CMSC5733 Social Computing

04-Graph Mining

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Outline

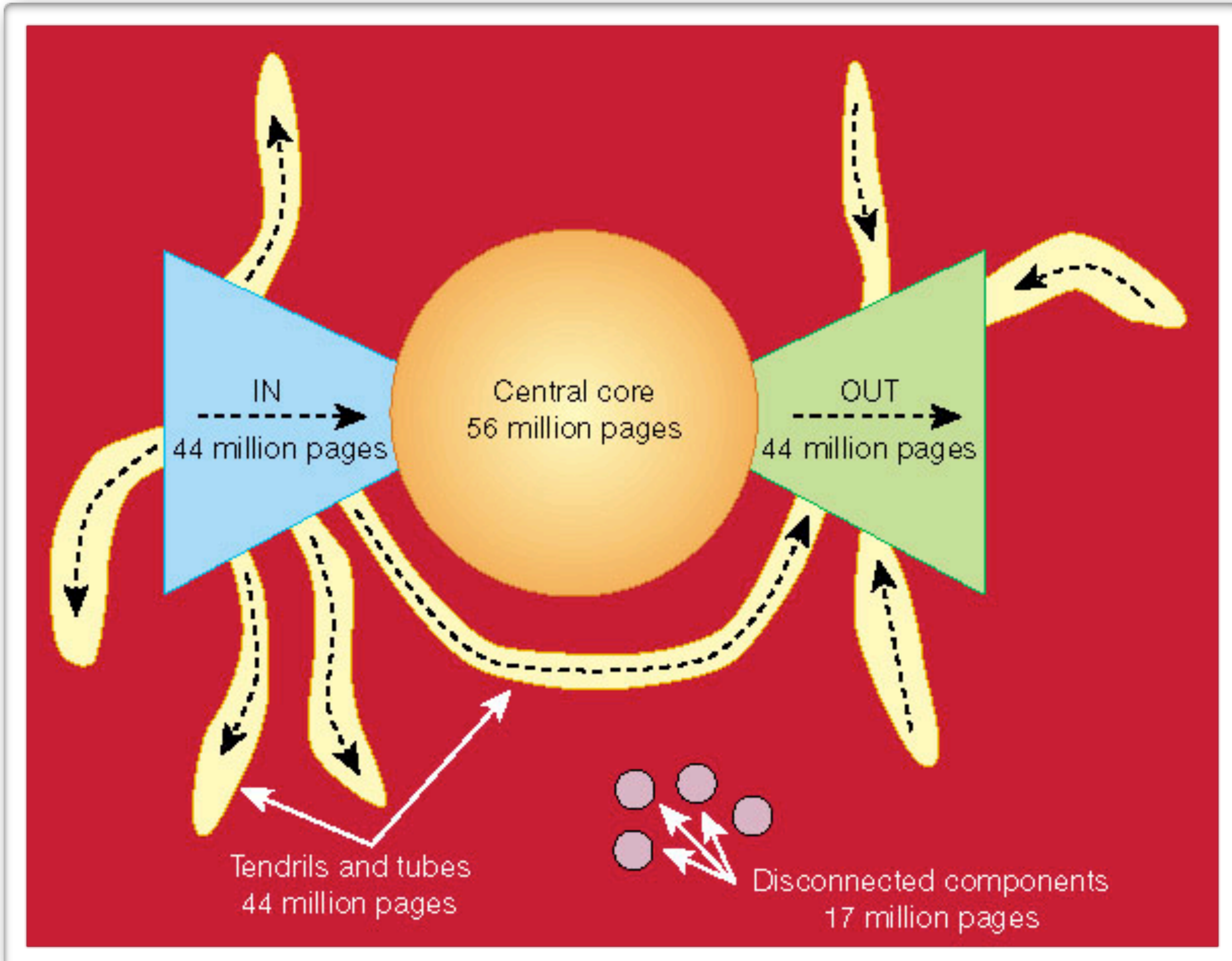
- Graph Characteristics, Patterns, and Structures
- Graph Generation & Information Propagation
- Graph Mining Algorithms

Graph Structures

Graph Patterns

- What are the characteristics of graphs?
- How can we compare graphs?
- What patterns hold for these graphs?
  - Power laws
  - Small diameters
  - Community effects
- How does the Internet graph look like?

What Does the Web Look Like?



- Recursive bowtie structure
- Ease of navigation
- Resilience

Introduction

- Graph mining is simply extraction of information from a massive graph
  - How does any network look like? The visualization of the relationship. One example is to look into how does the Internet or web look like.
  - Once we can characterize something, then we may be able to explore what is unique, abnormal, etc.
  - Are there any characteristics/principles/laws that hold?

Graph Distributions

- Two variables x and y are related by a power law when their scatter plot is linear on a log-log scale:

$$y(x) = cx^{-\gamma} \tag{1}$$

  where c and γ are positive constants.

- The constant γ is often called the **power law exponent**.

- **Power Law Distribution**. A random variable is distributed according to a power law when the probability density function (pdf) is given by

$$p(x) = cx^{-\gamma}, \gamma > 1, x \geq x_{\min} \tag{2}$$

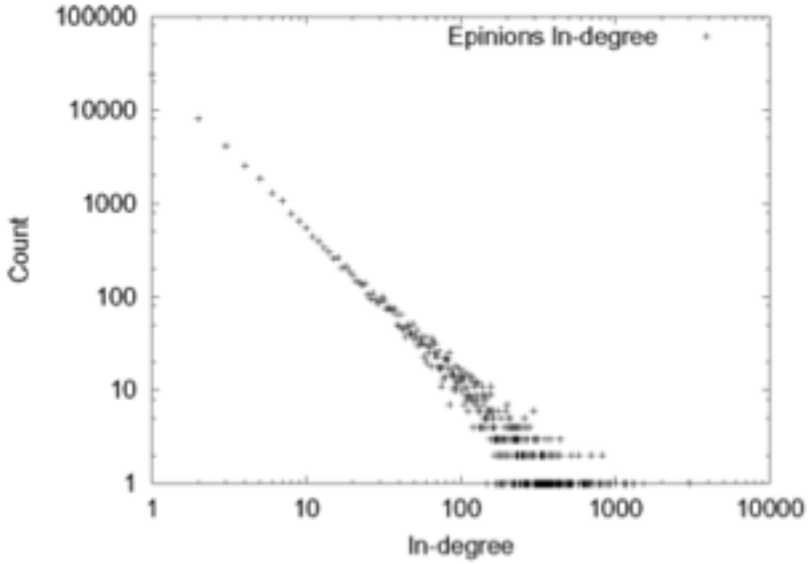
- $\gamma > 1$ ensures that $p(x)$ can be normalized.

- It is unusual to find $\gamma < 1$ in nature.

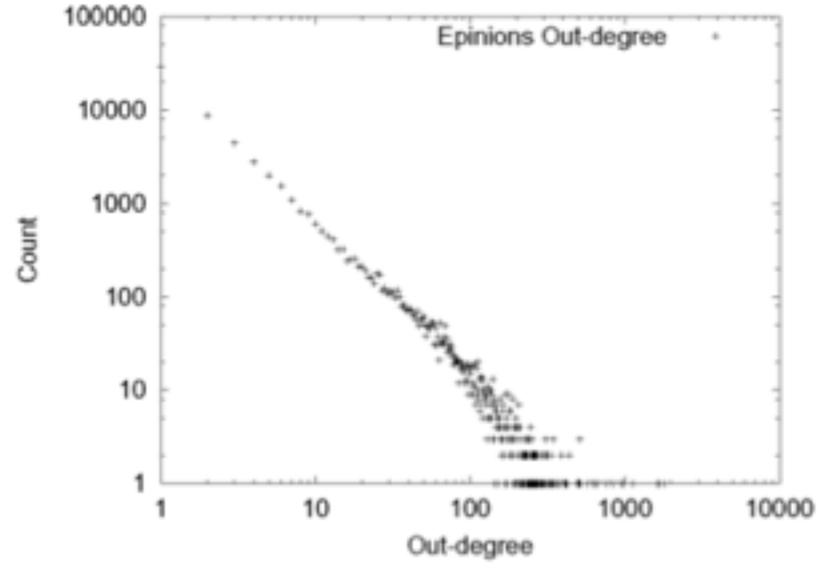
Degree Distribution

- The **Degree Distribution** of an undirected graph is a plot of the count c_k of nodes with degree k, versus the degree k, typically on a log-log scale.
- Occasionally, the fraction $\frac{c_k}{N}$ is used instead of c_k; however, this merely translates the log-log plot downwards.
- For directed graphs, out-degree and in-degree distributions are defined separately.
- Computational issues:
  1. Creating the scatter plot
  2. Computing the power law exponent
     – Regression models, maximum-likelihood estimation(MLE), non-parametric estimators, etc.
  3. Checking for goodness of fit
     – Correlation coefficient, statistical hypothesis methods, etc.

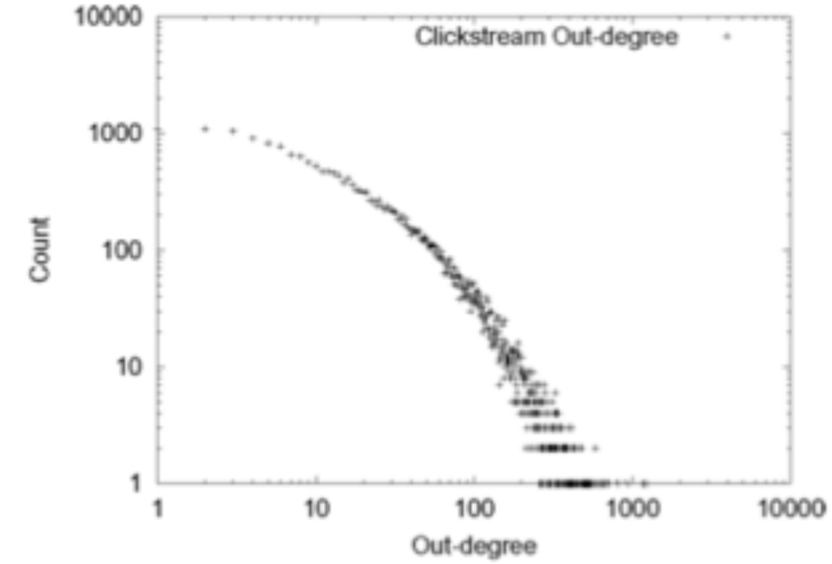
Examples of Power Law



(a) Epinions In-degree



(b) Epinions Out-degree



(c) Clickstream Out-degree

- Internet graph (2.1-2.2), Internet router (2.48), in-degree (2.1) and out-degree (2.38-2.72) of the WWW graph, PageRank, citation graph (3), etc.
- Power Law distributions are *heavy-tailed* so they decay more slowly than Gaussian distributions with exponential decay!

