Each of the problems is worth 10 points. Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you *must* write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please turn in the solutions by 11.59pm on Thursday 2 October. The homework should be dropped off in the box labeled CSC 3130 on the 9th floor of SHB. Late homeworks will not be accepted.

Problem 1

Which of the following statements are correct? If you think a statement is correct, give a proof. If you think it is incorrect, give a counterexample.

(a) Let $\Sigma_1 = \{a, b\}$ and $\Sigma_2 = \{a, b, c\}$. If language L is regular over Σ_1, then L is also regular over Σ_2.

(b) If L_1 is not regular and L_2 is not regular, then $L_1 \cup L_2$ is not regular.

(c) If L is regular over alphabet $\Sigma = \{a, b\}$, then the language

$$L' = \{x : ax \text{ is a string in } L\}$$

is also regular.

(d) If L is regular, then the language

$$L' = \{wxw : x, w \text{ are strings in } L\}$$

is also regular.

(e) If L is regular, then the language

$$L' = \{w : \text{for some string } x,\ xw \text{ is in } L\}$$

is also regular.

(f) **(Extra credit)** If L is regular, then the language

$$L' = \{w : ww \text{ is a string in } L\}$$

is also regular.

Problem 2

Which of the following languages are regular, and which are not? To show a language is regular, give a DFA, NFA, or regular expression for it (with explanation). To show a language is not regular, prove it using the pumping lemma.

(a) $L_1 = \{0^n 1 0^n : n \geq 0\}$, $\Sigma = \{0, 1\}$.

(b) $L_2 = \{0^n 1 0^m : n, m \geq 0\}$, $\Sigma = \{0, 1\}$.

(c) $L_3 = \{x : x \text{ has more } b\text{s than } a\text{s and } c\text{s together}\}$, $\Sigma = \{a, b, c\}$.

(d) $L_4 = \{x : \text{all symbols in } x \text{ occur a different number of times}\}$, $\Sigma = \{a, b, c\}$.

(e) $L_5 = \{x : x \text{ has the same number of patterns 001 and 110}\}$, $\Sigma = \{0, 1\}$.

(f) $L_6 = \{x : x \text{ has the same number of patterns 010 and 101}\}$, $\Sigma = \{0, 1\}$.

Problem 3

Run the minimization algorithm on the DFA given by the following transition table:

	0	1
$\rightarrow q_0$	q_1	q_4
q_1	q_1	q_2
$*q_2$	q_3	q_3
q_3	q_3	q_3
$*q_4$	q_3	q_3

Clearly show the different stages that the minimization algorithm goes through. After you perform the minimization, prove that all pairs of states of the minimized DFA are distinguishable.

Problem 4

You have a file `c.txt` that lists the latitudes, longitudes, and time zones of various cities around the world:

```
Aberdeen, UK           57.4N    2.9W      12.00pm
Adelaide, Australia    34.55S   138.36E   9.30pm
Algiers, Algeria       36.50N   3E        1.00pm
Ankara, Turkey         39.55N   32.55E    2.00pm
Ascuncion, Paraguay    25.15S   57.40W    8.00am
...
```

Write `grep` commands (including a short explanation about how they work) that will search for the following information in the file:

(a) Any cities in the US in the 3.00am time zone.

(b) Any cities in the Western hemisphere and South of the equator.

(c) Any cities in the Arctic Circle (latitude 66N to 90N).

(d) Any cities whose time zone does not fall on the hour, for example:

```
Adelaide, Australia    34.55S   138.36E   9.30pm
Kathmandu, Nepal       27.43N   25.19E    5.45pm
Rangoon, Myanmar       16.50N   96E       6.30pm
```