Quick Sort and Merge Sort

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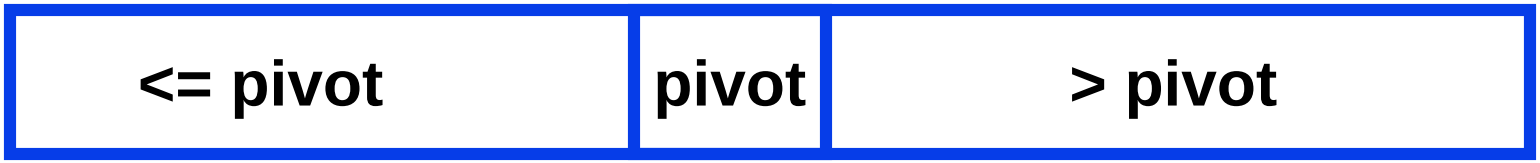
CSCI2100A Data Structures

Quick Sort

- Efficient sorting algorithm
- Example of Divide and Conquer algorithm
- Two phases
  - Partition phase
    - Divides the work into half
  - Sort phase
    - Conquers the halves!

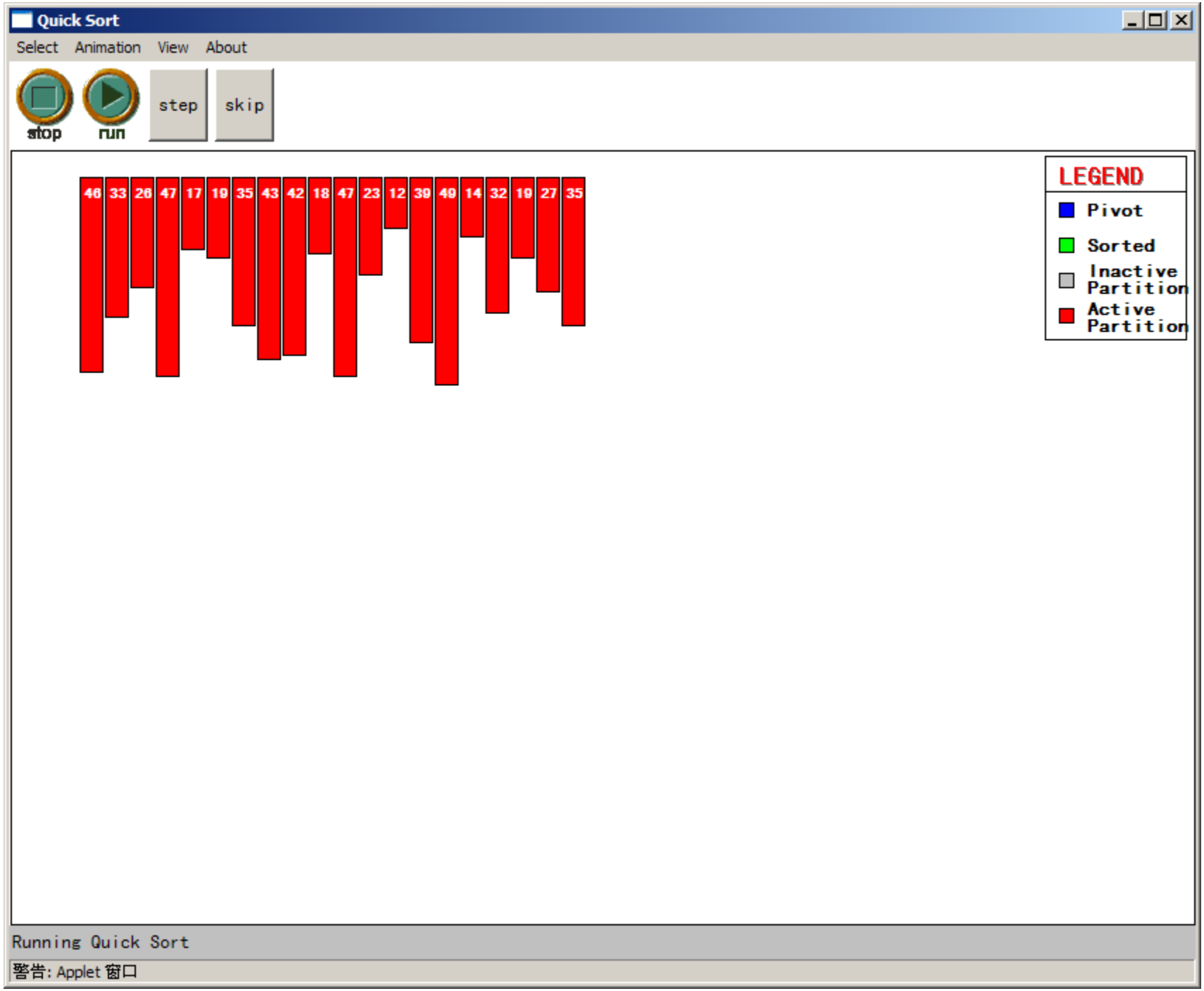
Quicksort

- Partition
  - Choose a pivot
  - Find the position for the pivot so that
    - all elements to the left are less/equal
    - all elements to the right are greater