Other Distributions

- **Exponential Cutoffs**. Looks like power law over the lower range of values, but decays very fast for higher values. It is defined as,

$$y(x = k) \propto e^{-k/\kappa} k^{-\gamma}$$

  where $e^{-k/\kappa}$ is the exponential cutoff term, and $k^{-\gamma}$ is the power law term.

- The airport network, electric power grid of Souther California are examples of the exponential cutoffs distribution.

- **Longnormals**. Sometimes subsets of a power law graph can deviate significantly. It looks like a truncated parabolas on log-log scale.

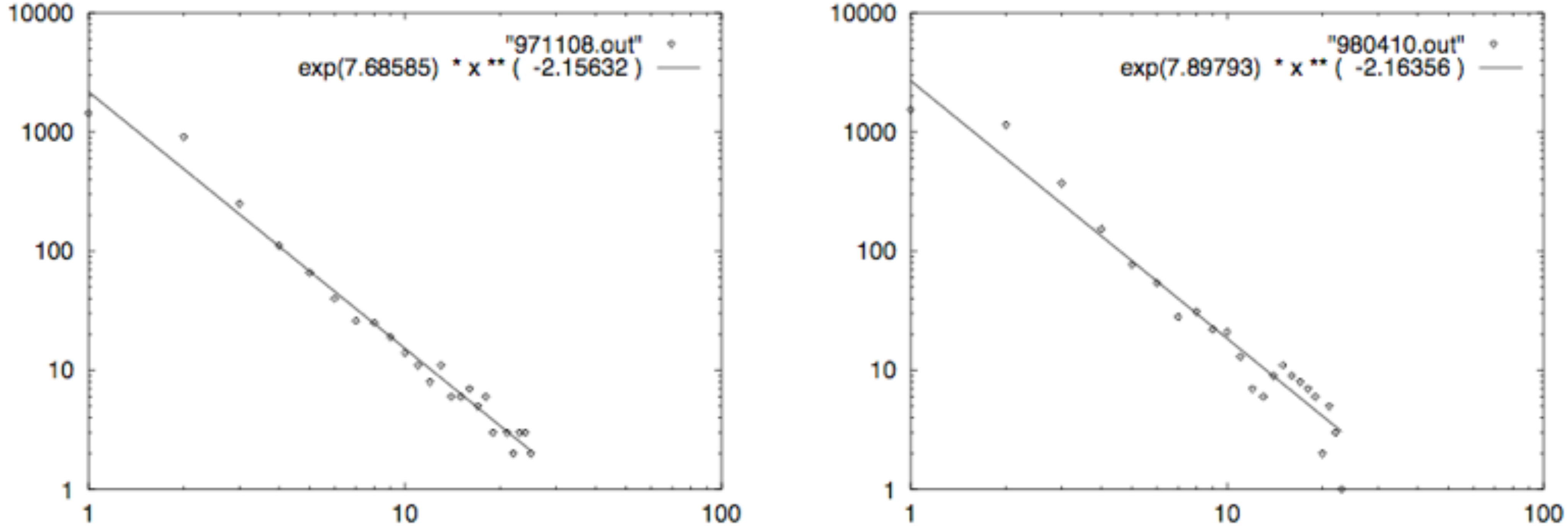
- It has unimodal distributions on the log-log scale and a discrete truncated lognormal (Discrete Gaussian Exponential, DGX) has a good fit.

$$y(x = k) = \frac{A(\mu, \sigma)}{k} \exp\left[-\frac{(\ln k - \mu)^2}{2\sigma^2}\right], k = 1, 2, \ldots,$$

  where μ and σ are parameters and $A(\mu, \sigma)$ is a constant.

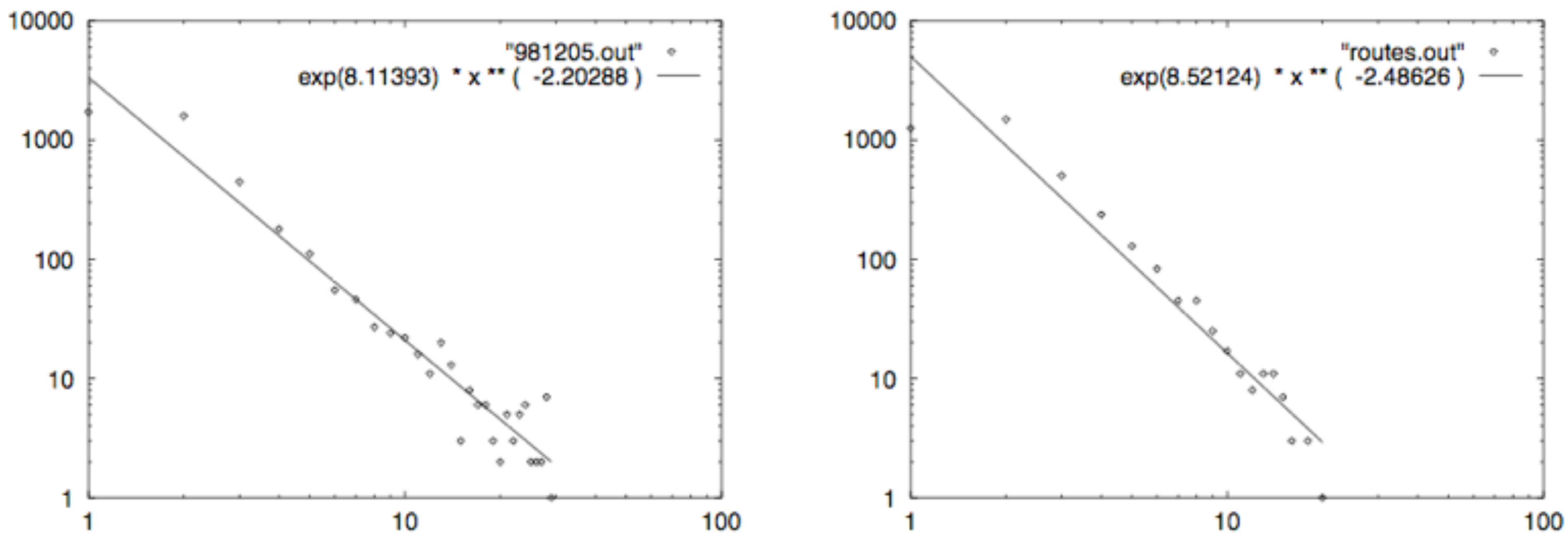
- The topic-based subsets of the WWW, Web clickstream data, sales data in retail chains, file size distributions, and phone usages are some examples of the Longnormals distribution.

Outdegree Plots



(a) Int-11-97 (b) Int-04-98

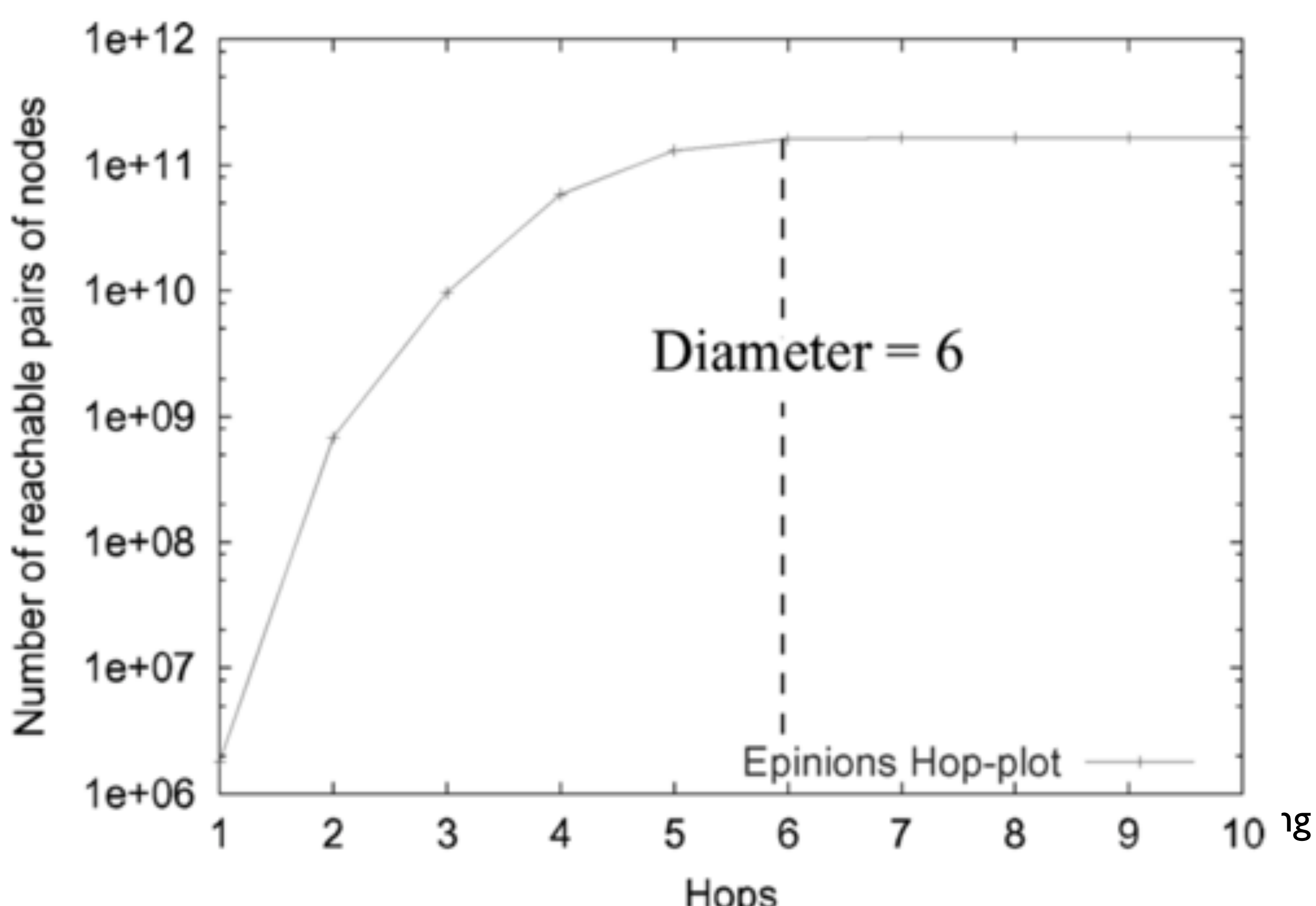
Figure 5: The outdegree plots: Log-log plot of frequency f_d versus the outdegree d.



