Problem 1

Design a TM M to accept the language $L = \{0^n 1^n \mid n \geq 1\}$.

Solution

Initially, the type of M contains 0^n1^n followed by an infinity of blanks. Repeatedly, M replaces the leftmost 0 by X, moves right to the leftmost 1, replacing it by Y, moves left to find the rightmost X, then moves one cell right to the leftmost 0 and repeats the cycle. If, however, when searching for a 1, M finds a blank instead, then M halts without accepting. If, after changing a 1 to a Y, M finds no more 0's, then M checks that no more 1's remain, acceppting if there are none.

Let $Q = \{q_0, q_1, q_2, q_3, q_4\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, X, Y, B\}$, and $F = \{q_4\}$. Informally, each state represents a statement or a group of statements in a program. State q_0 is entered initially and also immediately prior to each replacement of a leftmost 0 by an X. State q_1 is used to search right, skipping over 0's and Y's untial it finds the leftmost 1. If M finds a 1 it changes it to Y, entering state q_2. State q_2 searches left for an X and enters state q_0 upon finding it, moving right, to the leftmost 0, as it changes sstate. As M searches right in state q_1, if a B or X is encountered before a 1, then the input is rejected; either there are too many 0's or the input is not in 0^*1^*.

State q_0 has another role. If, after state q_2 finds the rightmost X, there is a Y immediately to its right, then the 0's are exhausted. From q_0, scanning Y, state q_3 is entered to scan over Y's and check that no 1's remain. If the Y's are followed by a B, state q_4 is entered and acceptance occurs; otherwise the string is rejected. The function is shown below.

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)	$-$	$-$	(q_3, Y, R)	$-$
q_1	$(q_1, 0, R)$	(q_2, Y, L)	$-$	(q_1, Y, R)	$-$
q_2	$(q_2, 0, L)$	$-$	(q_0, X, R)	(q_2, Y, L)	$-$
q_3	$-$	$-$	$-$	(q_3, Y, R)	(q_4, B, R)
$* \ q_4$	$-$	$-$	$-$	$-$	$-$

Problem 2

Design Turing machines to recognize $\{ww^R | w \text{ is in } (0+1)^*\}$

Solution

	0	1	X	B
$\rightarrow q_0$	(q_1, X, R)	(q_2, X, R)	(q_6, X, R)	(q_6, B, R)
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	(q_4, X, L)	(q_4, B, L)
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$	(q_5, X, L)	(q_5, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, X, R)	$-$
q_4	(q_3, X, L)	$-$	$-$	$-$
q_5	$-$	(q_3, X, L)	$-$	$-$
$* q_6$	$-$	$-$	$-$	$-$

The state q_0 goes right on the tape and find the first one that is not X, say a, replace it by X. Then goes right to find the first X or B, if its left symbol is a, replace it by X, otherwise, reject.