These are the examples for Tutorial 2 with solutions. The alphabet is $\Sigma = \{0, 1\}$ in all the examples.

Problem 1

$L_1 = \{0^{n^2} \mid n \text{ is an integer and } n \geq 0\}$

Solution

Suppose it is regular, then there exist a DFA that accept L_1 and there are N states in the minimal DFA, say S. Let us choose the string $z = 0^{N^2}$. By pumping lemma, $z = uvw$, where $|uv| \leq N$ and $|v| \geq 1$, uv^iw can be accepted by S for every integer $i \geq 0$.Especially, when $i = 2$, uv^2w can be accepted by S. Let us check whether uv^2w is in L_1.

$$N^2 = |uvw| < |uv^2w| = |uvw| + |v| \leq |uvw| + |uv| \leq N^2 + N < N^2 + 2N + 1 = (N+1)^2$$

The length of uv^2w is not square of any integers, then uv^2w is not in L_1, contradiction.

Problem 2

$L_2 = \{0^m1^n \mid m > n \geq 0\}$

Solution

Suppose L_2 is regular, then there exist a DFA, say S, with N states that can accept L_2. Choose $z = 0^N1^{N-1}$ and $z = uvw$, where $|uv| \leq N$ and v is the pumping, thus $|v| \geq 1$. With only pumping deleted, uw also can be accepted by S. Notice that all symbols in v are 0s, then $uw \notin L_2$, contradiction.

Problem 3

$L_3 = \{0^{2n} \mid n \geq 1\}$

Solution

It is easy to construct a DFA for L_3, so it is regular.

Problem 4

$L_4 = \{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$

Solution

Suppose L_4 is regular, then there is a DFA, say S, with N states that can accept L_4. Choose $z = 0^N 1^N 0^{2N}$ and $z = uvw$, where $|uv| \leq N$ and v is the pumping, thus $|v| \geq 1$. With only pumping deleted, uw also can be accepted by S. Notice that all symbols in v are 0s, then $uw \notin L_4$, contradiction.

Problem 5

$L_5 = \{0^n \mid n \text{ is a prime}\}$

Solution

Suppose it is regular, then there is a DFA, say S, with N states that can accept L_5. Choose $z = 0^p$, where p is a prime and $p \geq N$. We have the patition $z = uvw$, where $|uv| \leq N$ and v is the pumping, thus $|v| \geq 1$. Consider the string $z' = uv^{|z|+1}w$, it can be accepted by L_5, but $|z'| = |z| + |z||v| = |z|(1 + |v|)$ is not a prime, contradiction.

Problem 6

$L_6 = \{x \mid x \text{ does not have three consecutive 0s}\}$.

Solution

L_6 is regular. You can construct a DFA for it.

Problem 7

$L_7 = \{x \mid x \text{ has an equal number of 0s and 1s}\}$.

Solution

Suppose L_7 is regular, then there is a DFA, say S, with N states that can accept L_7. Choose $z = 0^N 1^N$ and $z = uvw$, where $|uv| \leq N$ and v is the pumping, thus $|v| \geq 1$. With only pumping deleted, uw also can be accepted by S. Notice that all symbols in v are 0s, then there are less 0's than 1's in uw, $uw \notin L_7$, contradiction.

Problem 8

$L_8 = \{x \mid x = x^R\}$. Recall that x^R is x written backwards; for example, $(011)^R = 110$.

Solution

Suppose L_8 is regular, then there is a DFA, say S, with N states that can accept L_8. Choose $z = 0^N 110^N$ and $z = uvw$, where $|uv| \leq N$ and v is the pumping, thus $|v| \geq 1$. With only pumping deleted, uw also can be accepted by S. Notice that all symbols in v are 0s, then $uw \notin L_8$, contradiction.

Problem 9

$L_9 = \{x \mid x \text{ has a different number of 0s and 1s}\}$.

Solution

The easier way to prove L_9 is not regular goes like this. Suppose it is regular, then L_7 is L_9's complement, hence L_7 is regular, contradiction.

If you want to prove L_9 is not regular using the pumping lemma, it is also possible, but a bit more difficult. Suppose it is regular, then there is a DFA, say S, with N states that can accept L_9. Choose $z = 0^N 1^{N+N!}$ and $z = uvw$, where $|uv| \leq N$ and v is the pumping, thus $|v| \geq 1$. (Here $N! = 1 \cdot 2 \cdots N$.) Then $z' = uv^i w$ can be accepted by S for every nonnegtive integer i. Set $i = N!/|v| + 1$, then $z' = uv^{N!/|v|+1} w$. Notice that all symbols in v are 0s, it is easy to check $z' \notin L_9$, contradiction.