(a) Int-12-98 (b) Rout-95

The Hop Plot

- The **Hop-plot** is the plot of N_h versus h, where $N_h = \sum_u N_h(u)$, u is a node in the graph and $N_h(u)$ is the number of nodes in a neighborhood of h hops.
- The hop-plot can be used to calculate the *effective diameter* (or the eccentricity) of the graph.
- The effective diameter is defined as the minimum number of hops in which some fraction of all connected pairs of nodes can reach each other.



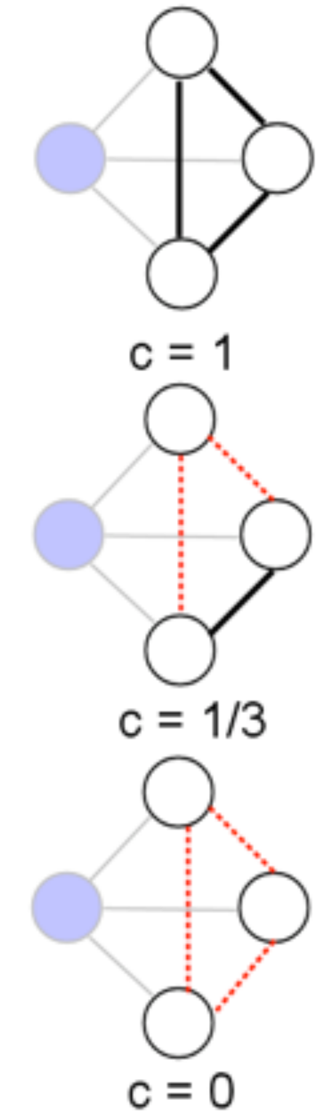
Clustering Coefficient

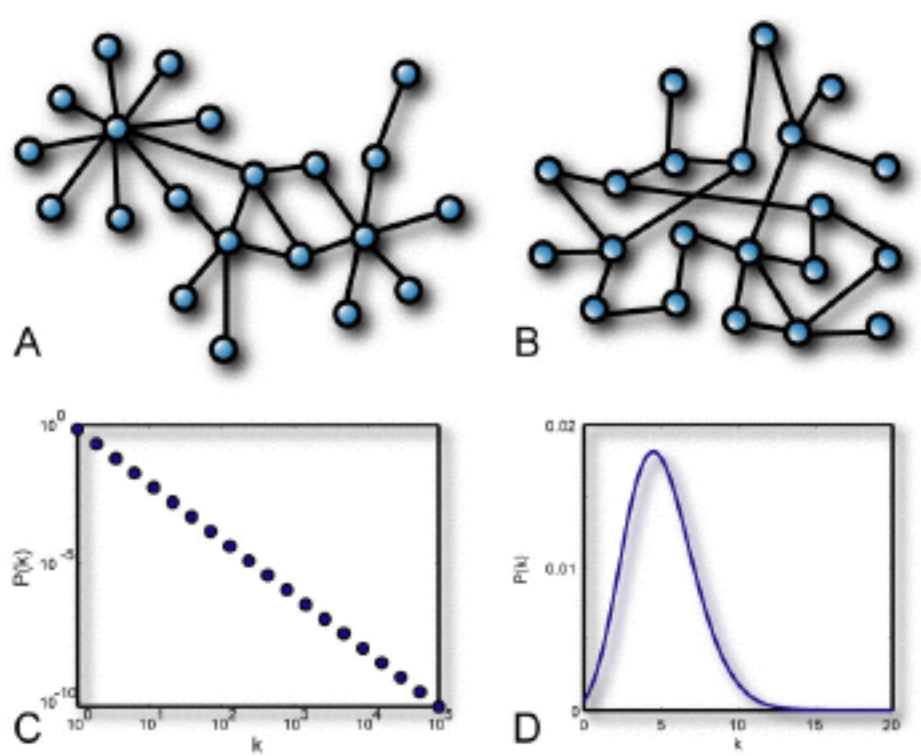
- **Clustering Coefficient**. Given that a node i has k_i neighbors, and there are n_i edges between the neighbors. The clustering coefficient of node i is defined as

$$C_i = \begin{cases} \frac{2n_i}{k_i(k_i-1)} & k_i > 1 \\ 0 & k_i = 0 \text{ or } 1 \end{cases}.$$

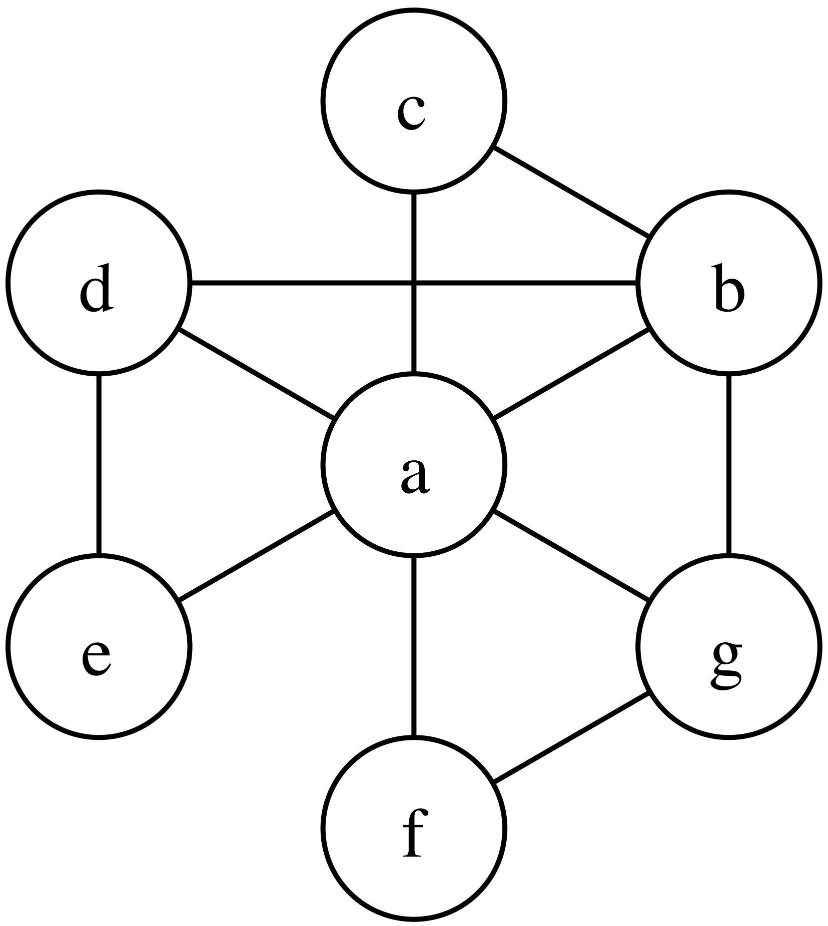
- For a node v with edges (u, v) and (v, w), the **Clustering Coefficient** of v measures the probability of existence of the third edge (u, w).

- The clustering coefficient of the entire graph (Global clustering coefficient) is found by averaging over all nodes in the graph.





An Example of Clustering Coefficient



Node a has 6 neighbors.

These neighbors could have been connected by 15 edges (6 x 5 / 2).

But with only 5 edges ({(c,b), (b,g), (g,f), (d,e), (d,b)}) exist so the local clustering coefficient of node a is 5/15 = 1/3

What is the global clustering coefficient?

Information Propagation

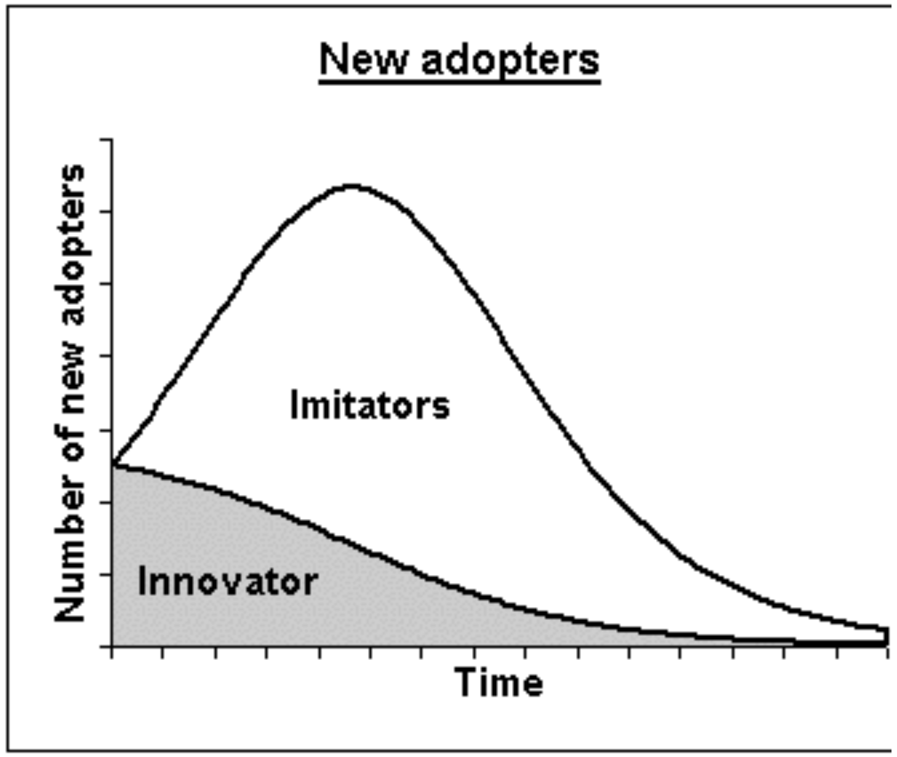
- Propagation Attributes
  - Propagation medium
  - Propagation rate
  - State of the node
  - Connectivity patterns
- Models
  - Threshold models
  - Viral propagation models
  - Diffusion models

