

Each of the problems is worth 10 points. Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you *must* write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please turn in the solutions by 11.59pm on Thursday 8 October. The homework should be dropped off in the box labeled CSC 3130 on the 9th floor of SHB. Late homeworks will not be accepted.

Problem 1

Which of the following statements are correct? If you think a statement is correct, give a proof. If you think it is incorrect, give a counterexample.

- (a) If L is regular, then $L' = \{w : w \text{ is a string in } L \text{ of odd length}\}$ is also regular.
- (b) If L is regular, then $L' = \{wxw : w, x \text{ are strings in } L\}$ is also regular.
- (c) For a string $w = w_1w_2 \dots w_n$, let $w^{\&} = w_1w_1w_2w_2 \dots w_nw_n$. For example, $(abb)^{\&} = aabbbb$. If L is regular, then $L' = \{w^{\&} : w \in L\}$ is also regular.
- (d) If L is regular over alphabet $\Sigma = \{0, 1\}$, then $L' = \{w : 0w1 \text{ is a string in } L\}$ is also regular.
- (e) If L is regular, then

$$L' = \{ww' : w, w' \text{ are strings in } L \text{ such that } w \neq w'\}$$

is also regular.

- (f) **(Extra credit)** For a string w , let w^{odd} be the string obtained by keeping only those symbols of w in odd positions. For instance, if $w = \text{springs}$, then $w^{\text{odd}} = \text{srns}$; if $w = \text{mama}$, then $w^{\text{odd}} = \text{mm}$. If L is regular, then

$$L' = \{w^{\text{odd}} : w \text{ is a string in } L\}$$

is also regular.

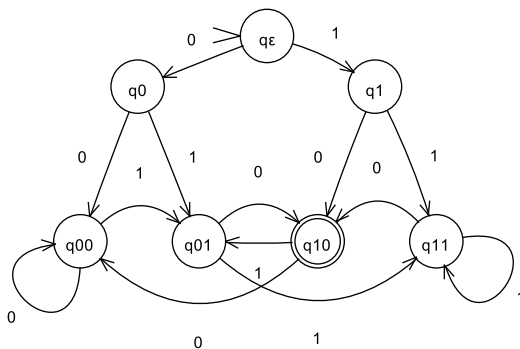
Problem 2

Which of the following languages are regular, and which are not? If a language is regular, draw the minimal DFA for it and show that every pair of states is distinguishable. If a language is not regular, prove it using the pumping lemma.

- (a) $L_1 = \{0^n 10^n : n \geq 0\}$, $\Sigma = \{0, 1\}$.
- (b) $L_2 = \{0^n 10^m : m \neq n; m, n \geq 0\}$, $\Sigma = \{0, 1\}$.
- (c) $L_3 = \{w : w \text{ has fewer 0s than 1s and 2s together}\}$, $\Sigma = \{0, 1, 2\}$.
- (d) $L_4 = \{w : w \text{ has at least one 1 and at least two 2s}\}$, $\Sigma = \{1, 2\}$.
- (e) **(Extra credit)** $L_5 = \{w : w \text{ has the same number of patterns 011 and 110}\}$, $\Sigma = \{0, 1\}$.
- (f) $L_6 = \{w : w \text{ has the same number of patterns 010 and 101}\}$, $\Sigma = \{0, 1\}$.

Problem 3

This problem concerns the following DFA.



- (a) Run the minimization algorithm on this DFA. Clearly show the different stages that the minimization algorithm goes through.
- (b) Show that every pair of states in the minimized DFA is distinguishable.
- (c) Convert the minimized DFA into a regular expression using the conversion algorithm from class.

Problem 4

You have a file `cities.txt` that lists the latitudes, longitudes, and time zones of various cities around the world:

```
Aberdeen, UK          57.4N   2.9W   12.00pm
Adelaide, Australia  34.55S 138.36E 9.30pm
Algiers, Algeria     36.50N  3E     1.00pm
Ankara, Turkey       39.55N 32.55E 2.00pm
Ascuncion, Paraguay  25.15S 57.40W 8.00am
...
```

Write `grep` commands (including a short explanation about how they work) that will search for the following information in the file:

- All cities in the US and Canada in the 6.00am time zone.
- All cities in the tropics (latitude 33S to 33N).
- All cities with longitude between 10E and 20E *not* in the 1.00pm time zone.
- All cities whose time zone does not fall on the hour, for example:

```
Adelaide, Australia  34.55S 138.36E 9.30pm
Kathmandu, Nepal    27.43N 25.19E 5.45pm
Rangoon, Myanmar    16.50N 96E   6.30pm
```