Quicksort

- Conquer
  - Apply the same algorithm to each half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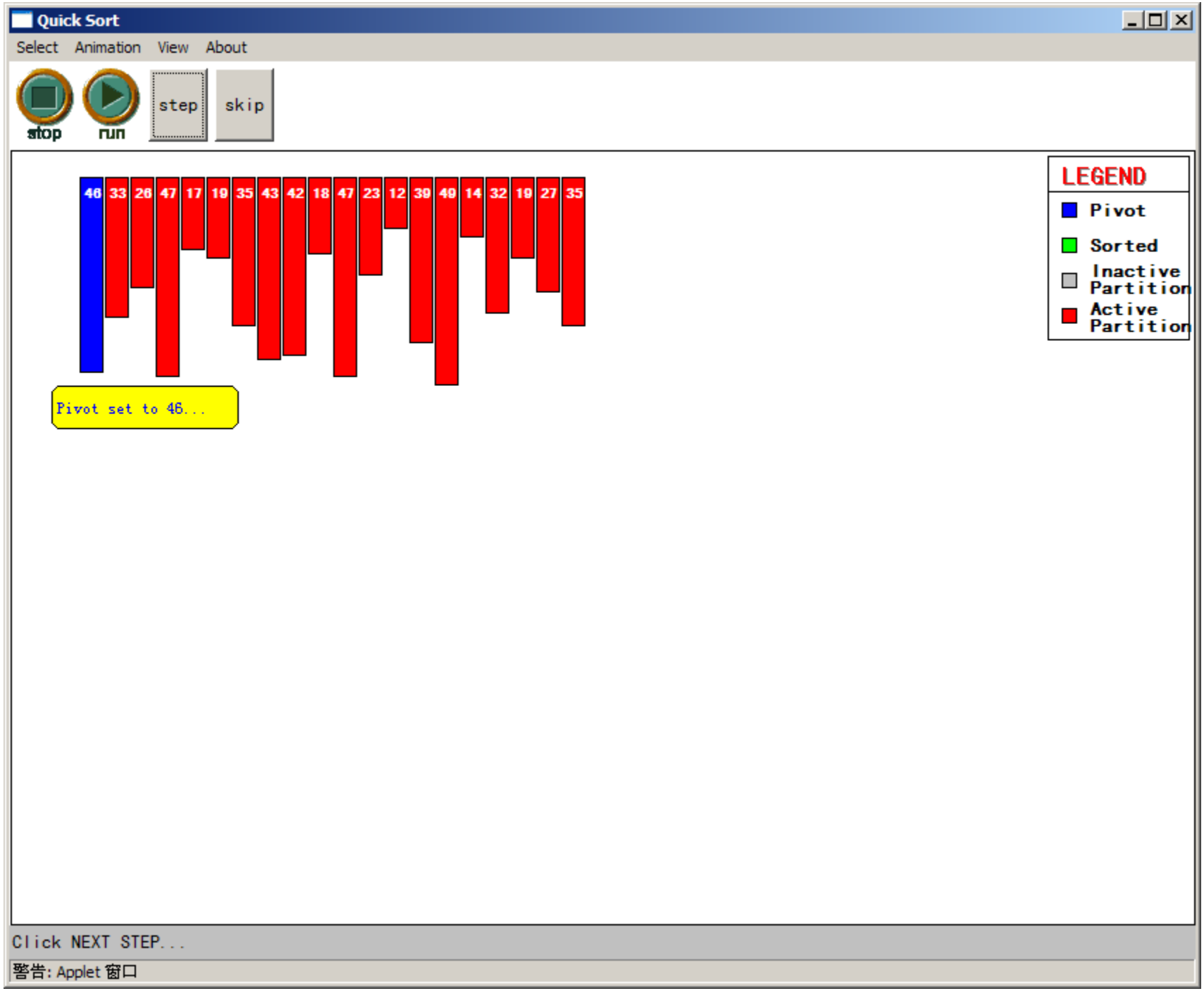
LEGEND
Pivot
Sorted
Inactive Partition
Active Partition
Running Quick Sort
警告: Applet 窗口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Pivot set to 46...
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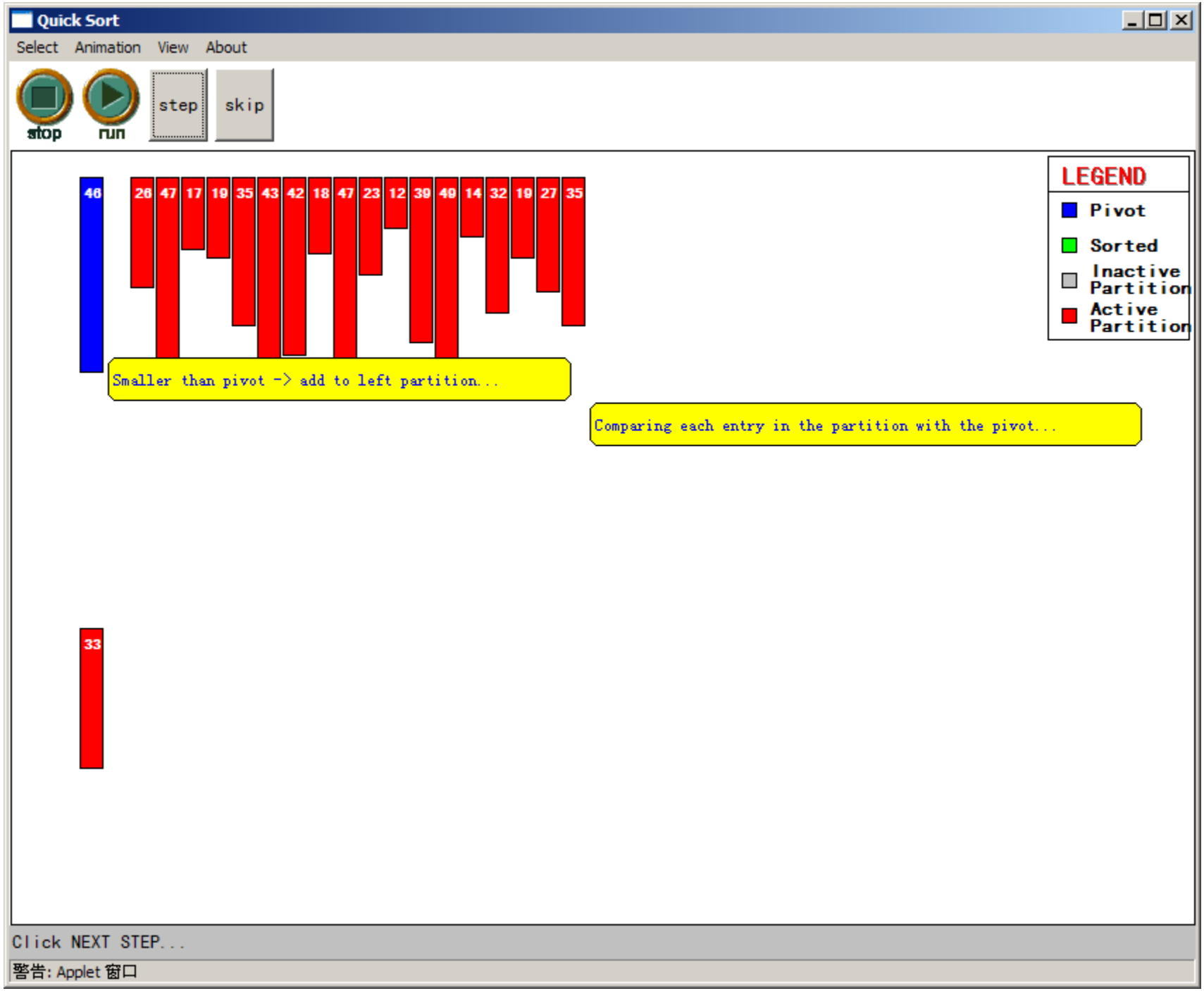
Click NEXT STEP...
警告: Applet 窗口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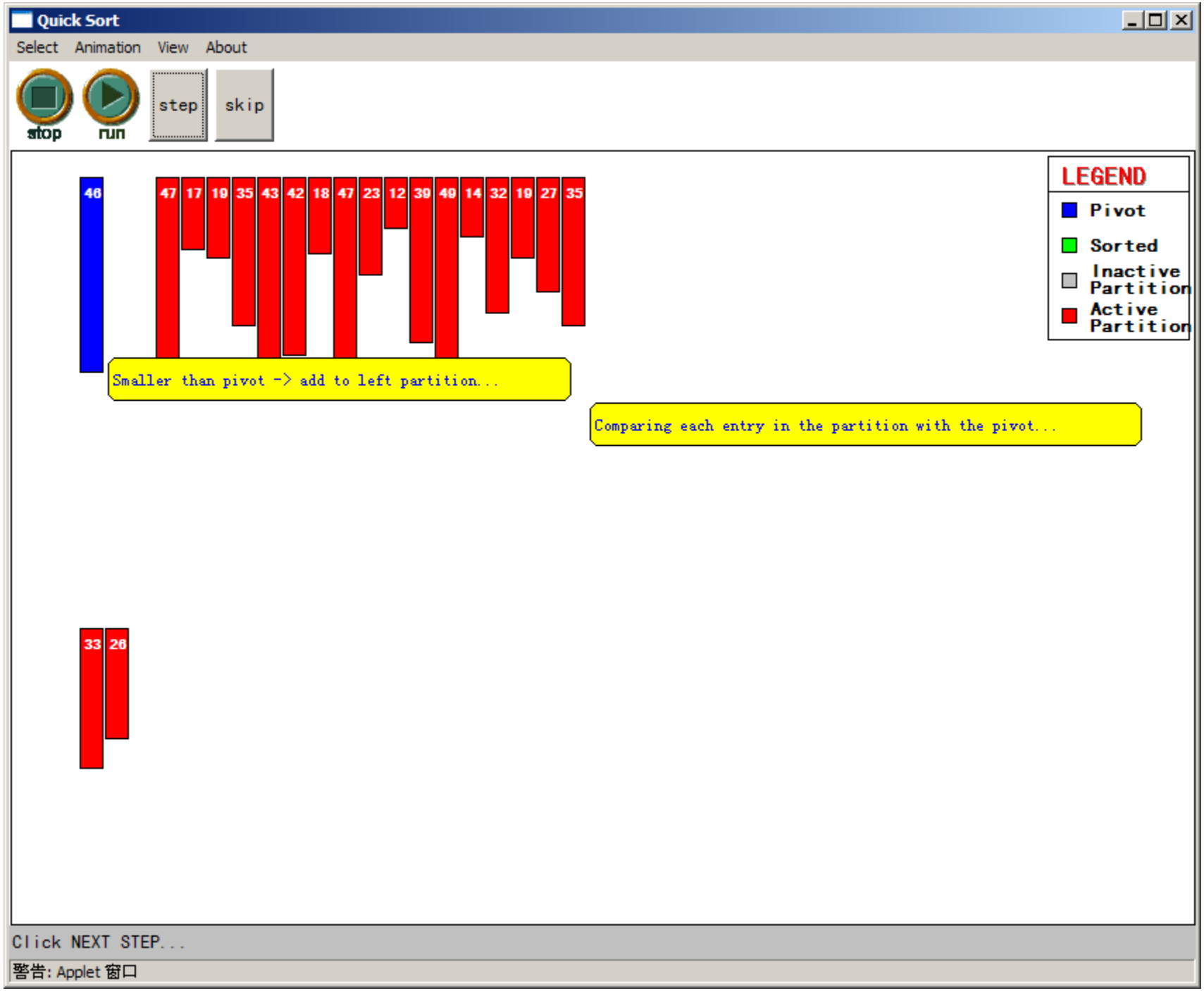
LEGEND
Pivot
Sorted
Inactive Partition
Active Partition
Smaller than pivot -> add to left partition...
Comparing each entry in the partition with the pivot...
33
Click NEXT STEP...
警告: Applet 窗口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LEGEND
Pivot
Sorted
Inactive Partition
Active Partition
Smaller than pivot -> add to left partition...
Comparing each entry in the partition with the pivot...