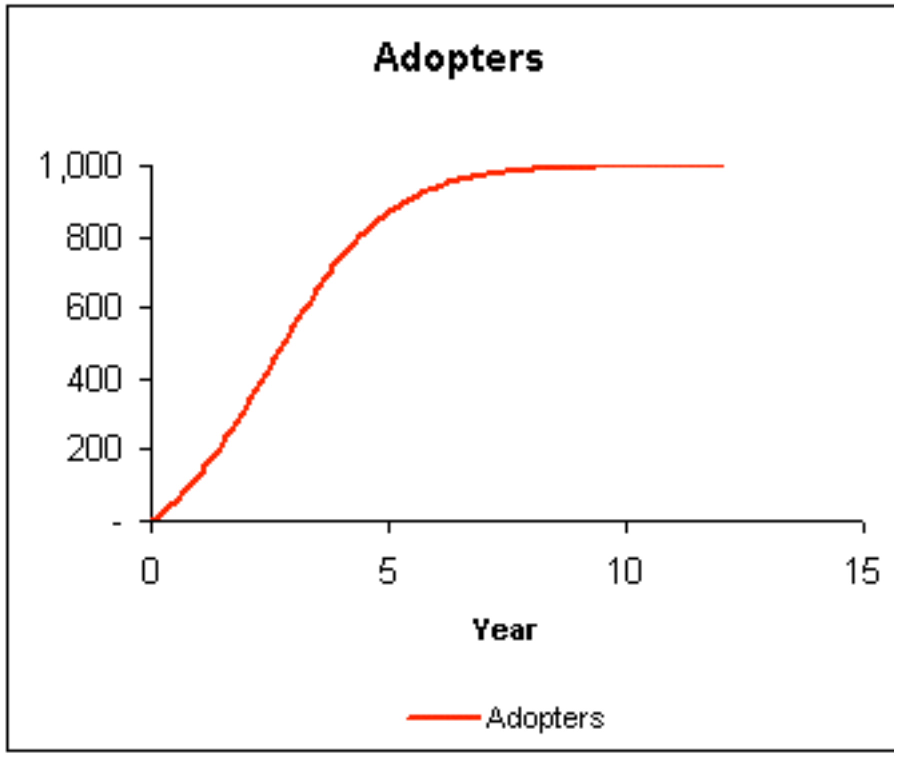
Viral Propagation

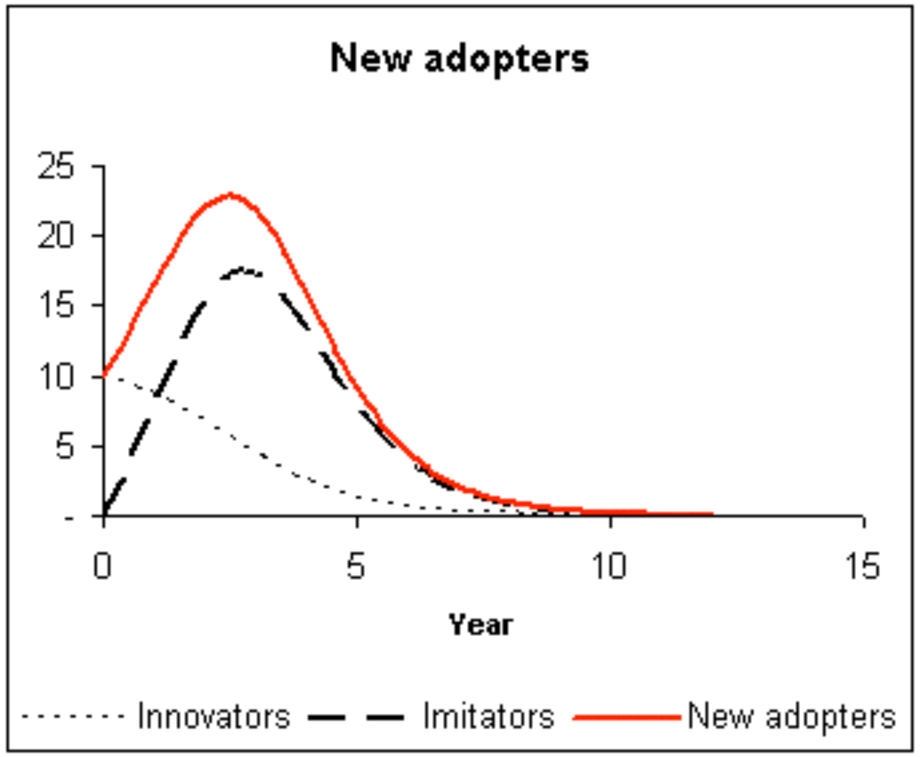
- SIR Model
  - Susceptible (S), Infective (I), and Removed (R)
  - Each edge (i, j) has a spreading function (birth rate) β_{ij}
  - Each Infective node *u* has a rate of getting cured (death rate) δ_u
  - The spread of infections depends on $\tau = \beta / \delta$
- SIS Model
  - Similar to the SIR model except that once an infective node is cured, it goes back to the susceptible state

Bass Diffusion Model

- The process of how new products get adopted as an interaction between users and potential users







Bass Diffusion Formulation

The **Bass Diffusion Model** is defined as

$$\frac{f(t)}{1 - F(t)} = p + qF(t)$$

where

- $f(t)$ is the rate of change of the installed base fraction
- $F(t)$ is the installed base fraction
- p is the coefficient of innovation
- q is the coefficient of imitation

Sales $S(t)$ is the rate of change of installed base (i.e., adoption) $f(t)$ multiplied by the ultimate market potential m

$$\begin{aligned} S(t) &= mf(t) \\ S(t) &= m\tfrac{(p+q)^2}{p} \tfrac{e^{-(p+q)t}}{(1+\frac{q}{p}e^{-(p+q)t})^2} \end{aligned}$$

The time of peak sales t^* is defined as

$$t^* = \frac{\ln q - \ln p}{p + q}$$

Discussions

- Properties to consider
  - Degree distributions
  - Clustering coefficient
  - Community structure
  - Implementation issues
- How do you make friends?
- How can one recommend friends?
- How does information propagate among friends?

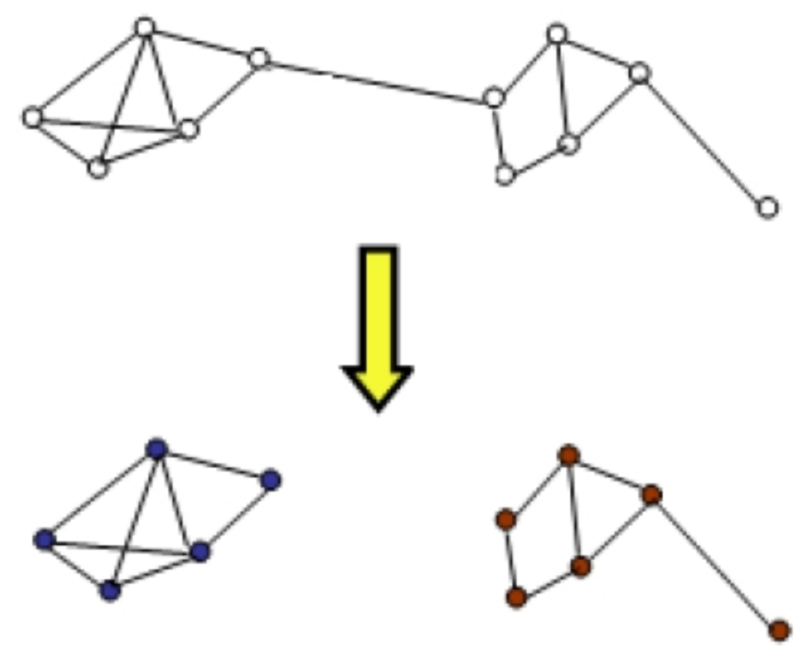
Graph Mining

Clustering

- Finding patterns in data, or grouping similar groups of data-points together into clusters.
- Clustering algorithms for numeric data
  - Lloyd's K-means, EM clustering, spectral clustering etc.
- Traditional definition of a "good" clustering
  - Points assigned to same cluster should be highly similar
  - Points assigned to different clusters should be highly dissimilar

Graph Clustering

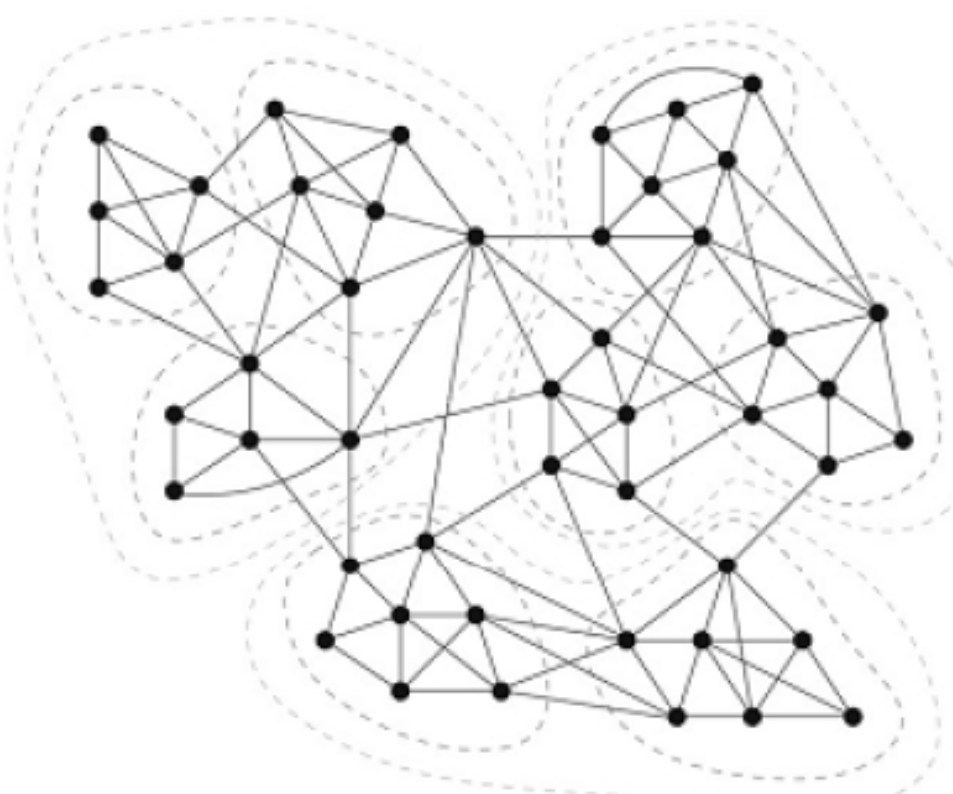
- Graphical representation of data as undirected graphs
- Clustering of vertices on basis of edge structure
- Defining a graph cluster
  - In its loosest sense, a graph cluster is a connected component
  - In its strictest sense, it's a maximal clique of a graph
- Many vertices within each cluster
- Few edges between clusters

