1 Turing Machine

1.1 DFA and PDA and TM

(a) DFA

- DFA contains finite states.
- When staying in the current state, read a symbol from input string, then move to another state according to transition function \( \sigma : Q \times \Sigma \to Q \).
- Accept input string when reach an accept state after read whole input string.

(b) PDA

- PDA contains finite states and an infinite stack (memory).
- When staying in the current state, read a symbol from input string and pop a symbol from the stack, then push a symbol to the stack and move to another state according to transition function \( \sigma : Q \times \Sigma \times \Gamma \to 2^{Q \times \Gamma} \).
- Accept input string when we reach an accept state after read whole input string.

(c) TM

- TM contains finite states and an infinite tape (input string + memory).
- When staying in the current state, read a symbol from the tape, then write a symbol to the tape, move to another state and move the head to the left or the right according to transition function \( \sigma : Q \times \Gamma \to Q \times \Gamma \times \{L, R\} \).
- Halt immediately when reach accept state or reject state.

1.2 Formal description of TM

A Turning machine is a 7-tuple, \( (Q, \Sigma, \Gamma, \sigma, q_0, q_{\text{accept}}, q_{\text{reject}}) \), where \( Q, \Sigma, \Gamma \) are all finite sets.

(a) \( Q \) is the set of states,

(b) \( \Sigma \) is the input alphabet not containing the blank symbol \( \sqcup \),

(c) \( \Gamma \) is the tape alphabet, where \( \sqcup \in \Gamma \) and \( \Sigma \subseteq \Gamma \).

(d) \( \sigma : Q \times \Gamma \to Q \times \Gamma \times \{L, R\} \) is the transition function.

(e) \( q_0 \in Q \) is the start state.
(f) $q_{\text{accept}} \in Q$ is the accept state, and 

(g) $q_{\text{reject}} \in Q$ is the reject state, where $q_{\text{reject}} \neq q_{\text{accept}}$.

2 An Example

Describe a Turning machine (TM) $M_2$ that decides $A = \{0^{2^n} | n \geq 0\}$, the language consisting of all string of 0s whose length is a power of 2.

- A high level description of $M_2 = "$ On input string $w$:
  
  (a) Sweep left to right across the tape, crossing off every other 0.
  
  (b) If in stage (a) the tape contained a single 0, accept.
  
  (c) If in stage (a) the tape contained more than a single and the number of 0s was odd, reject.
  
  (d) Return the head to the left-hand end of the tape. Go to stage (a)."

- State diagram for Turing machine $M_2$.

  ![State Diagram](image)

  Here $\Gamma = \{0, x, \square\}$. We can cross off every other 0 in part $q_3, q_4$. If the tape contains just a single 0 we will accept the string in part $q_1, q_2, q_{\text{acc}}$. And if the tape contains more than a single 0 and the number of 0s is odd we will reject the string in part $q_4, q_{\text{rej}}$. 