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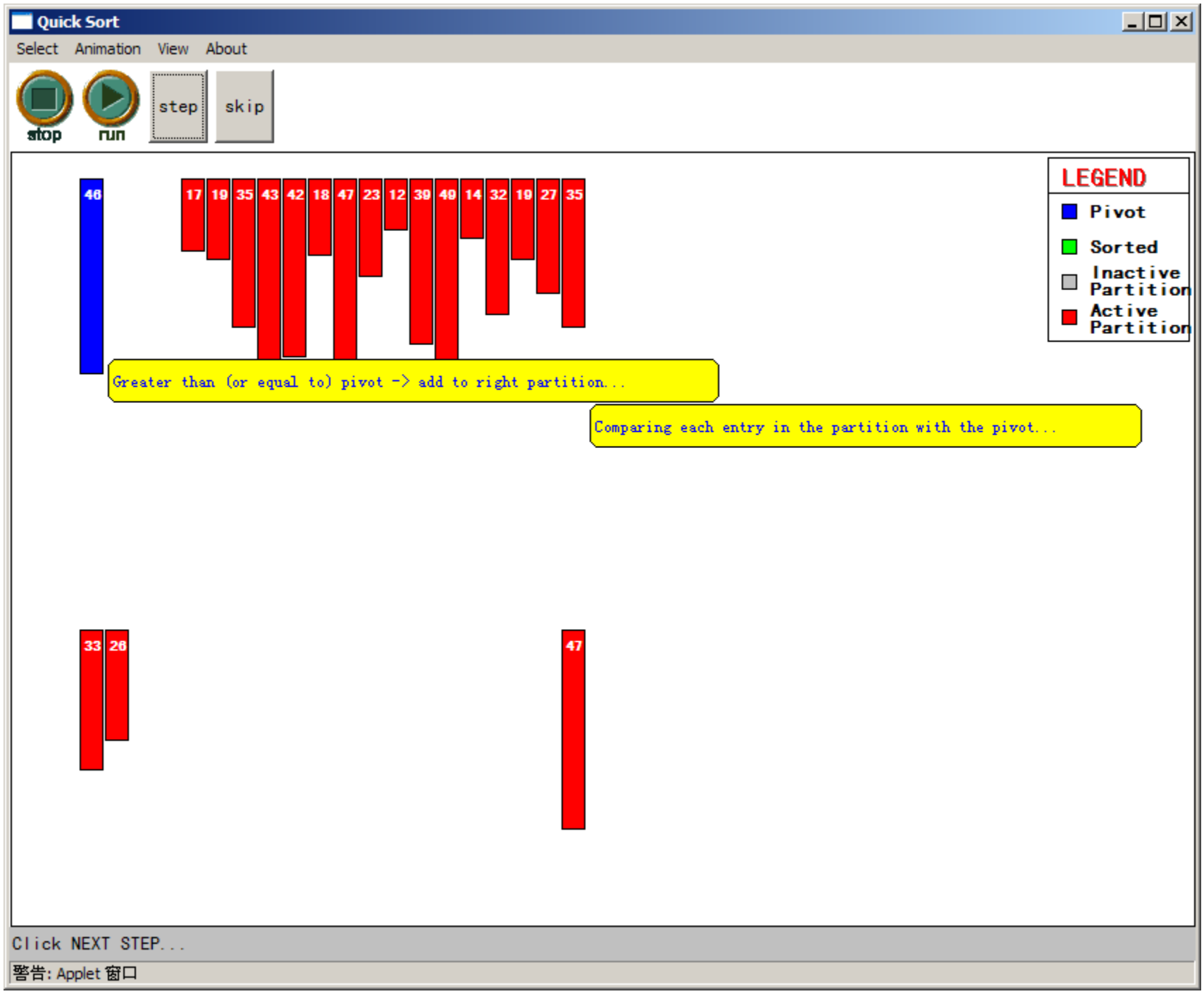
Click NEXT STEP...
警告: Applet 窗口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LEGEND
Pivot
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Greater than (or equal to) pivot -> add to right partition...
Comparing each entry in the partition with the pivot...
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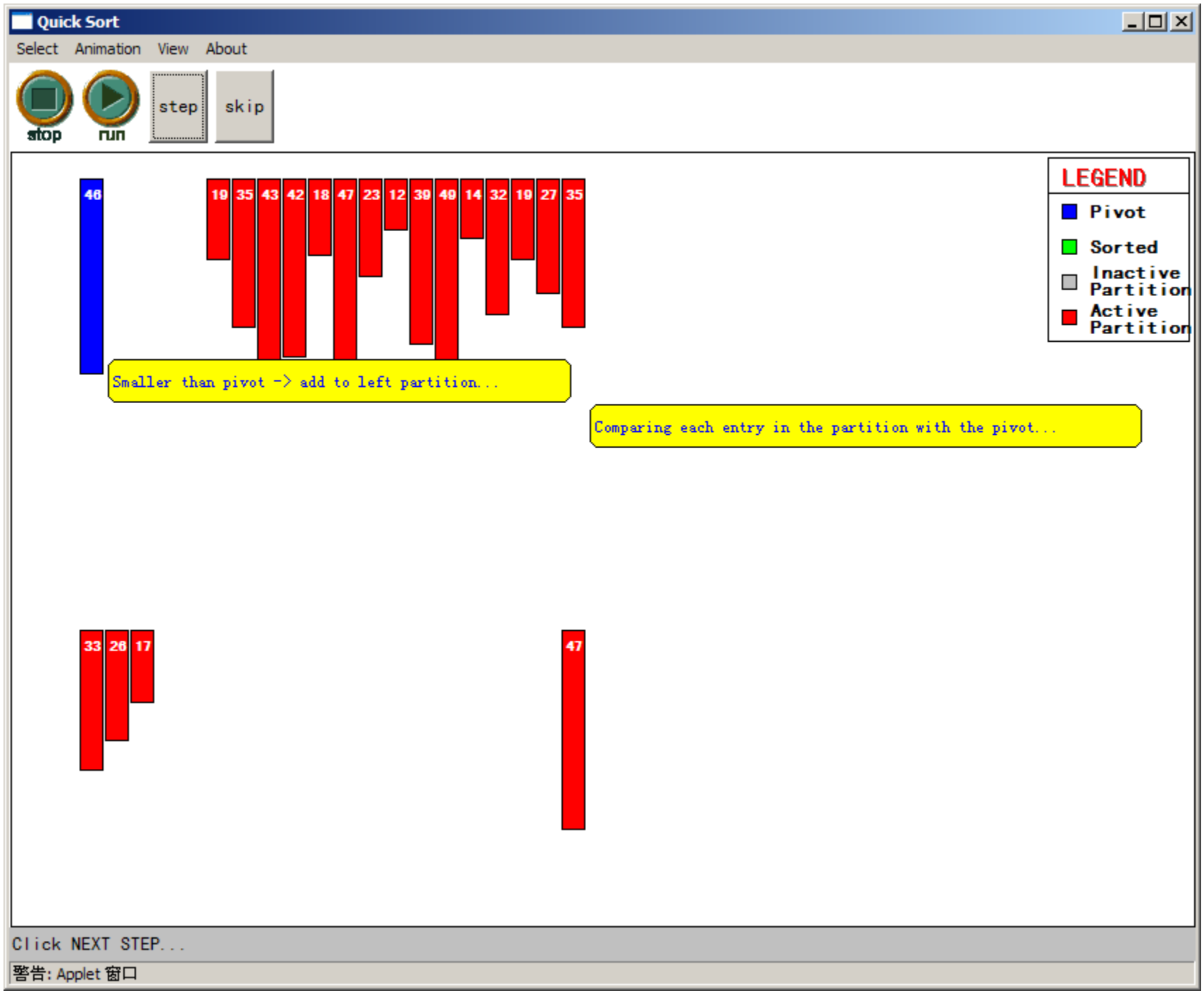
警告: Applet 窗口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Smaller than pivot -> add to left partition...
Comparing each entry in the partition with the pivot...
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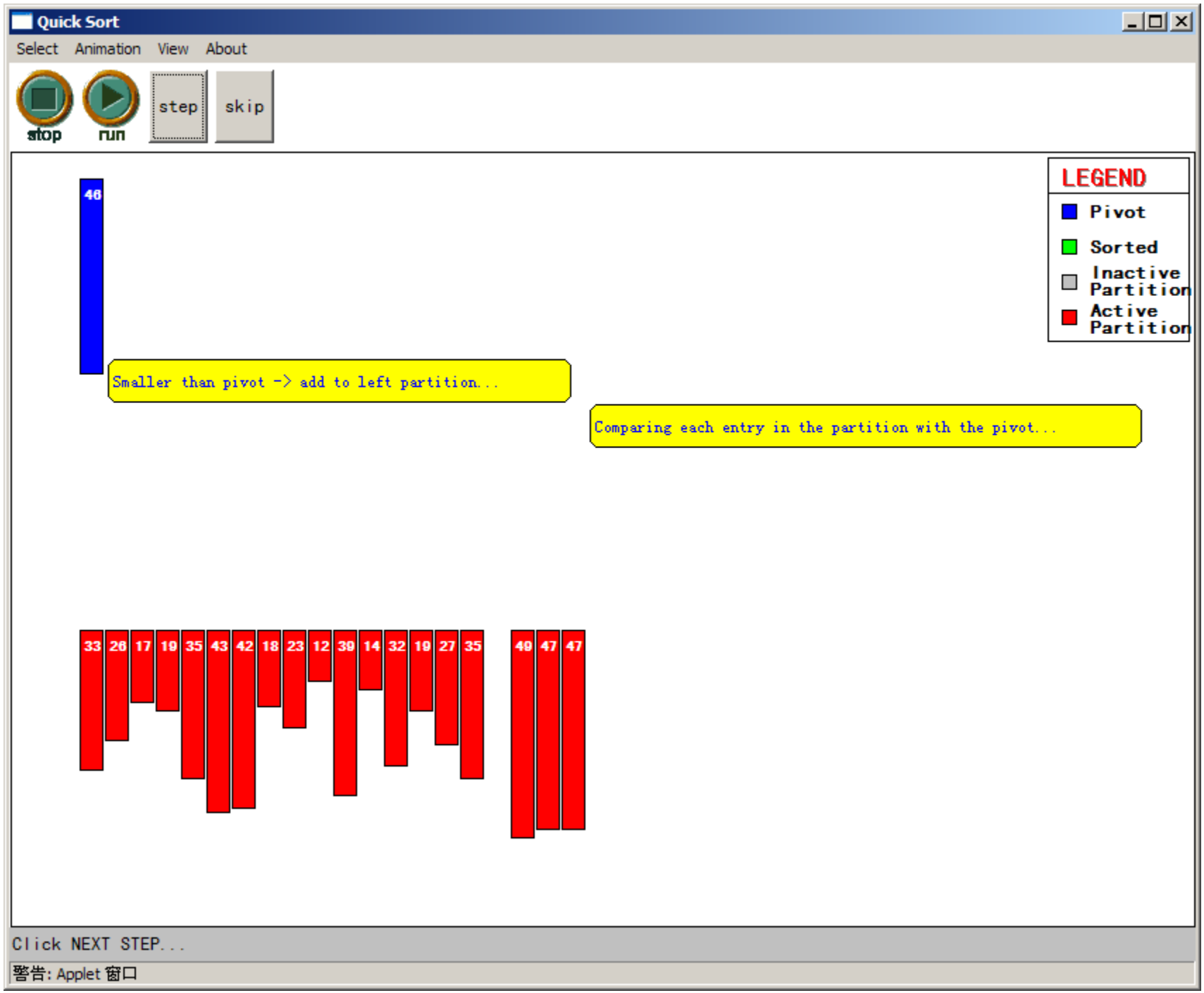
警告: Applet 窗口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Smaller than pivot -> add to left partition...
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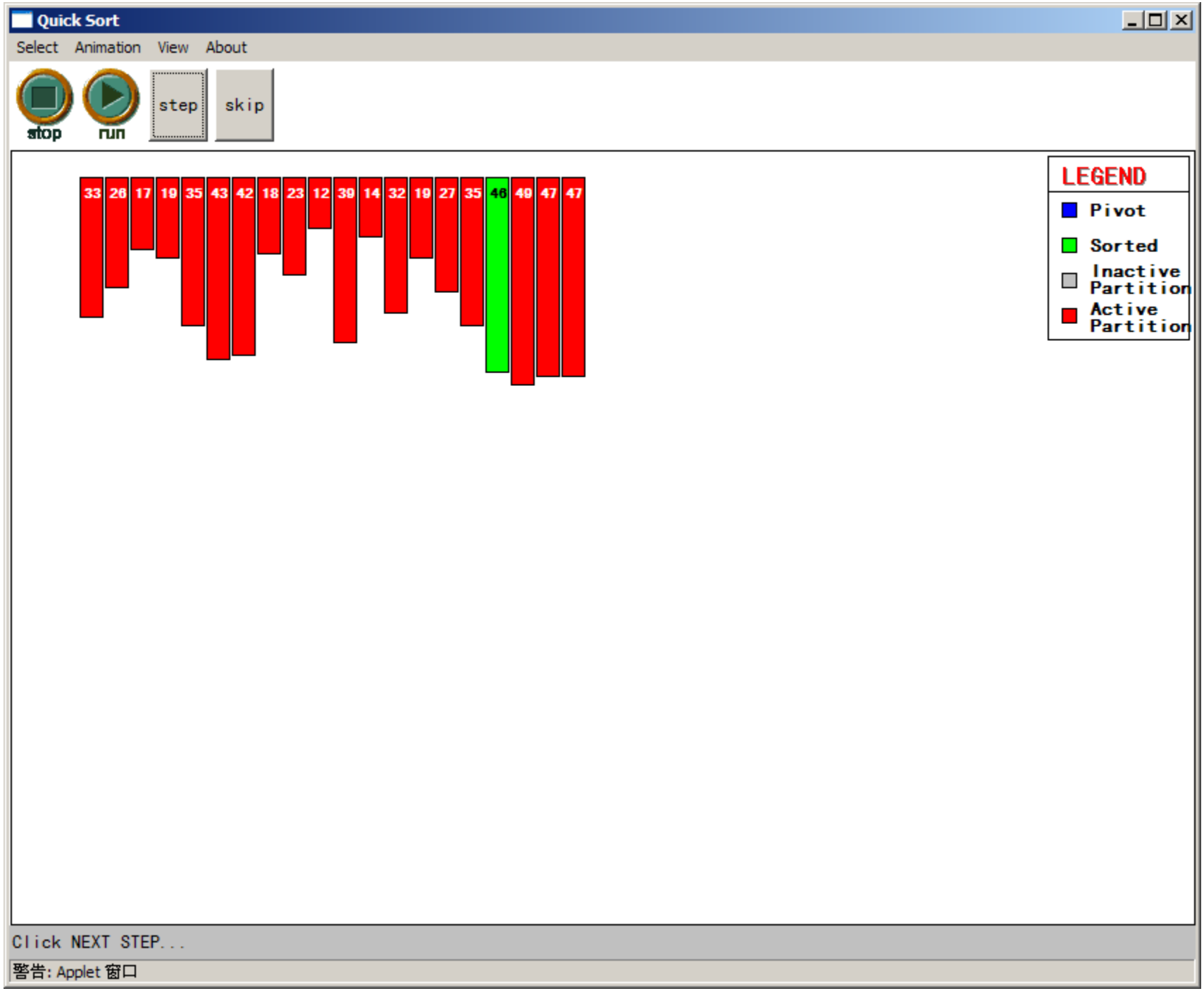