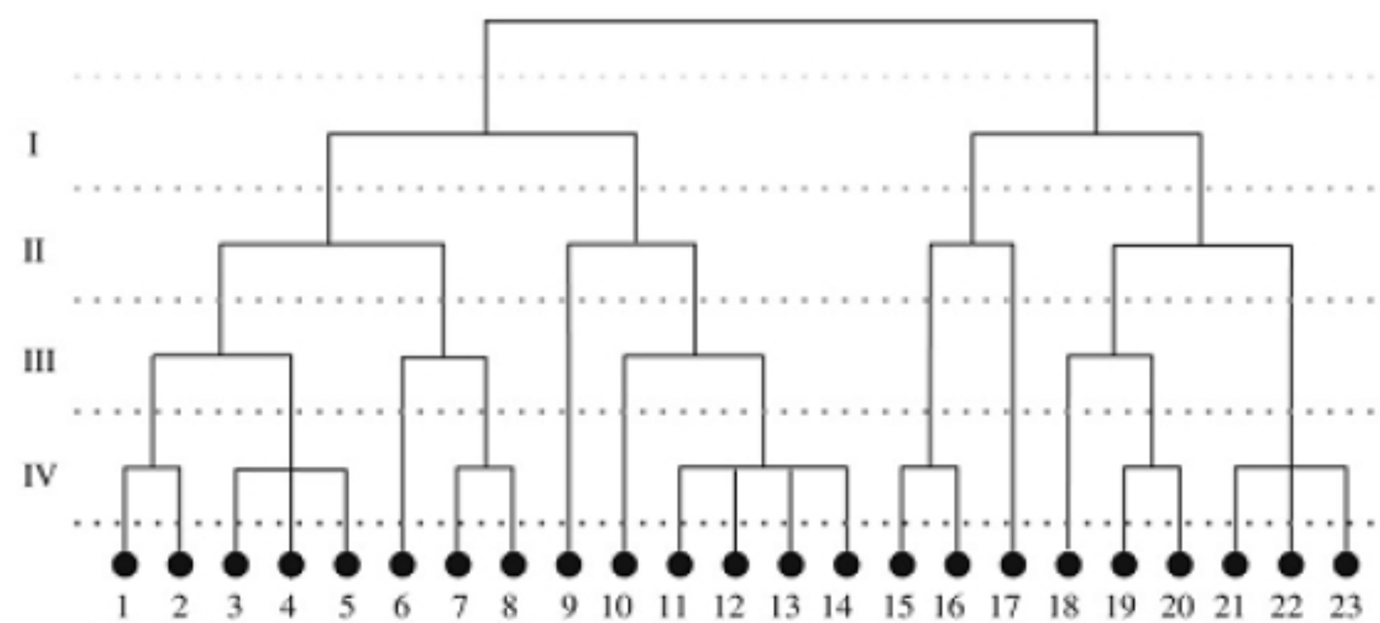


GRAPH PARTITIONING!!

Clustering Paradigm

- Hierarchical clustering vs. flat clustering
- Hierarchical:
  - Top down
  - Bottom up



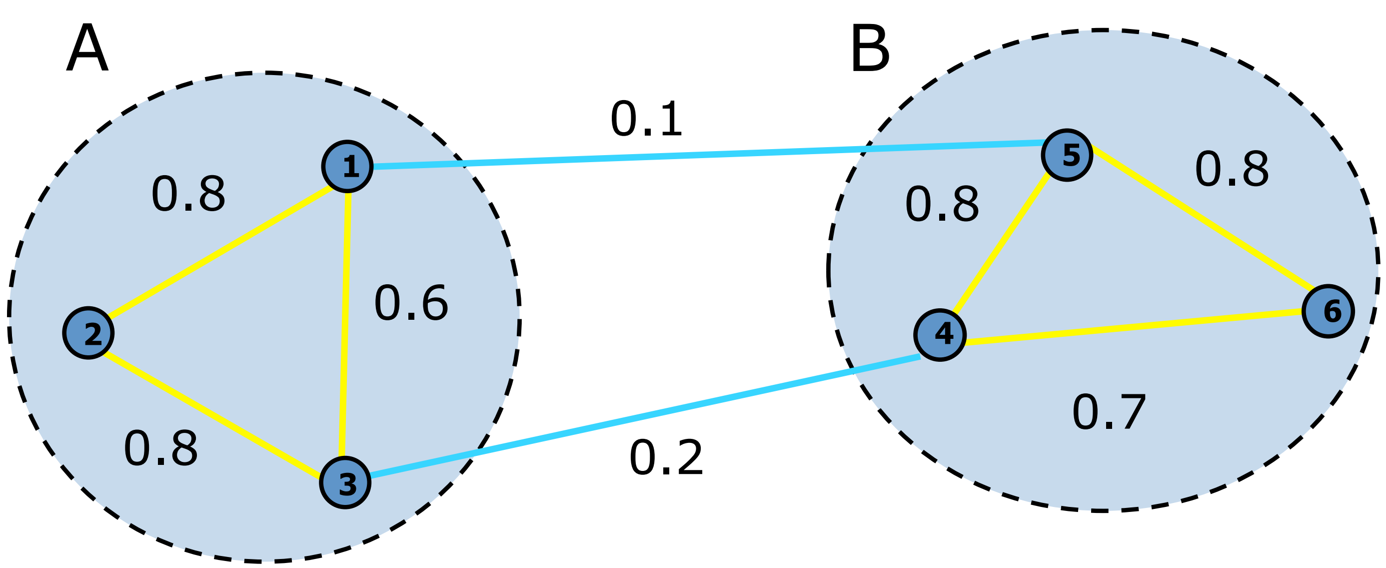


Overview

- Cut based methods
  - Become NP hard with introduction of size constraints
  - Approximation algorithms minimizing graph conductance
- Maximum flow
  - Using results by Golberg and Tarjan
  - Reasonable for small graphs
- Graph spectrum based methods
  - Stable perturbation analysis
  - Good even when graph is not exactly block diagonal
  - Typically, second smallest eigenvalue is taken as graph characteristic
  - Spectrum of graph transition matrix for blind walk

Graph Cuts

- Express partitioning objectives as a function of the "edge cut" of the partition
- *Cut*: Set of edges with only one vertex in a group.
  - We want to find the minimal cut between groups
  - The group that has the minimal cut would be the partition



$$cut(A,B) = \sum_{i \in A, j \in B} w_{ij}$$

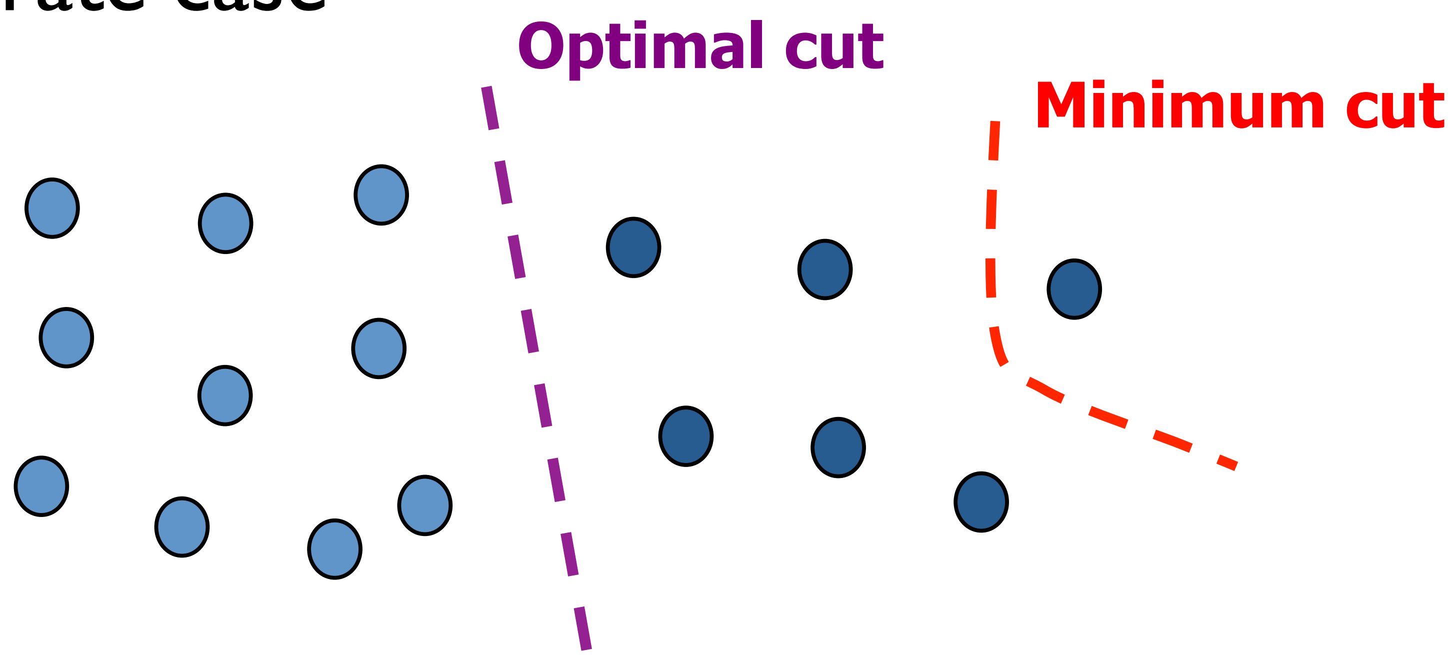
$$cut(A,B) = 0.3$$

Graph Cut Criteria

- Criterion: Minimum-cut
  - Minimize weight of connections between groups

$$\min cut(A, B)$$

- Degenerate case



- Issues
  - Only considers external cluster connections
  - Does not consider internal cluster density

Graph Cut Criteria

- Criterion: Normalized-cut [Shi & Malik,'97]
  - Consider the connectivity between groups relative to the density of each group

$$\min Ncut(A,B) = \frac{cut(A,B)}{vol(A)} + \frac{cut(A,B)}{vol(B)}$$

  - Normalize the association between groups by volume
    - *vol*(*A*): The total weight of the edges originating from group *A*
- Why use this criterion?
  - Minimizing the normalized cut is equivalent to maximizing normalized association
  - Produce more balanced partitions

Summary

- Clustering as a graph partitioning problem
  - Quality of a partition can be determined using graph cut criteria
  - Identifying an optimal partition is NP-hard
- Spectral clustering techniques
  - Efficient approach to calculate near-optimal bi-partitions and *k*-way partitions
  - Based on well-known cut criteria and strong theoretical background

Graph Cuts and Max-Flow/Min-Cut Algorithms

- A **flow network** is defined as a directed graph where an edge has a nonnegative capacity
- A **flow** in *G* is a real-valued (often integer) function that satisfies the following three properties:
  - Capacity Constraint:
    - For all $u, v \in V, f(u,v) \leq c(u,v)$
  - Skew Symmetry
    - For all $u, v \in V, f(u,v) = -f(v,u)$
  - Flow Conservation
    - For all $u \in (V \backslash \{s,t\}), \sum_{v \in V} f(u,v) = 0$

How to Find the Minimum Cut?

