

Each of the problems is worth 10 points. Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you *must* write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please turn in the solutions by 11.59pm on Thursday 27 November. The homework should be dropped off in the box labeled CSC 3130 on the 9th floor of SHB. Late homeworks will not be accepted.

Problem 1

For each of the following problems, say whether it is decidable or not. Justify your answer by describing an appropriate Turing Machine (algorithm), or by reducing from problems that were shown undecidable in class.

- (a) $L_1 = \{\langle G \rangle : \text{The empty string has a unique parse tree in CFG } G\}$.
- (b) $L_2 = \{\langle G \rangle : \text{CFG } G \text{ is an LR}(0) \text{ grammar}\}$.
- (c) $L_3 = \{\langle G \rangle : \text{CFG } G \text{ generates all strings that start and end with the same symbol}\}$.
- (d) $L_4 = \{\langle G \rangle : \text{CFG } G \text{ generates all strings of the form } w\%w^R, \% \notin w\}$.

Problem 2

For each of the following variants of the Post Correspondence Problem (PCP), say if it is decidable or not. Justify your answer by describing an appropriate algorithm or by reducing from PCP.

- (a) PCP_1 : PCP over the alphabet $\Sigma = \{1\}$.
- (b) PCP_2 : PCP over the alphabet $\Sigma = \{0, 1\}$.

The *alphabet* of a tile is the set of symbols that appear on the tiles. For instance in the following example the alphabet is $\Sigma = \{a, b, c, d\}$:

| | | | |
|----|----|----|---|
| ab | ab | d | b |
| bc | d | cd | d |

Problem 3

A *triangle* in a graph is a clique of size three (that is, a set of three vertices $\{u, v, w\}$ so that (u, v) , (v, w) , and (u, w) are all edges in the graph). A *triangle cover* is a subset of vertices that touches all the triangles in the graph. Let

$$TC = \{(G, k) : G \text{ is a graph with a triangle cover of size } k\}.$$

- (a) Show that TC is in NP.
- (b) Show that TC is NP-complete by giving a polynomial-time reduction from vertex-cover (VC).

Problem 4

Now it is time to reflect on the last few months and provide some feedback on your experience as a student in CSC 3130. You may use the following questions as guidelines, but feel free to write your own observations if you prefer.

- What was your favorite topic? What was your least favorite one?
- Are there any topics that piqued your interest and wished we had spent more time on? Any ones you found uninteresting, or wished we had skipped altogether? Anything you might want to learn more about in the future?
- Did you feel sufficiently prepared for the class? If not, what kind of preparation do you think would have been helpful to have? Were you overprepared? If so, were there any topics that were uninteresting because you had seen them before?
- Do you think what you learned here will be useful for you in your future studies or profession?
- Do you think the knowledge you acquired here is more conceptual (it will help you think the right way) or technical (it will help you solve specific problems that come along)? Or neither?
- Could the material have been made more compelling for you if we did things differently? If so, how? (E.g., more examples, a different textbook, more/less/different homeworks)
- Did you find the audio recordings of the lectures useful at all? Did you ever listen to them?