警告: Applet 窗口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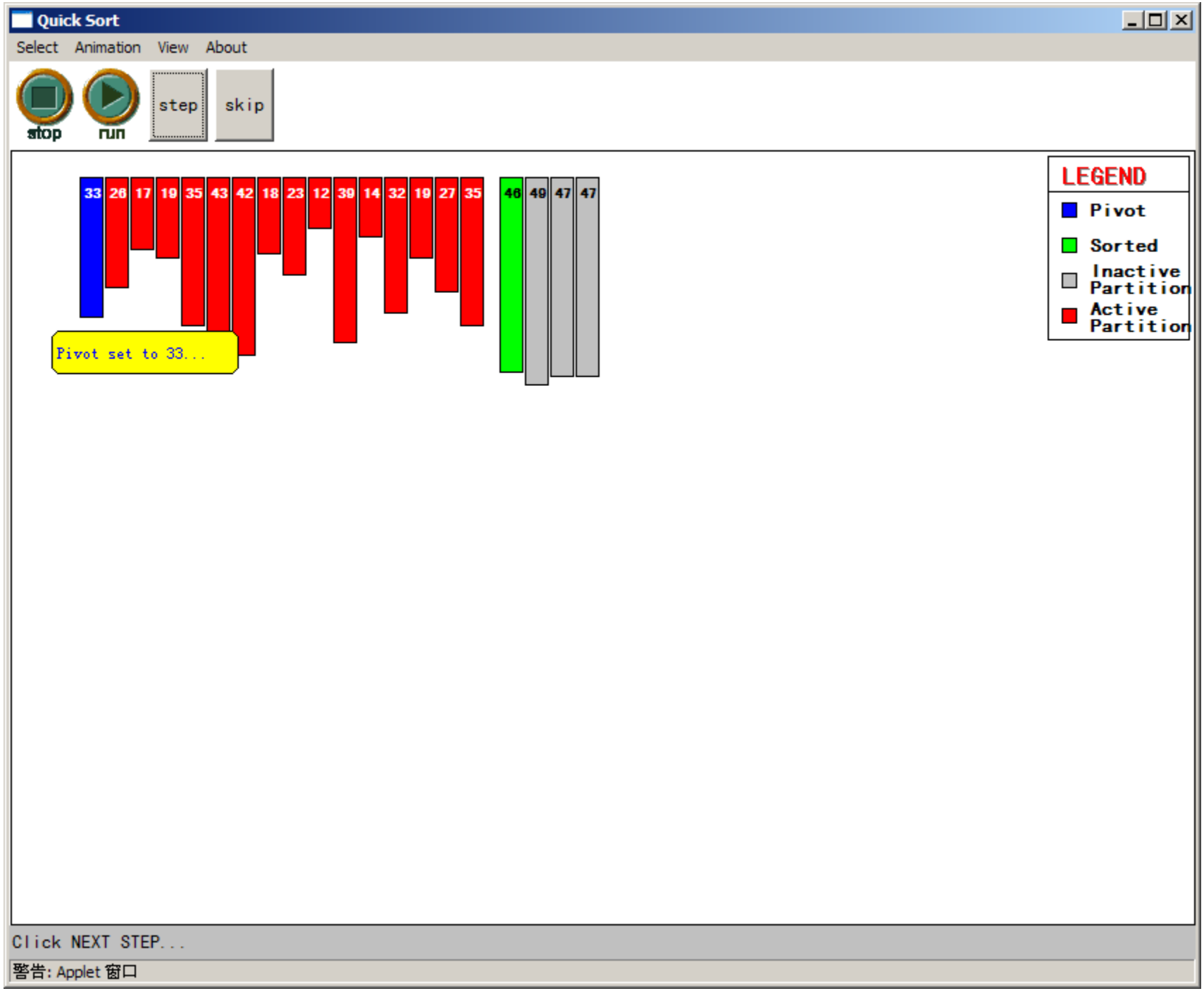
Inactive Partition
Active Partition
Click NEXT STEP...
警告: Applet 窗口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Pivot set to 33...
LEGEND
Pivot
Sorted
Inactive Partition
Active Partition
Click NEXT STEP...
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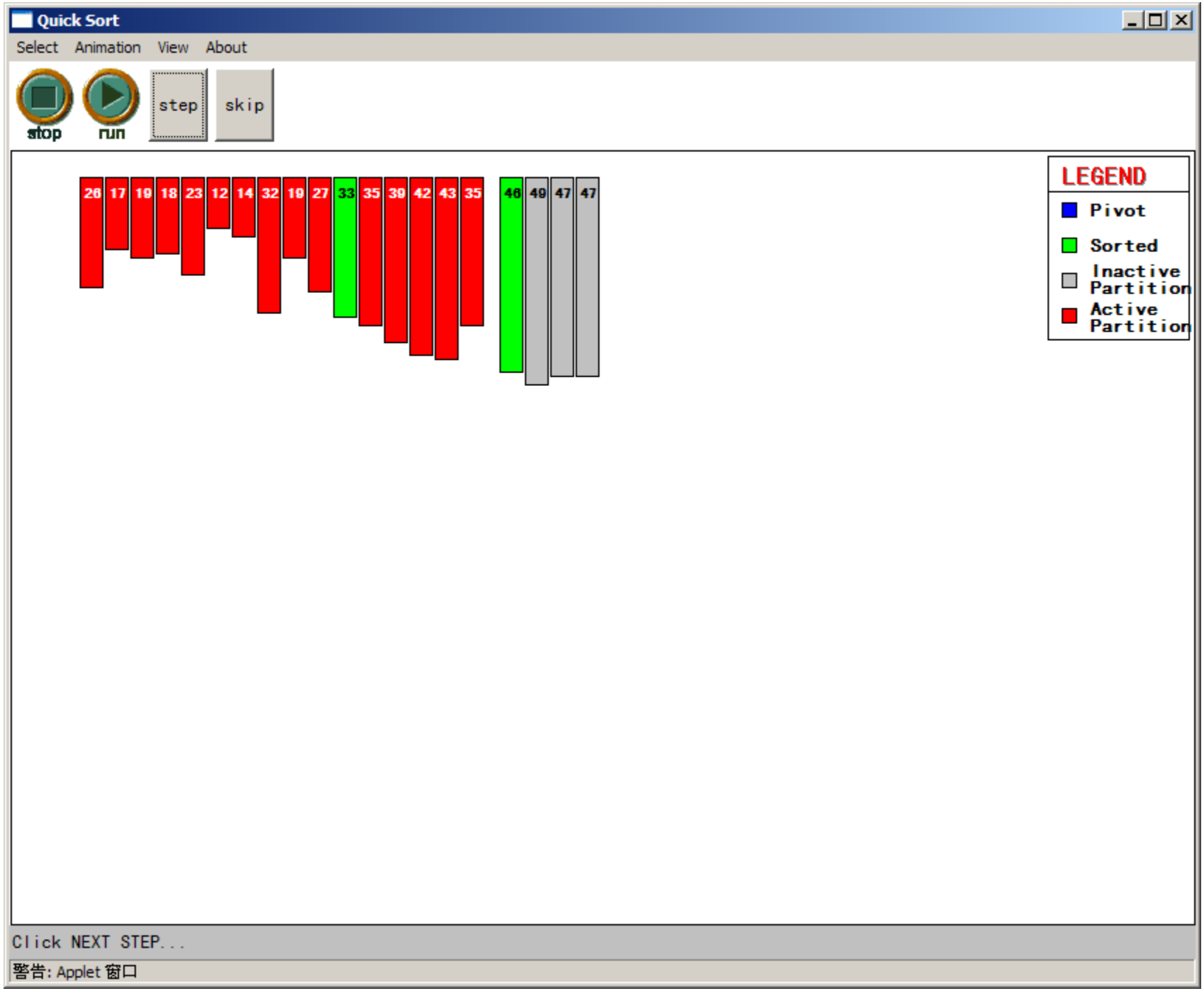
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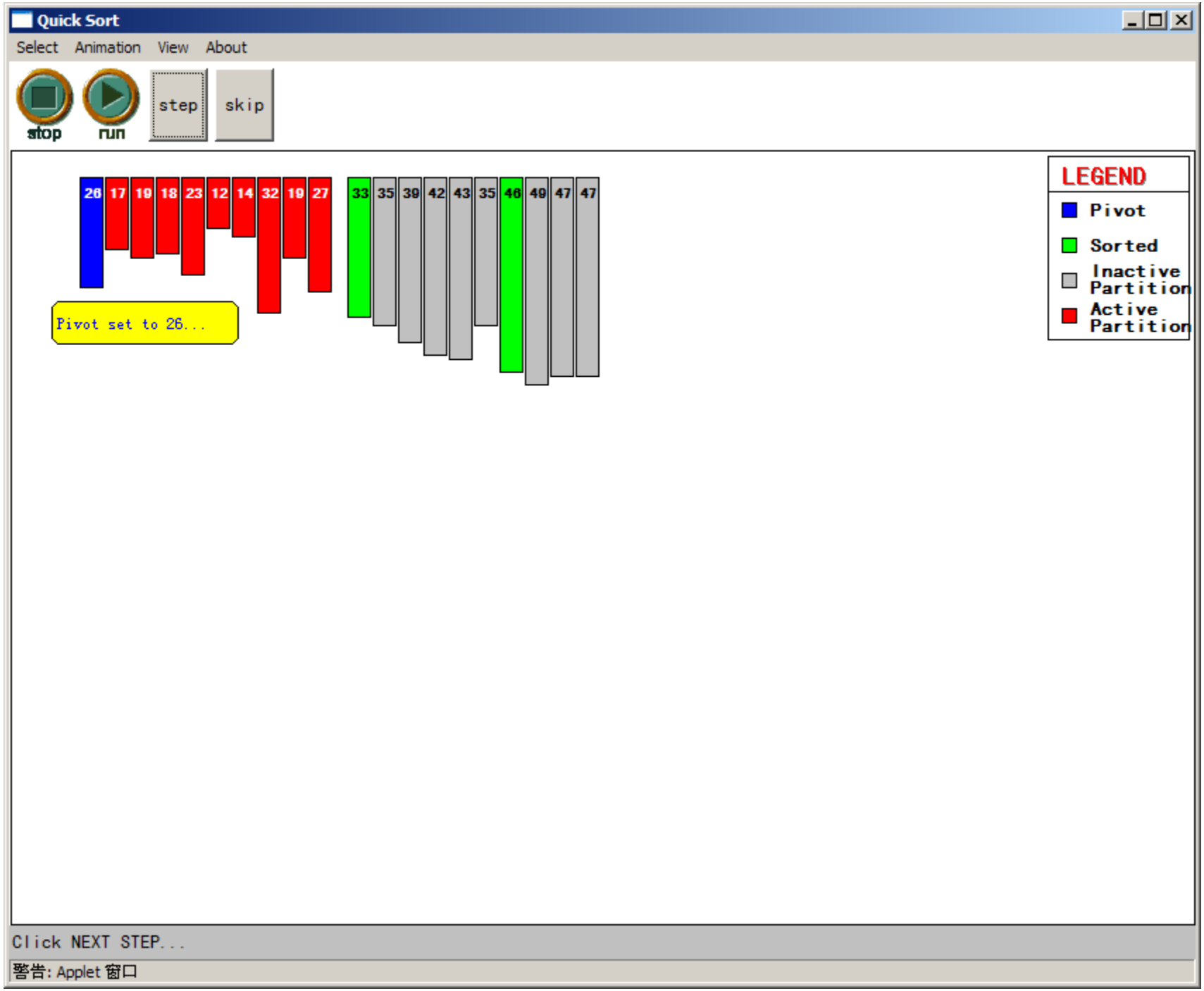
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Pivot set to 26...
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Active Partition
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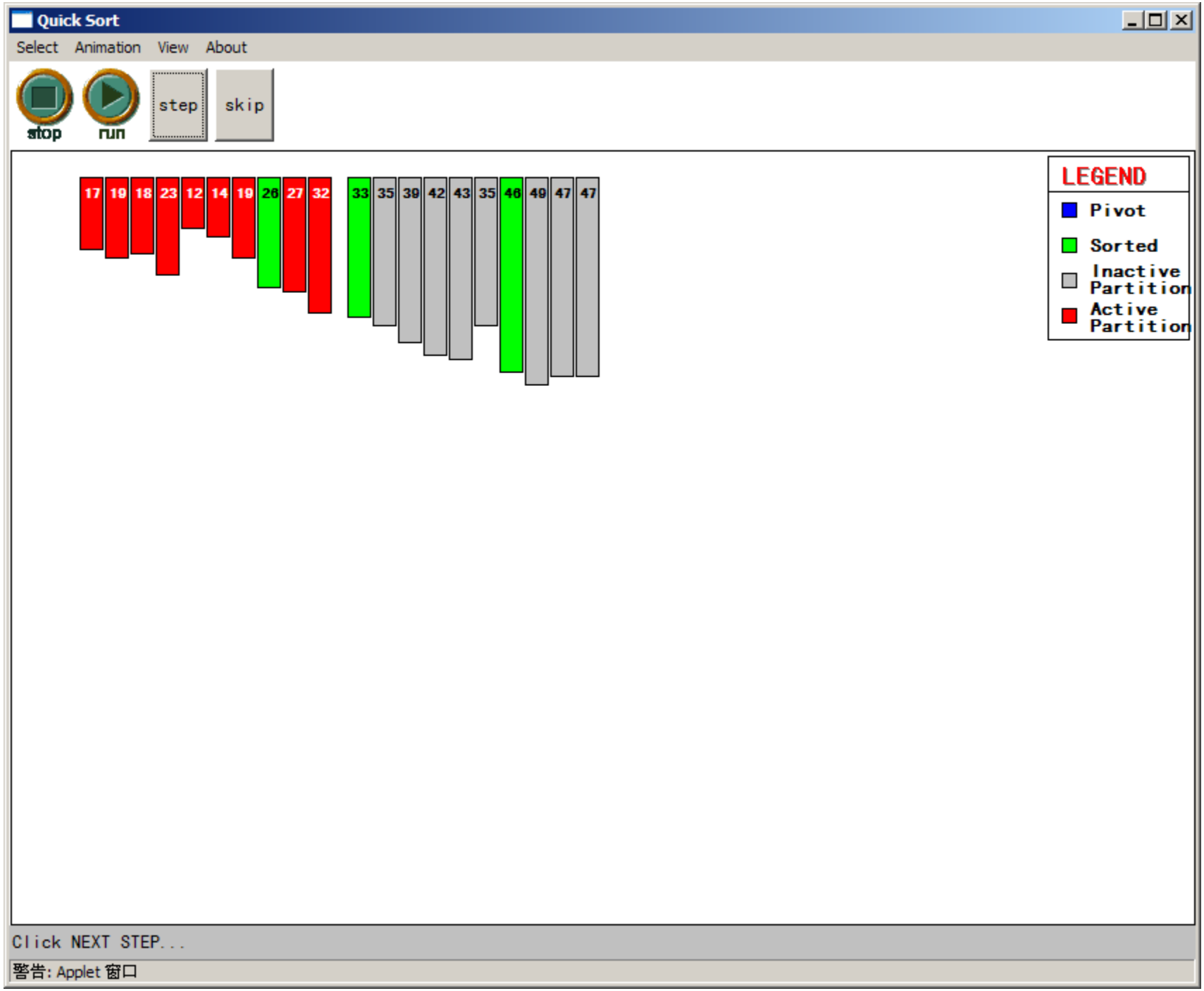
警告: Applet 窗口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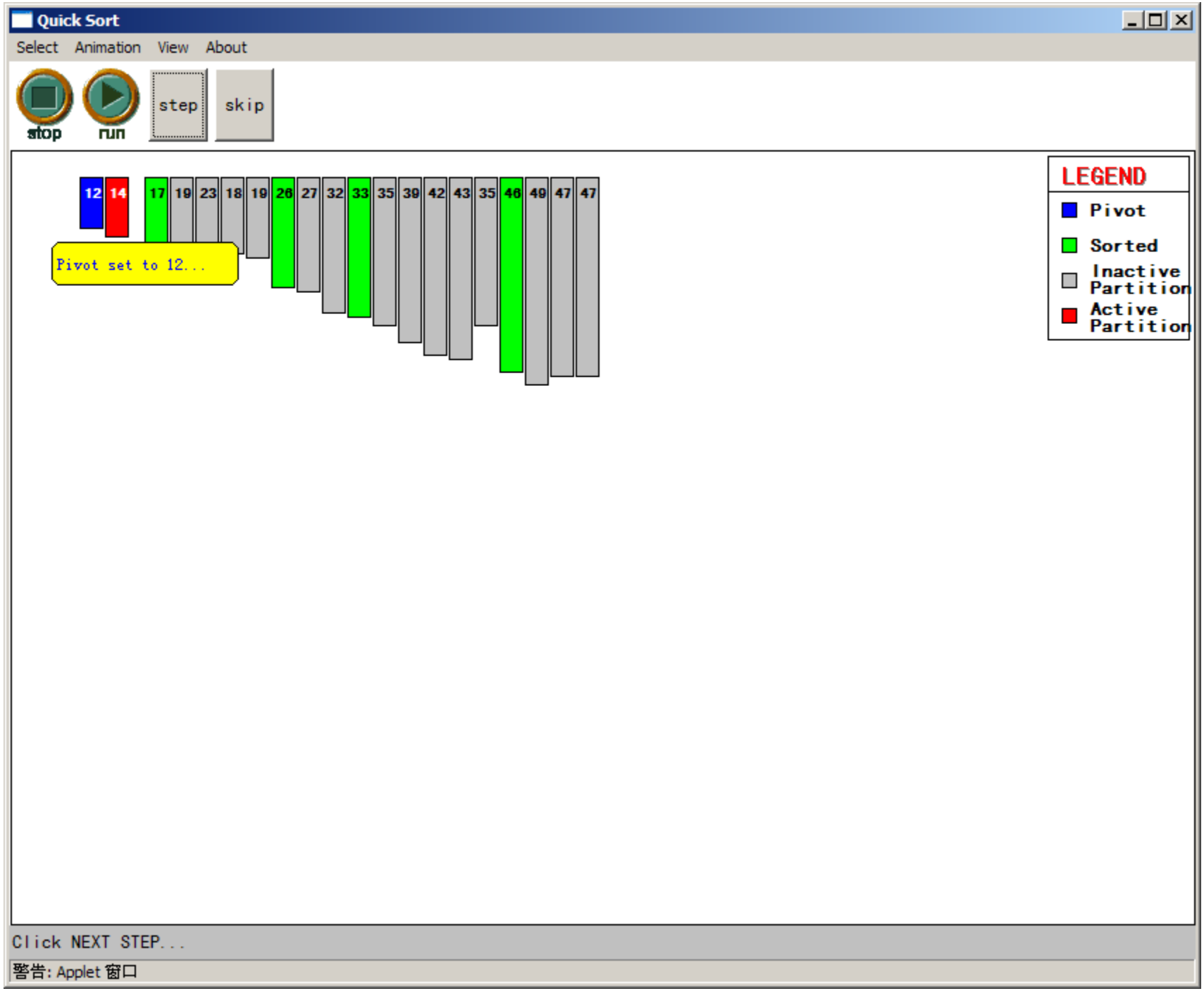
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Pivot set to 12...