- Theorem: In graph *G*, the maximum source-to-sink flow possible is equal to the capacity of the minimum cut in *G*

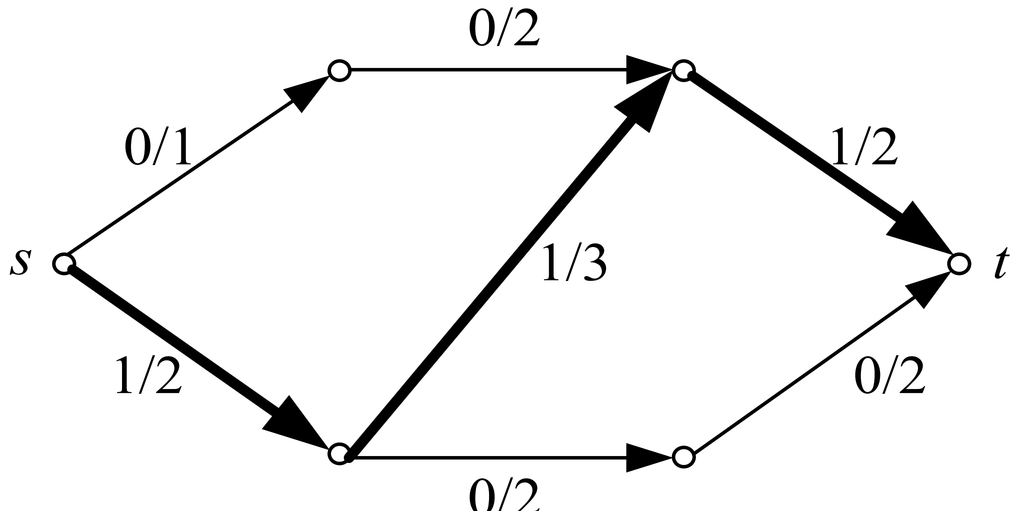
  [L. R. Foulds, Graph Theory Applications, 1992 Springer-Verlag New York Inc., 247-248]

Maximum Flow and Minimum Cut Problem

- Some basic concepts
  - If f is a flow, then the net flow across the cut (S, T) is defined to be $f(S, T)$, which is the sum of all edge capacities from S to T subtracted by the sum of all edge capacities from T to S
  - The capacity of the cut (S, T) is $c(S, T)$, which is the sum of the capacities of all edge from S to T
  - A minimum cut is a cut whose capacity is the minimum over all cuts of G
- Algorithms
  - Ford-Fulkerson Algorithm
  - Push-Relabel Algorithm
  - New Algorithm by Boykov, etc.

Ford-Fulkerson Algorithm

- Main Operation
  - Starting from zero flow, increase the flow gradually by finding a path from *s* to *t* along which more flow can be sent, until a max-flow is achieved
  - The path for flow to be pushed through is called an **augmenting path**
- The Ford-Fulkerson algorithm uses a residual network of flow in order to find the solution
- The residual network is defined as the network of edges containing flow that has already been sent
- For example, in the graph shown below, there is an initial path from the source to the sink, and the middle edge has a total capacity of 3, and a residual capacity of 3-1=2



Ford-Fulkerson Algorithm

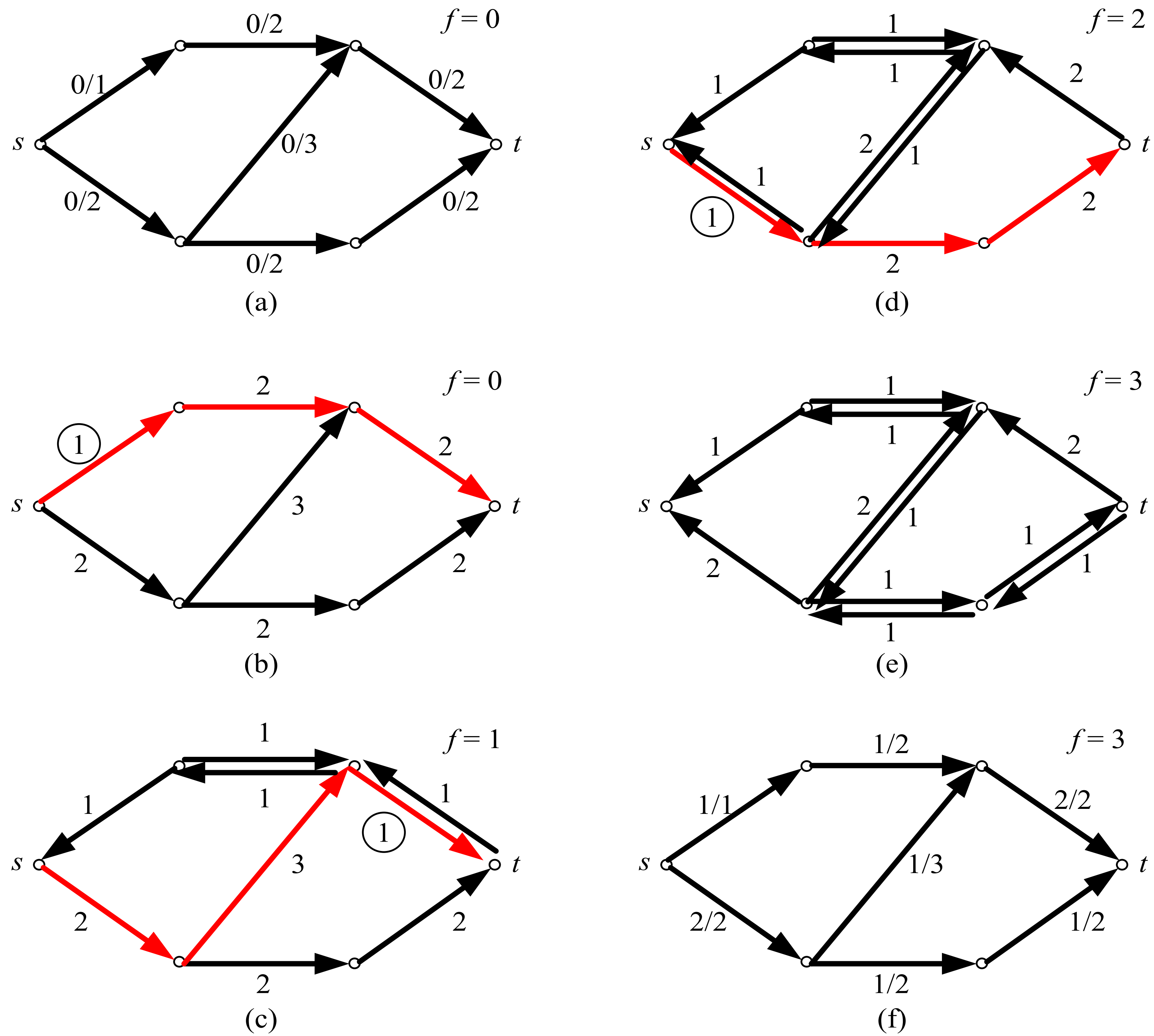
- Assuming there are two vertices, u and v, let $f(u, v)$ denote the flow between them, $c(u, v)$ be the total capacity, $c_f(u, v)$ be the residual capacity, and there should be,

$$c_f(u, v) = c(u, v) - f(u, v)$$

- Given a flow network and a flow f, the residual network of G is $G_f = (V, E_f)$, where $E_f = \{(u, v) \in V \times V : c_f(u, v) > 0\}$
- Given a flow network and a flow f, an augmenting path P is a simple path from s to t in the residual network
- We call the maximum amount by which we can increase the flow on each edge in an augmenting path P the residual capacity of P, given by,

