for $x \in \{0, 1\}$. Show that any DNF for $RMAJ$ must have width at least $n \log_3 2$.

Question 2

A read-once DNF is a DNF in which every variable appears at most once. Show the existence of a function $f$ that has a read-once DNF such that NOT $f$ does not have a read-once DNF.

Question 3

Assume the function $f: \{0, 1\}^n \to \{0, 1\}^n$ is $(2^{\Omega(n)}, 2^{-\Omega(n)})$-one-way. Show that the function $g: \{0, 1\}^{2n} \to \{0, 1\}^{2n}$

$$g(x, x') = \begin{cases} 0^{2n}, & \text{if } x' = 0^n, \\ (f(x), 0^n), & \text{otherwise} \end{cases}$$

is $(2^{\Omega(n)}, 2^{-\Omega(n)})$-one-way, but $h(x, x') = g(g(x, x'))$ is not $(p(n), 1/p(n))$-one-way for any polynomial $p$.

Question 4

Let $\delta > 0$ and suppose $C'$ is a randomized oracle circuit with oracle type $\{0, 1\}^n \to \{0, 1\}$ such that

- $\Pr[C' \text{ accepts}] \geq 2/3$ for every $f$ such that $f(x) = 1$ for at least $\delta 2^n$ values of $x$, and
- $\Pr[C^0 \text{ accepts}] \leq 1/3$ where $0$ is the all zero function.

Show that $C'$ must make $\Omega(1/\delta)$ oracle queries for sufficiently large $n$. 