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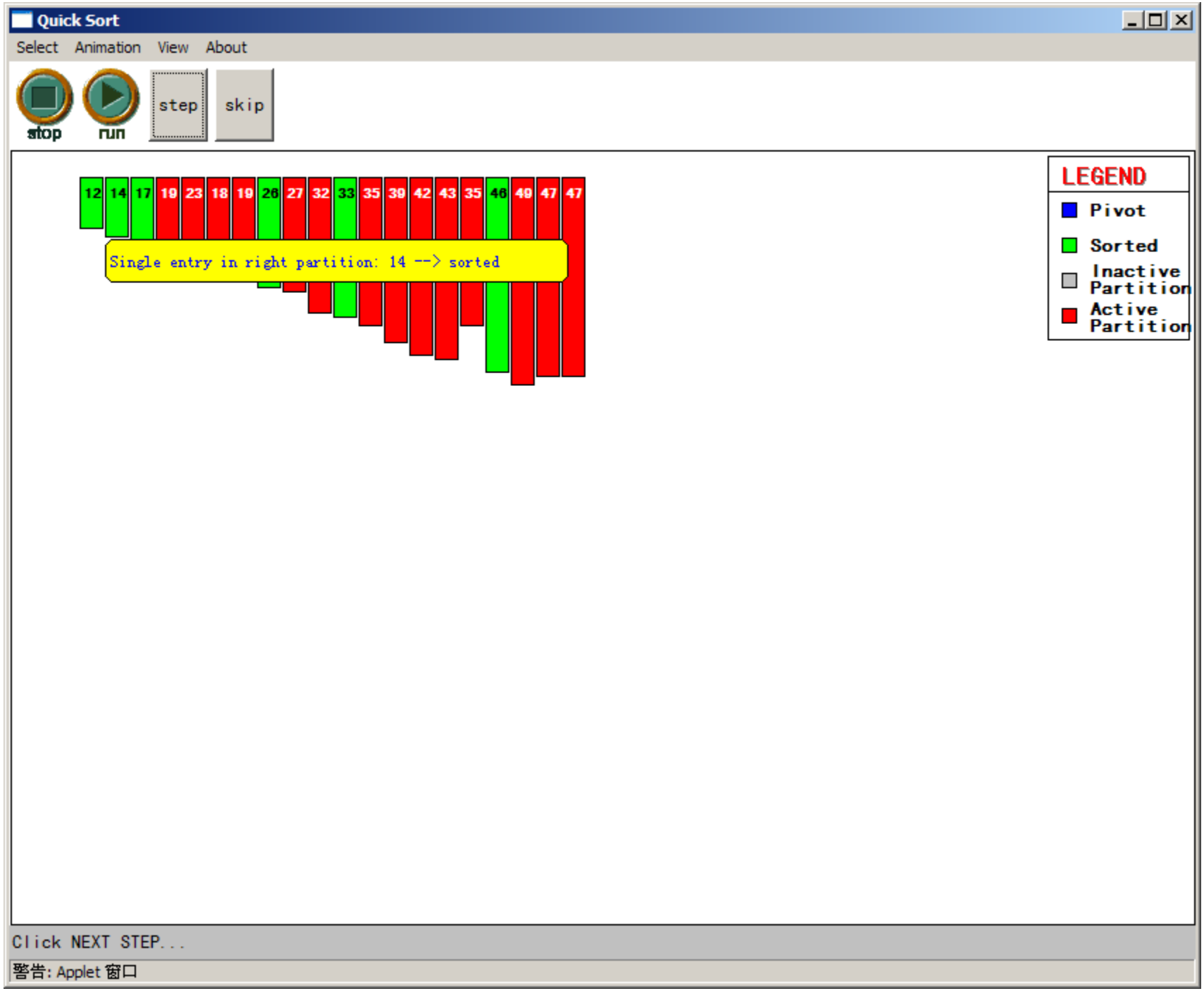
警告: Applet 窗口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Single entry in right partition: 14 --> sorted
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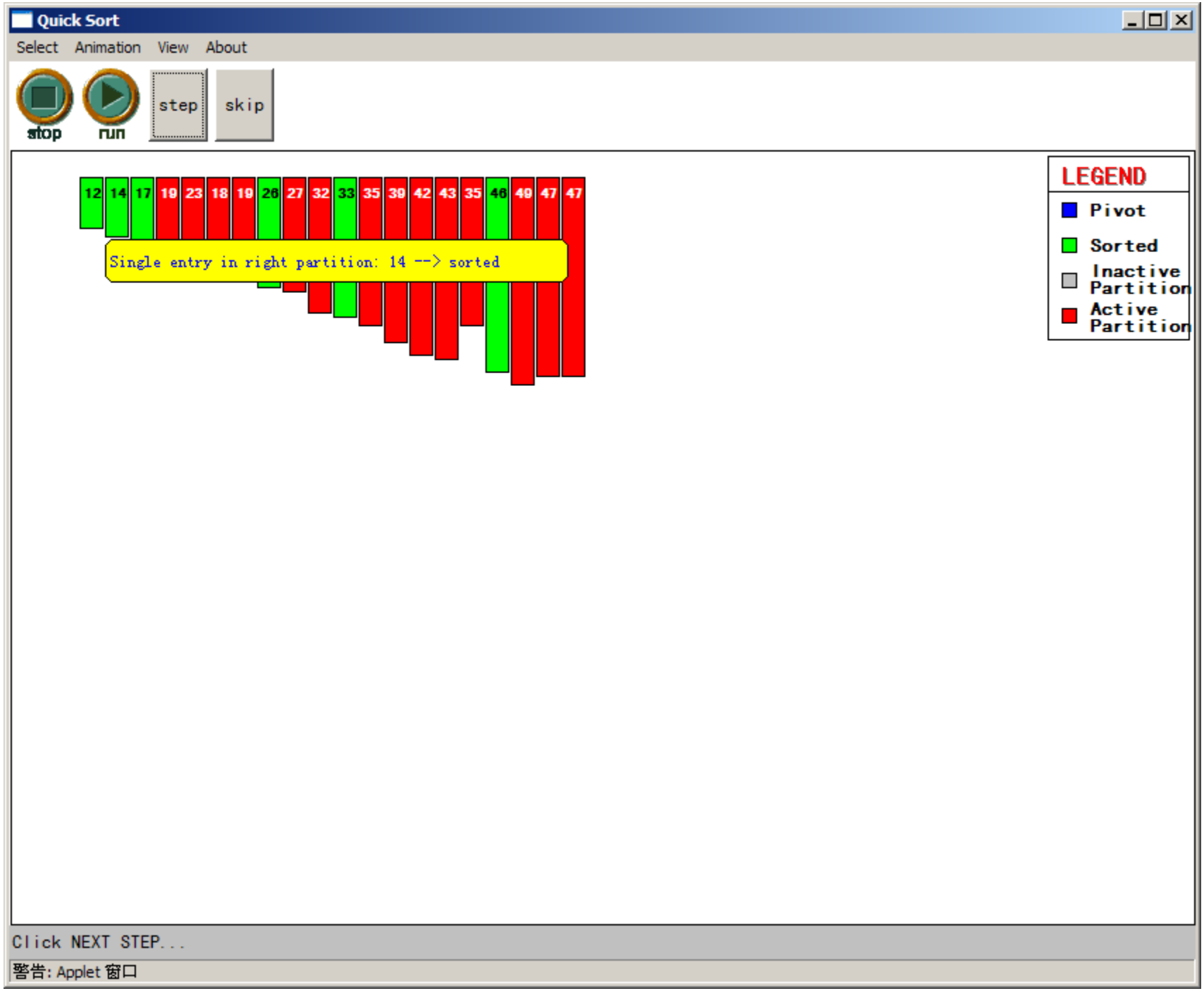
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Single entry in right partition: 14 --> sorted
LEGEND
Pivot
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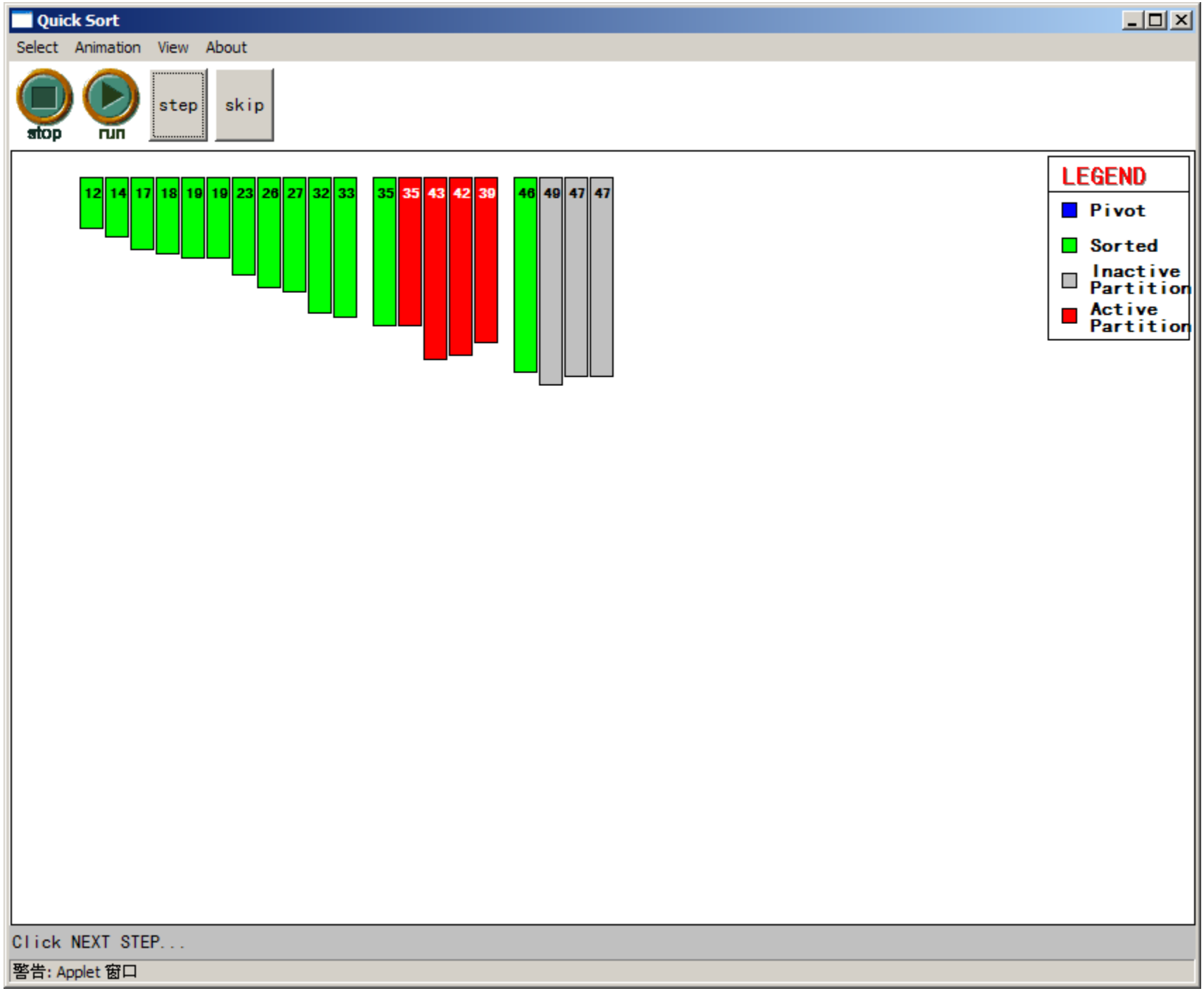
Inactive Partition
Active Partition
Click NEXT STEP...
警告: Applet 窗口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Active Partition
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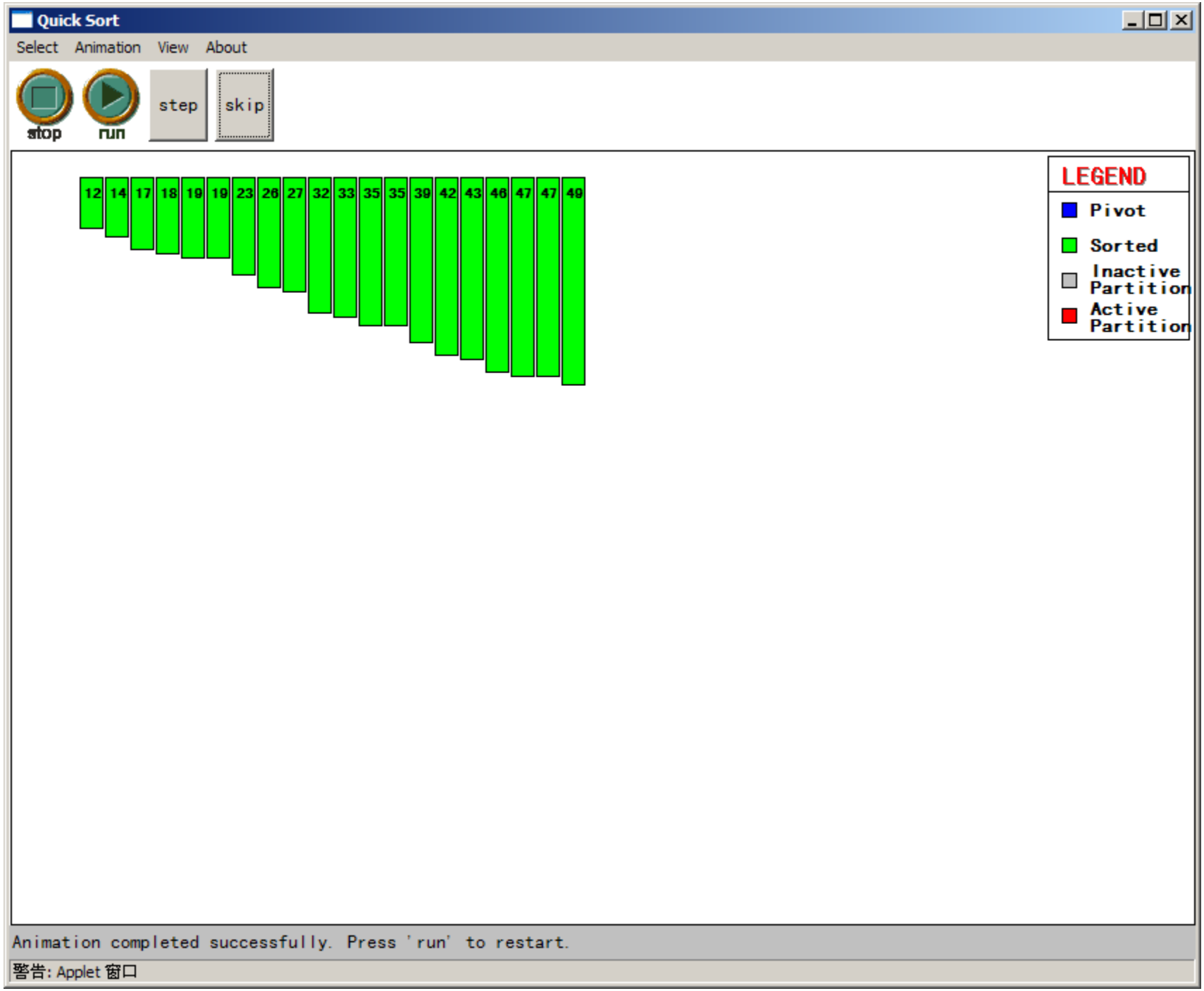
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LEGEND
Pivot
Sorted
Inactive Partition
Active Partition
Animation completed successfully. Press 'run' to restart.
警告: Applet 窗口