$$c_f(P) = \min\{c_f(u, v) : (u, v) \text{ is on } P\}$$

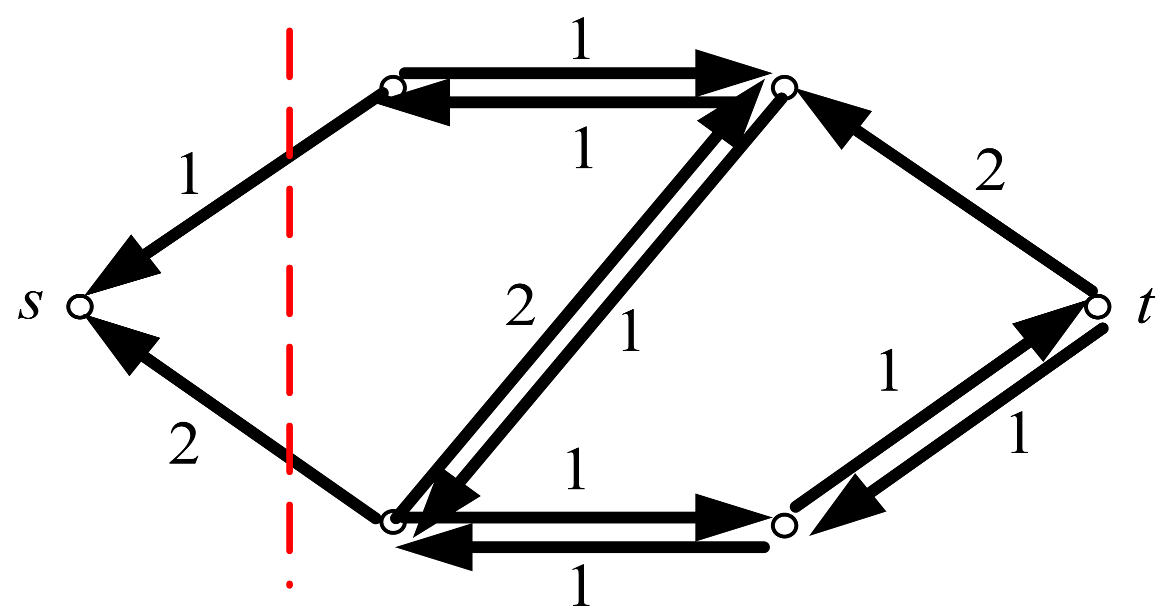
Example



Finding the Min-Cut

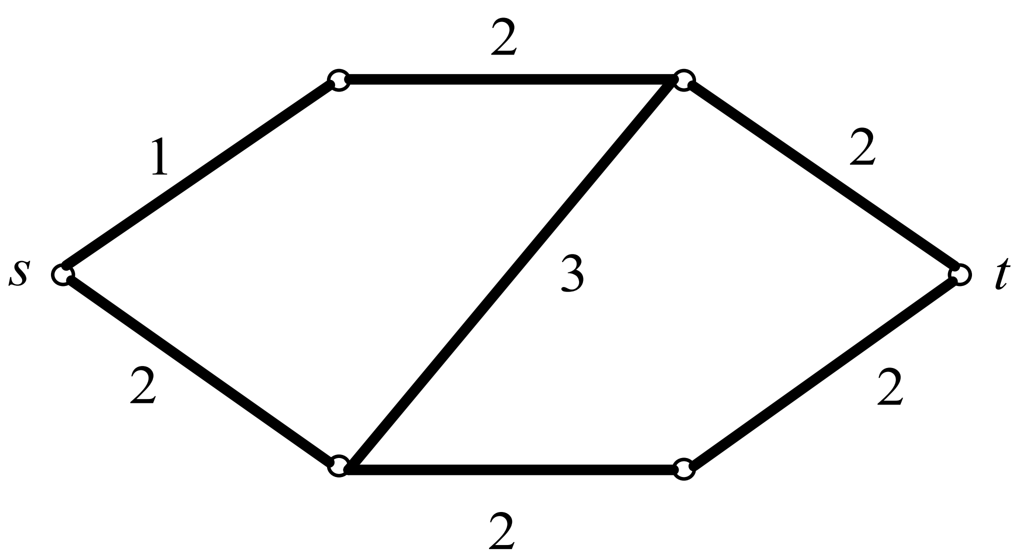
- After the max-flow is found, the minimum cut is determined by

$$\begin{aligned} S &= \{\text{All vertices reachable from } s\} \\ T &= G \backslash S \end{aligned}$$

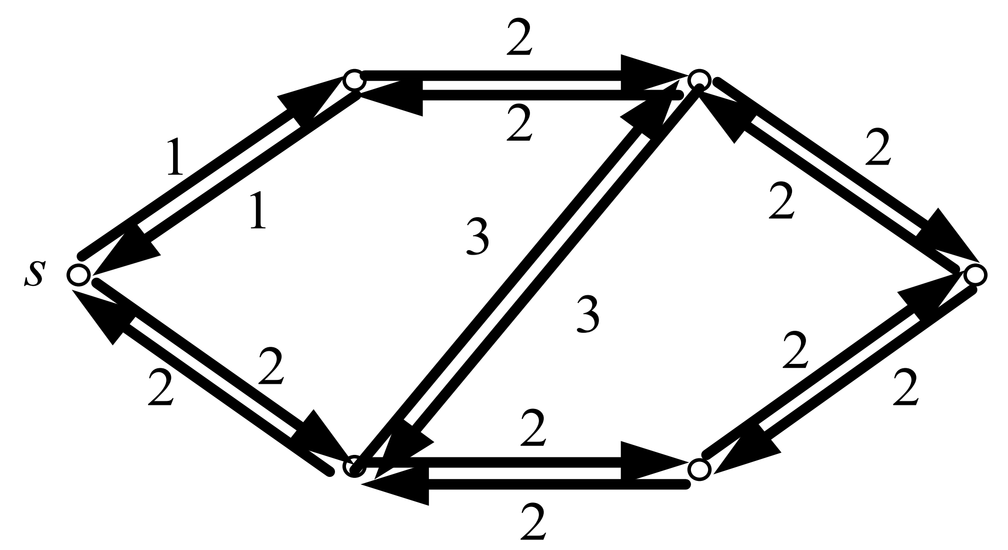


Special Case

- As in some applications only undirected graph is constructed, when we want to find the min-cut, we assign two edges with the same capacity to take the place of the original undirected edge



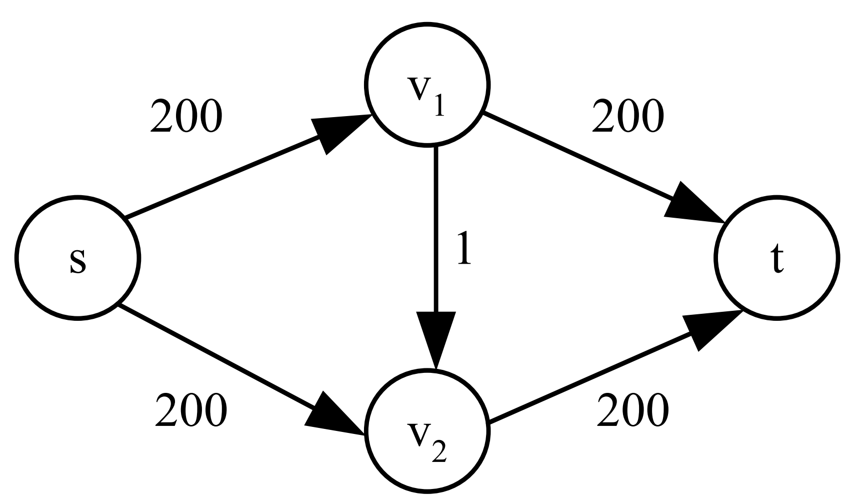
(a)



(b)

Ford-Fulkerson Algorithm Analysis

- The running time of the algorithm depends on how the augmenting path is determined
  - If the searching for augmenting path is realized by a breadth-first search, the algorithm runs in polynomial time of $O(E\ |f_{\max}|\ )$
- Under some extreme cases the efficiency of the algorithm can be reduced drastically
  - One example is shown in the figure below, applying Ford-Fulkerson algorithm needs 400 iterations to get the max flow of 400



CMSC5733 Social Computing

04-Link Analysis

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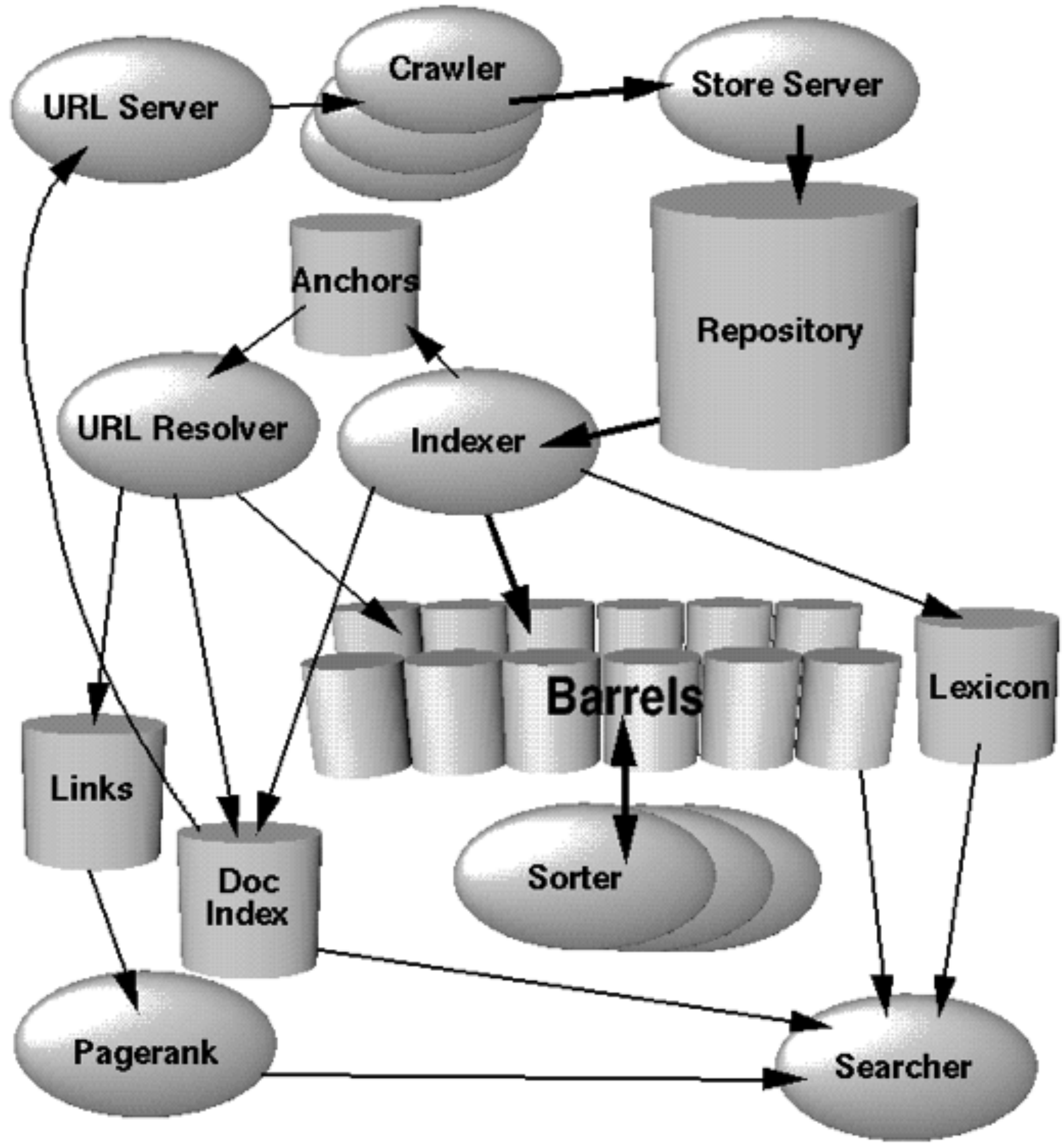
Traditional Information Retrieval

- Content matching against the query
  - Occurrence of query words
  - Location of query words
  - Document weighting
- Not much of ranking
- Science Citation Index and Impact Factor

Challenges of Web Search

- Voluminous
- Dynamic (generated deep web)
- Self-organized
- Hyperlinked
- Quality of Information
- Accessibility

Information Retrieval and Search Engine



Crawler

- Page Repository
- Indexing Module
- Indices

- Query Module
- Ranking Module

Information Retrieval Basics

- Vector Space Model
- Relevance Scoring and Relevance Feedback
- Meta-search Engines
- Precision vs. Recall

The InDegree Algorithm

- A simple heuristic
- Rank the pages according to popularity (indegree) of the page
- Issues?

