

Exact and Stable Recovery of Pairwise Interaction Tensor

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*a scalable algorithm for recovering pairwise
interaction tensors from limited observations*

Recovery of Pairwise Interaction Tensor

| Object | Decomposition | Recovery |
|---|--|---|
| rank- k matrix $\mathbf{M} \in \mathbb{R}^{n_1 \times n_2}$ | $M_{ij} = \langle u_i, v_j \rangle$ | [Candes et al. 2009] guaranteed recovery of \mathbf{M} from $O(nk \log^2(n))$ observations |
| rank- k tensor $\mathbf{T} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ | $T_{ijk} = \langle u_i, v_j, w_k \rangle$ | computing the rank is NP-hard best rank-1 approximation is NP-hard |
| pairwise interaction tensor $\mathbf{T} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ | T_{ijk} $= \langle u_i^{(a)}, v_j^{(a)} \rangle + \langle v_j^{(b)}, w_k^{(b)} \rangle$ $+ \langle w_k^{(c)}, u_i^{(c)} \rangle$ | this paper: guaranteed recovery of \mathbf{T} from $O(nk \log^2(n))$ observations. |

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- Pairwise interaction tensor is a special case of tensor.
 - Good performance in various applications
 - Tag recommendation [Rendle et al. 2009]
 - Sequential data analysis [Rendle et al. 2010]
 - Existing recovery algorithms are local optimization heuristics.

Key ideas and results

- Reduce to a **matrix completion** problem.
- Formulate a **constrained trace norm minimization** objective.
- Optimize using **SVT**.



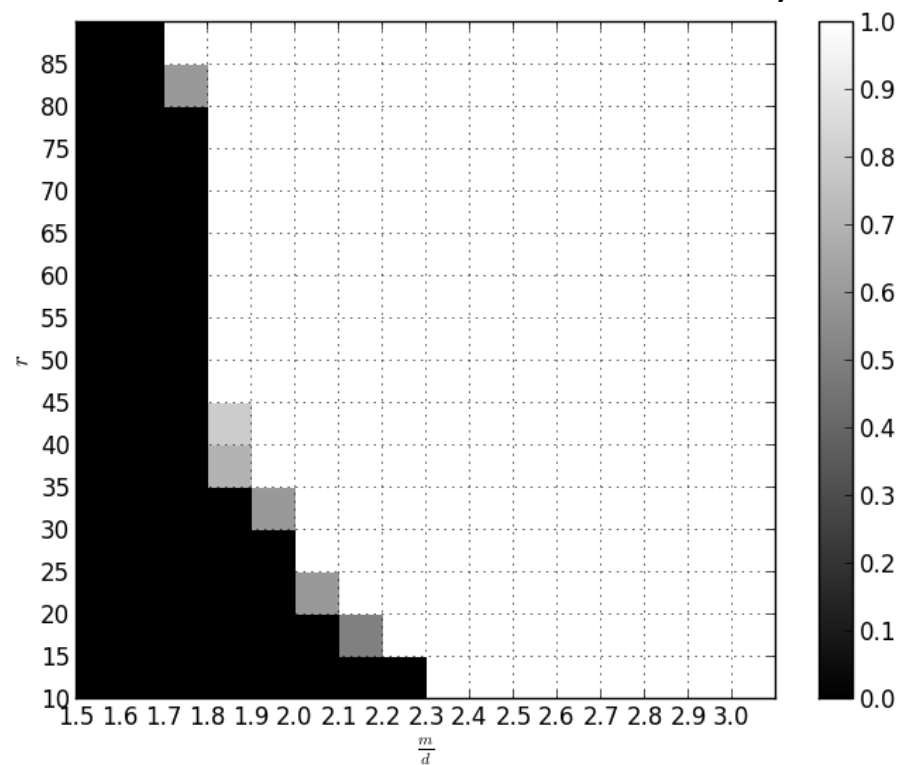
$$\min_{(X,Y,Z) \in S} \sqrt{n_3} \|X\|_* + \sqrt{n_1} \|Y\|_* + \sqrt{n_2} \|Z\|_*$$
$$\text{s. t. } X_{ij} + Y_{jk} + Z_{ki} = T_{ijk}, \forall ijk \in \Omega$$

- S is a constraint for ensuring **uniqueness** of recovery.
- Ω is the set of observations
- (formulation of exact recovery)

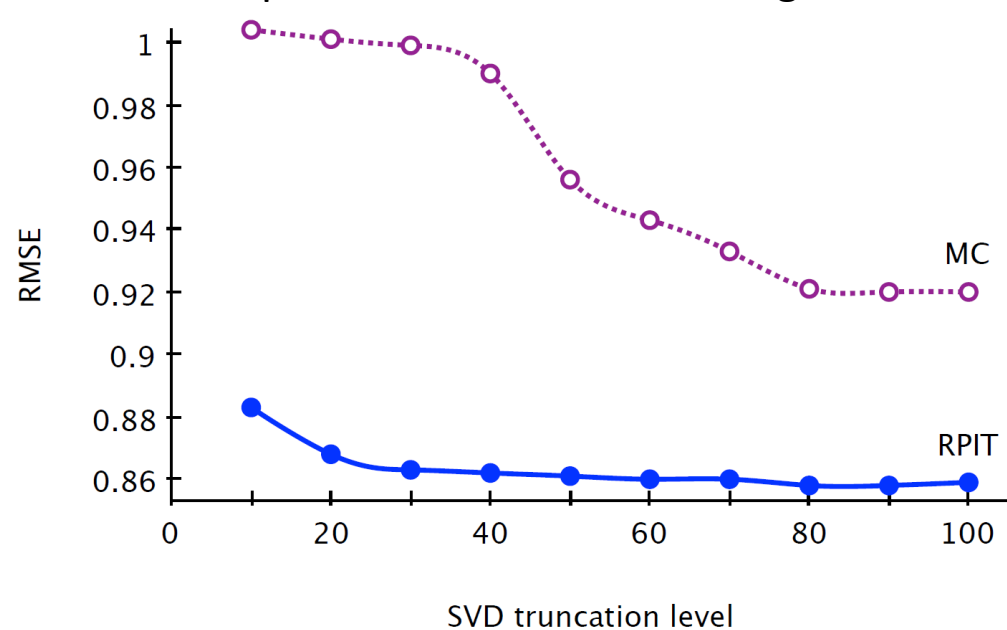
Theoretical results

- **Exact observations.**
 - Recovery is **exact** from $O(nk \log^2(n))$ observations
- **Noisy observations**
 - Provable approximation guarantees.

Phase Transition of Exact Recovery



Temporal Collaborative Filtering



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