Quicksort

- Implementation

```
quicksort( void *a, int low, int high )
    {
    int pivot;
    /* Termination condition! */
    if ( high > low )
       {
       pivot = partition( a, low, high );
       quicksort( a, low, pivot-1 );
       quicksort( a, pivot+1, high );
       }
    }
```

Divide

Conquer

Quicksort - Partition

```
int partition( int *a, int low, int high ) {
   int left, right;
   int pivot_item;
   pivot_item = a[low];
   pivot = left = low;
   right = high;
   while ( left < right ) {
     /* Move left while item < pivot */
     while( a[left] <= pivot_item && left < right ) left++;
     /* Move right while item > pivot */
     while( a[right] > pivot_item) right--;
     if ( left < right ) {
         SWAP(a,left,right);
       }
     }
     /* right is final position for the pivot */
     a[low] = a[right];
     a[right] = pivot_item;
     return right;
   }
```

Quicksort - Analysis

- Non stable sorting algorithm
- Average case time complexity: O(n log n)
- Worst case time complexity: $O(n^2)$
  - Better pivot selection reduces probability
- Use when you need good performance on average

Questions in previous exam

- 7, 6, 1, 4, 8, 2, 3, 5
- Pivot: median of the first, middle, and last element of the array
- With a cutoff of 3
- Size of array is n, middle element index is n/2
- Index starts from 0 to n-1
- Demonstrate ONLY the process of partition the sequence into two parts of the QuickSort

Merge Sort - Definition

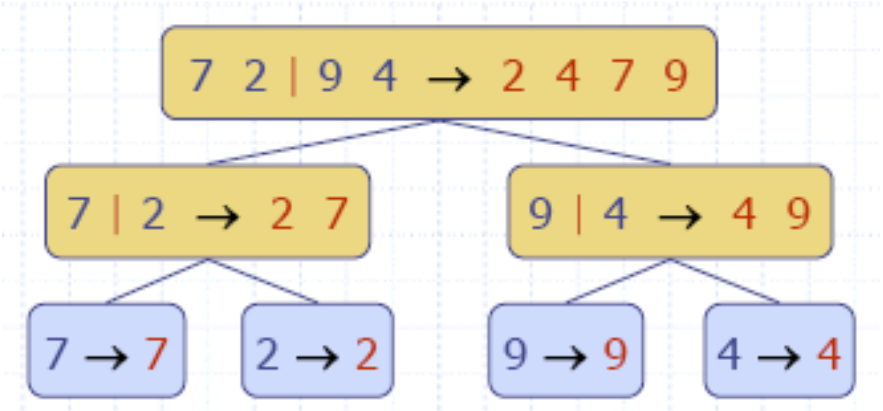
- **Definition:**
- A *sort* algorithm that
- Splits the items to be sorted into two groups
- *Recursively* sorts each group
- *Merge* them into a final sorted sequence.