The PageRank Algorithm

- Hyperlinked documents are different!
  - Similar to academic papers
  - In-links = authorities
  - Out-links = citations
  - Citations give better approximation of the quality of pages

Define PageRank

The PageRank calculation is defined as follows. We assume page A has pages $T_1, \cdots, T_n$ which point to it (i.e., are citations). The parameter d is a damping factor which can be set between 0 and 1. $C(A)$ is defined as the number of links going out of page A. The PageRank of a page A is given as follows:

$$PR(A) = (1-d) + d(PR(T_1)/C(T_1) + \cdots + PR(T_n)/C(T_n)). \qquad (1)$$

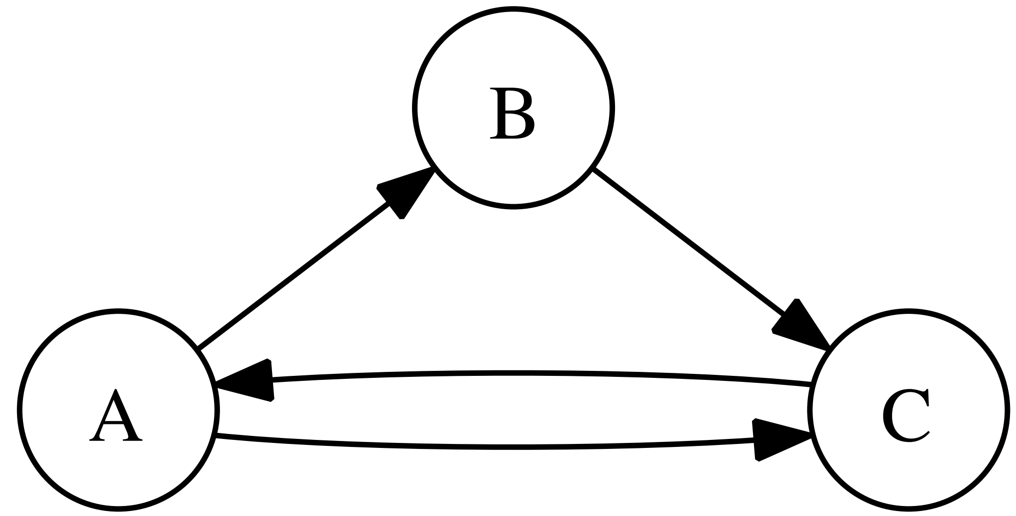
$$PR(A) = (1-d) + d\sum_{i}^{n} \frac{PR(T_i)}{C(T_i)}.$$

- PageRanks form a probability distribution over web pages, so the sum of all web pages' PageRanks will be one
- It can be calculated using a simple iterative algorithm, and corresponds to the principal eigenvector of the normalized link matrix of the web

Assumptions

- A "random surfer" who is given a web page at random
- The surfer keeps clicking on links, never hitting "back"
- The surfer gets bored and starts on another random page
- The probability that the random surfer visits a page is its PageRank
- The *d* damping factor is the probability at each page the Surfer will get bored and request another random page.
- Instead of a global *d*, one may consider a page damping factor d_i for each individual page or a group of pages

Examples

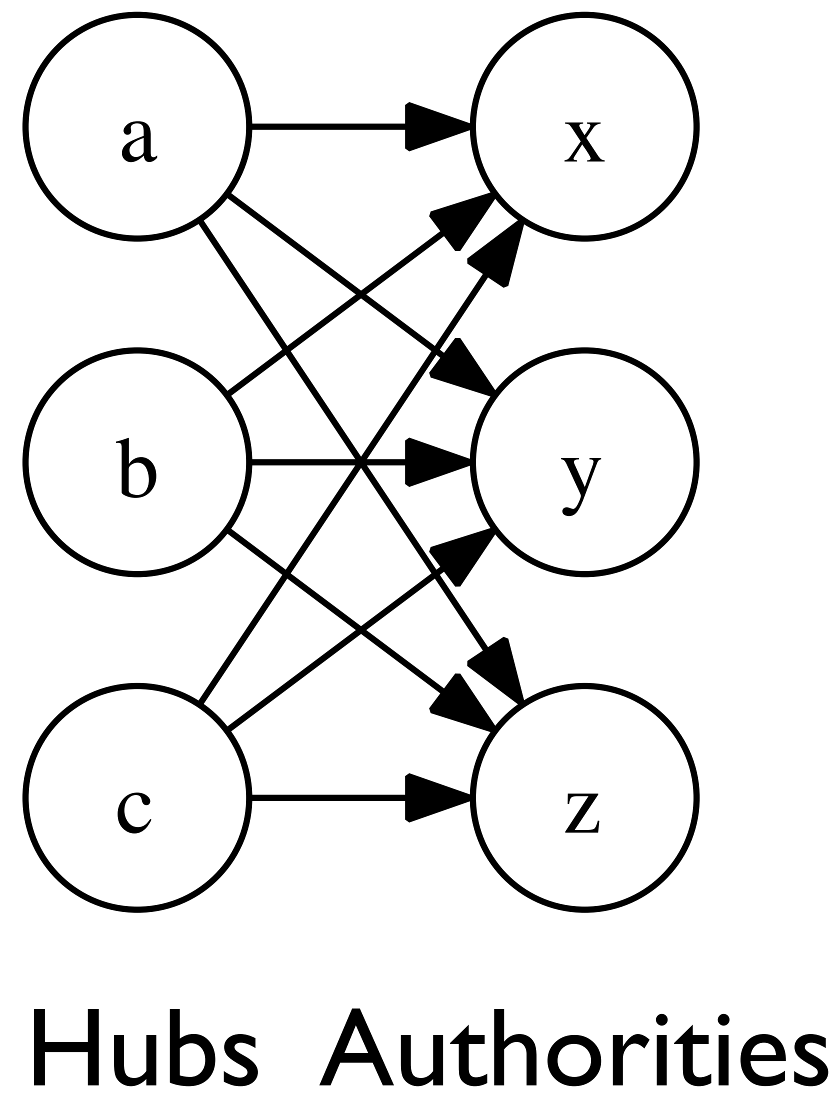


$$
\begin{aligned}
d &= 0.5 && (1)\\
PR(A) &= 0.5 + 0.5(PR(A)/2) && (2)\\
PR(C) &= 0.5 + 0.5(PR(A)/2 + PR(B)) && (3)
\end{aligned}
$$

$$
\begin{aligned}
PR(A) &= 14/13 = 1.07692308 && (4)\\
PR(B) &= 10/13 = 0.76923077 && (5)\\
PR(C) &= 15/13 = 1.15384615 && (6)
\end{aligned}
$$

Kleinberg's Algorithm

- Web page importance should depend on the search query being performed
- Each page should have a separate "authority" rating (based on the links going to the page) that captures the quality of the page as a resource itself
- Each page should also have a "hub" rating (based on the links going from the page) that captures the quality of the pages as a pointer to useful resources



Define HITS Algorithm

- The HITS (Hyperlink Induced Topic Distillation) algorithm computes lists of hubs and authorities for WWW search topics
- Start with a search topic, specified by one or more query terms
  - Sampling Stage--constructs a focused collection of several thousand Web pages likely to be rich in relevant authorities
  - Weight-propagation Stage-- determines numerical estimates of hub and authority weights by an iterative procedure
- The pages with the highest weights are returned as hubs and authorities for the search topic

The HITS Algorithm

Let the Web be a digraph $G = (V, E)$. Given a subgraph $S \subseteq V$ with $u, v \in S$ and $(u, v) \in E$. The authority and hub weights are updated as follows.

1. If a page is pointed to by many good hubs, we would like to increase its authority weight.
$$x_p = \sum_{q \text{ such that } q \to p} y_q, \tag{1}$$
where the notation $q \to p$ indicates taht q links to p.

2. If a page points to many good authorities, we increase its hub weight
$$y_p = \sum_{q \text{ such that } p \to q} x_q. \tag{2}$$

The above can be rewritten in a matrix notation as

$$x \leftarrow A^T y \leftarrow A^T A x = (A^T A)x \tag{3}$$

and

$$y \leftarrow Ax \leftarrow AA^T y = (AA^T)y \tag{4}$$

The HITS Pseudocode

- It is executed at query time, not at indexing time
- The hub and authority scores assigned to a page are query-specific.
- It computes two scores per document, hub and authority, as opposed to a single score.
- It is processed on a small subset of ‘relevant’ documents, not all documents as was the case with PageRank.

```
G := set of pages
for each page p in G do
  p.auth = 1 // p.auth is the authority score of the page p
  p.hub = 1 // p.hub is the hub score of the page p
function HubsAndAuthorities(G)
  for step from 1 to k do // run the algorithm for k steps
    for each page p in G do  // update all authority values first
      for each page q in p.incomingNeighbors do // p.incomingNeighbors is the set of pages that link to p
        p.auth += q.hub
    for each page p in G do  // then update all hub values
      for each page r in p.outgoingNeighbors do // p.outgoingNeighbors is the set of pages that p links to
        p.hub += r.auth
```

Conclusion

- Information propagation is heavily related to the network structure
- In addition to contents, links are powerful indicators to express the importance of an object