Merge Sort – Divide-and-Conquer

- Divide-and-conquer is a general algorithm design paradigm:
  - Divide: divide the input data S in two disjoint subsets S1 and S2
  - Conquer: solve the sub-problems associated with S1 and S2 recursively
  - Combine: combine the solutions for S1 and S2 into a solution for S
- The base case for the recursion are sub-problems of size 0 or 1

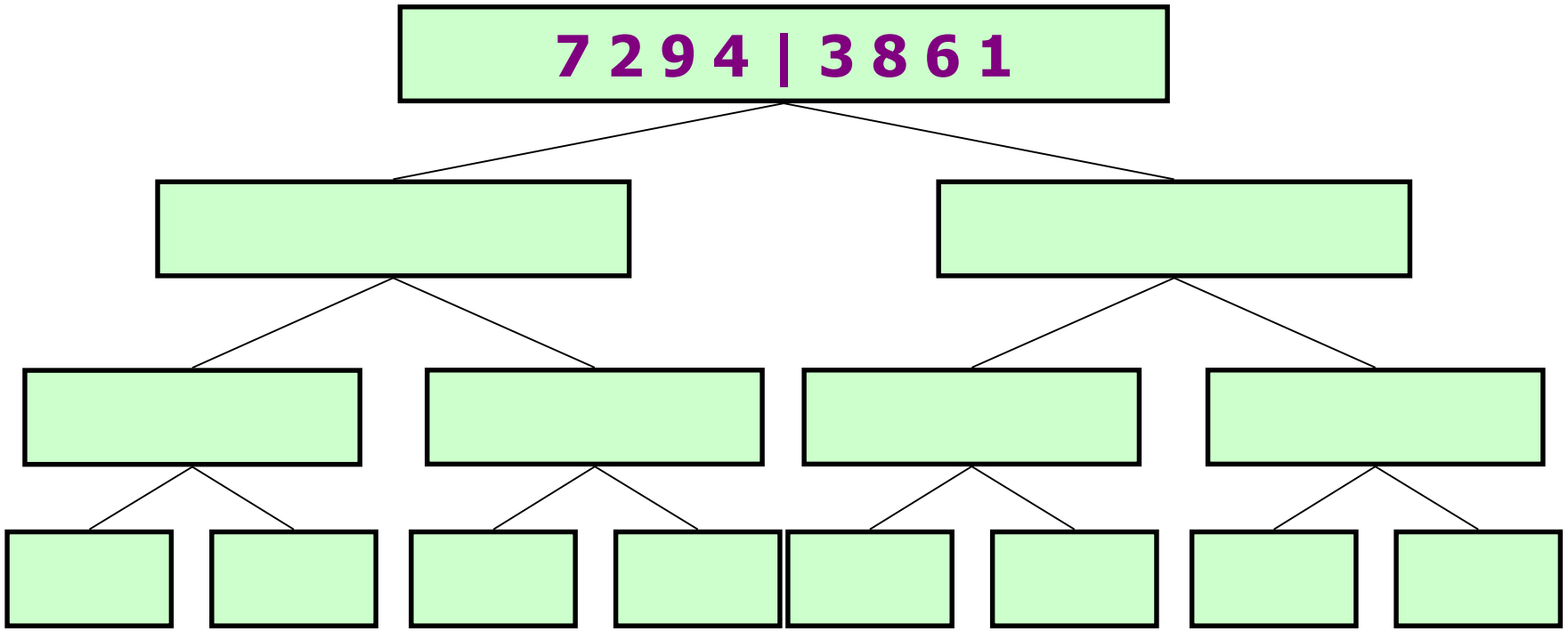
Merge Sort Tree

- An execution of merge-sort is depicted by a binary tree
  - each node represents a recursive call of merge-sort, and it stores
    - unsorted sequence before the execution
    - sorted sequence at the end of the execution
  - the root is the initial call
  - the leaves are calls on subsequences of size 0 or 1



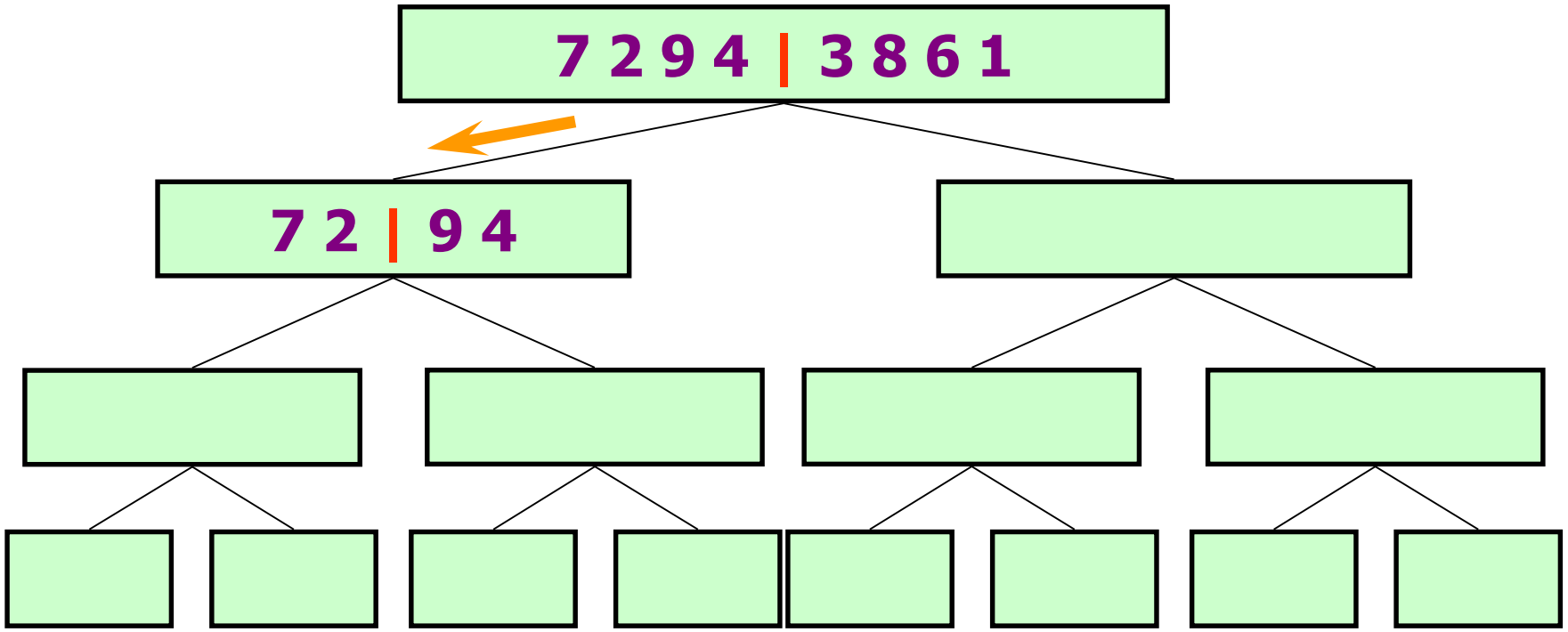
Merge Sort - example

Partition



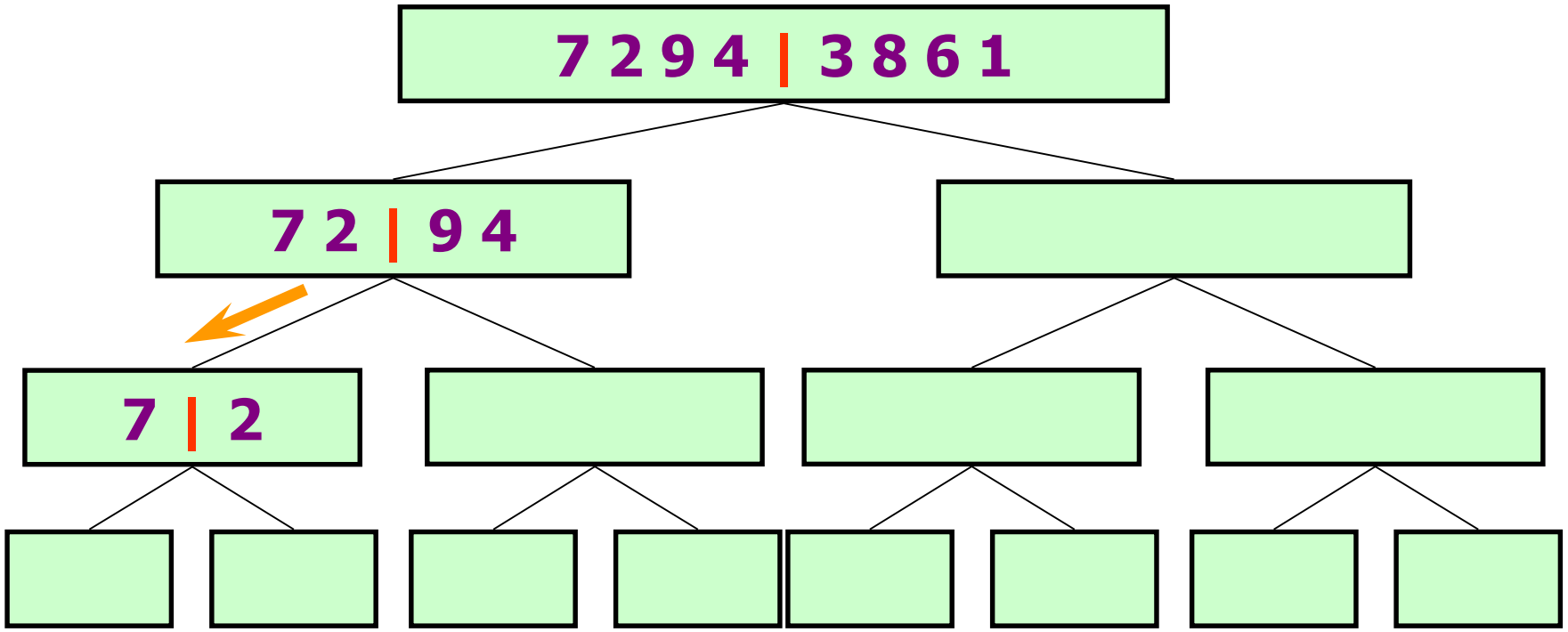
Merge Sort - example

Recursive call, partition



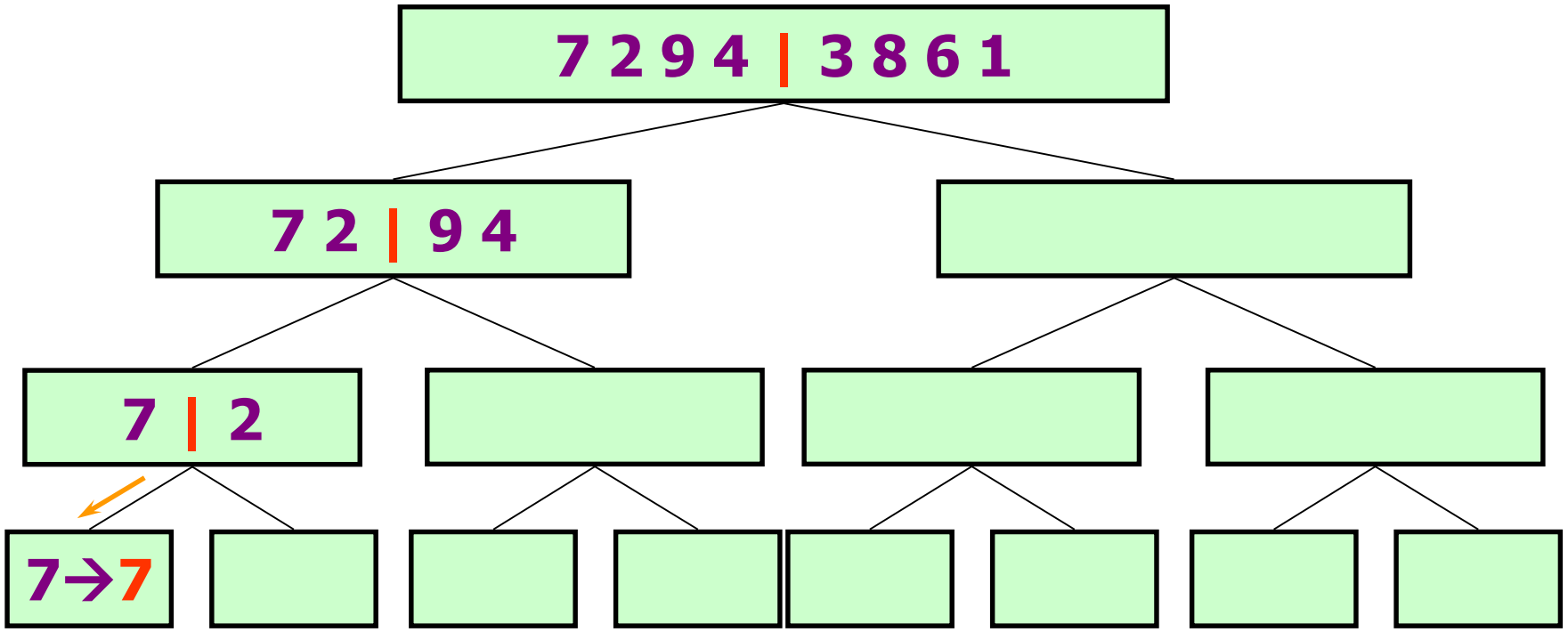
Merge Sort - example

Recursive call, partition



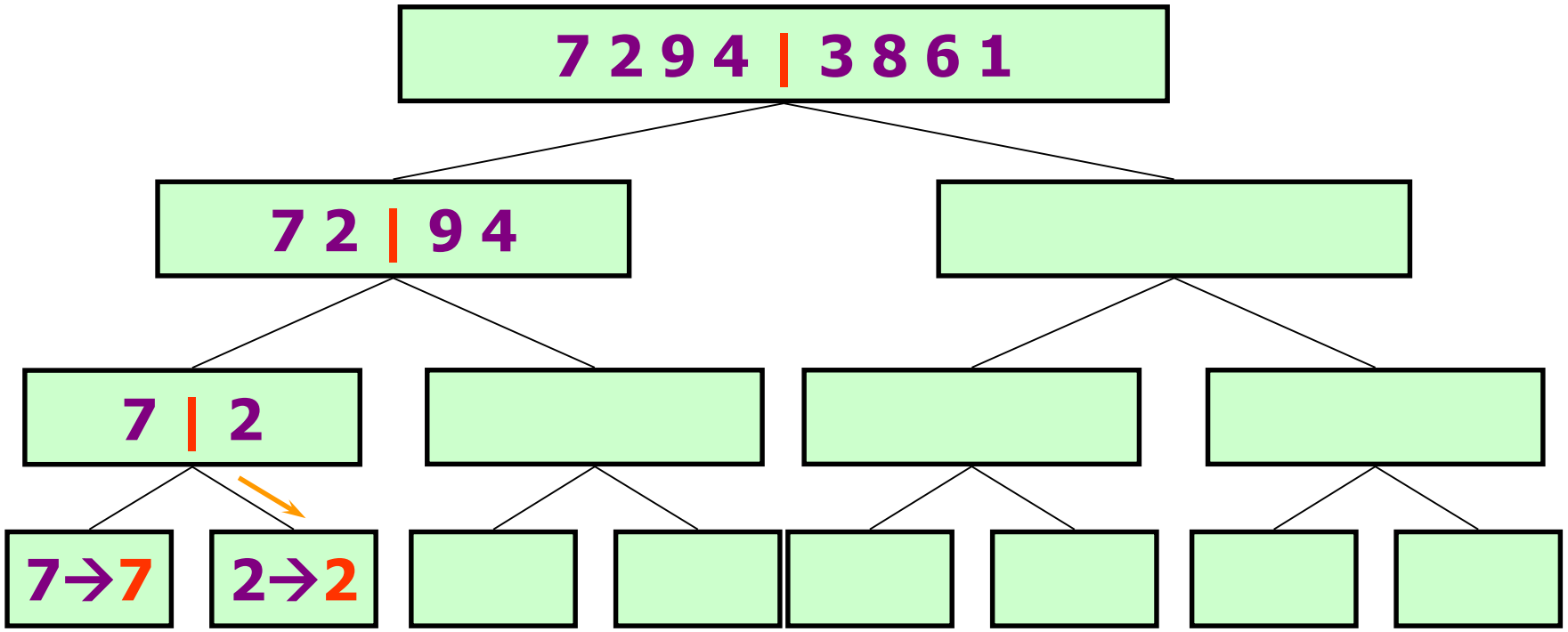
Merge Sort - example

Recursive call, base case



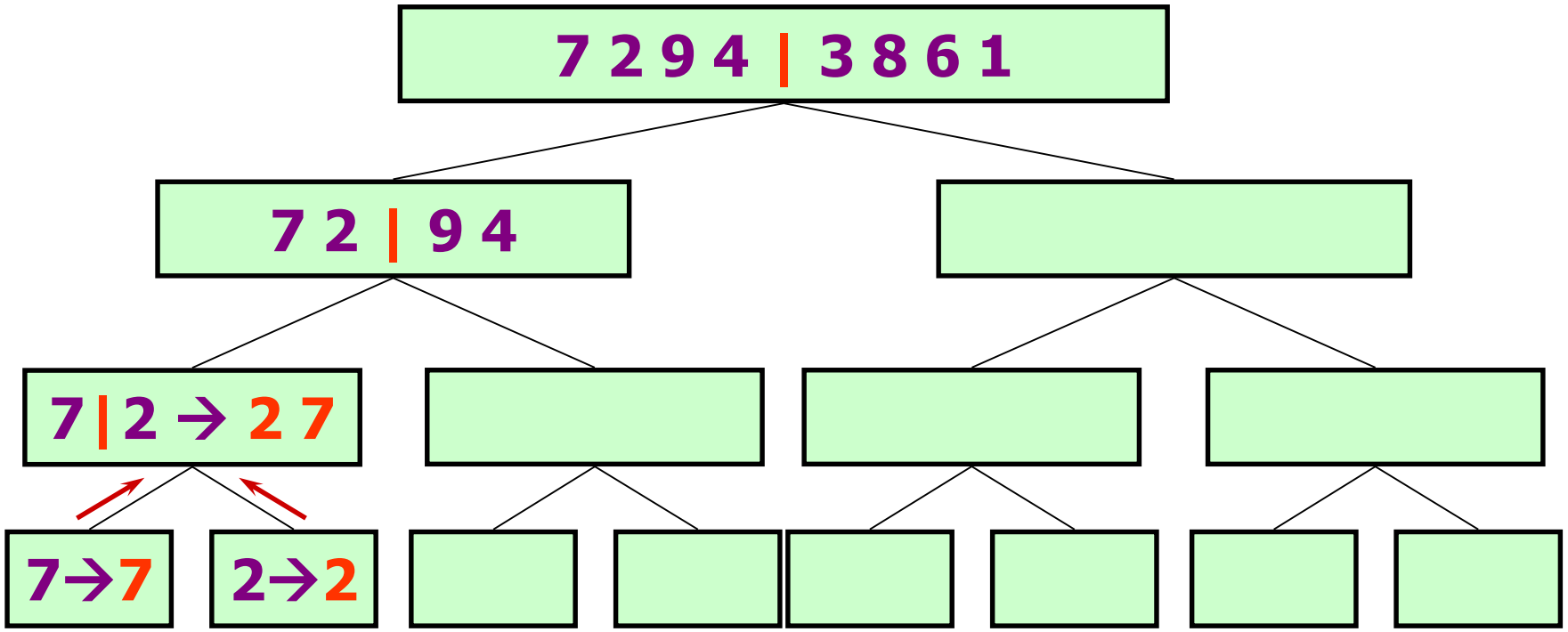
Merge Sort - example

Recursive call, base case



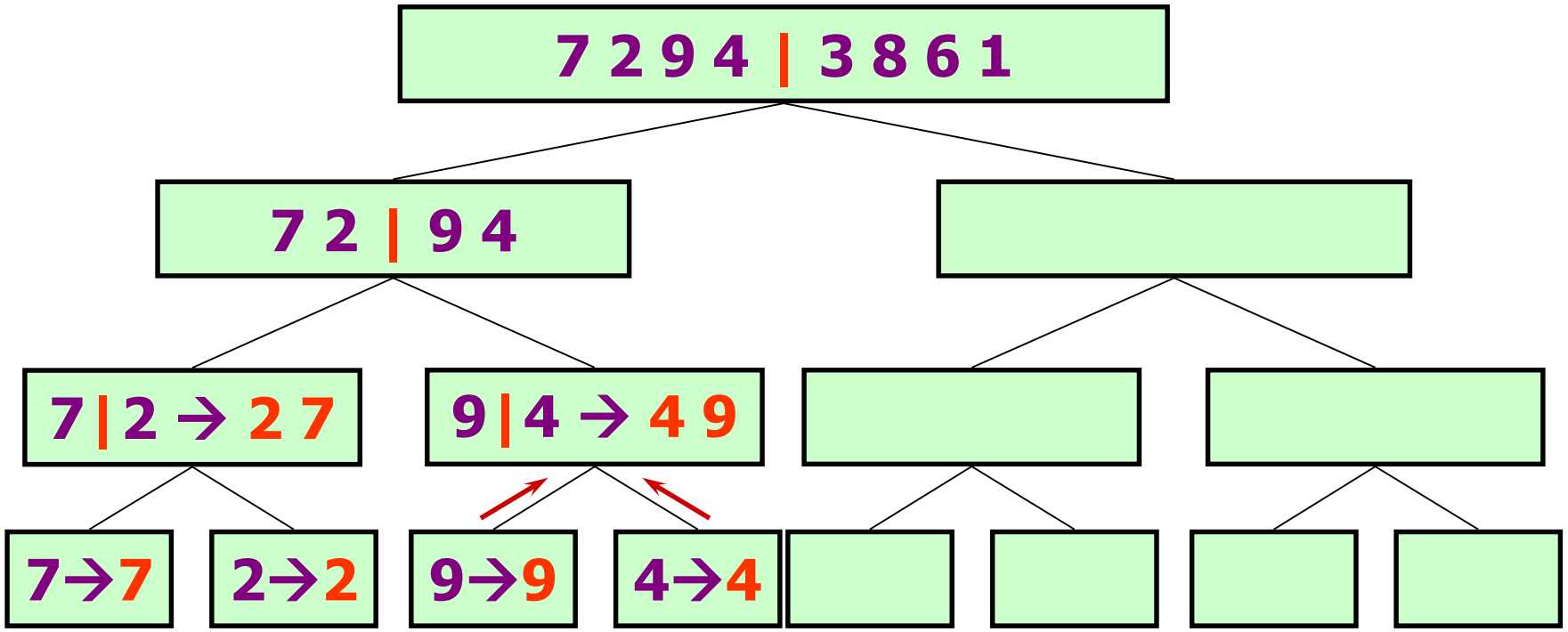
Merge Sort - example

Merge



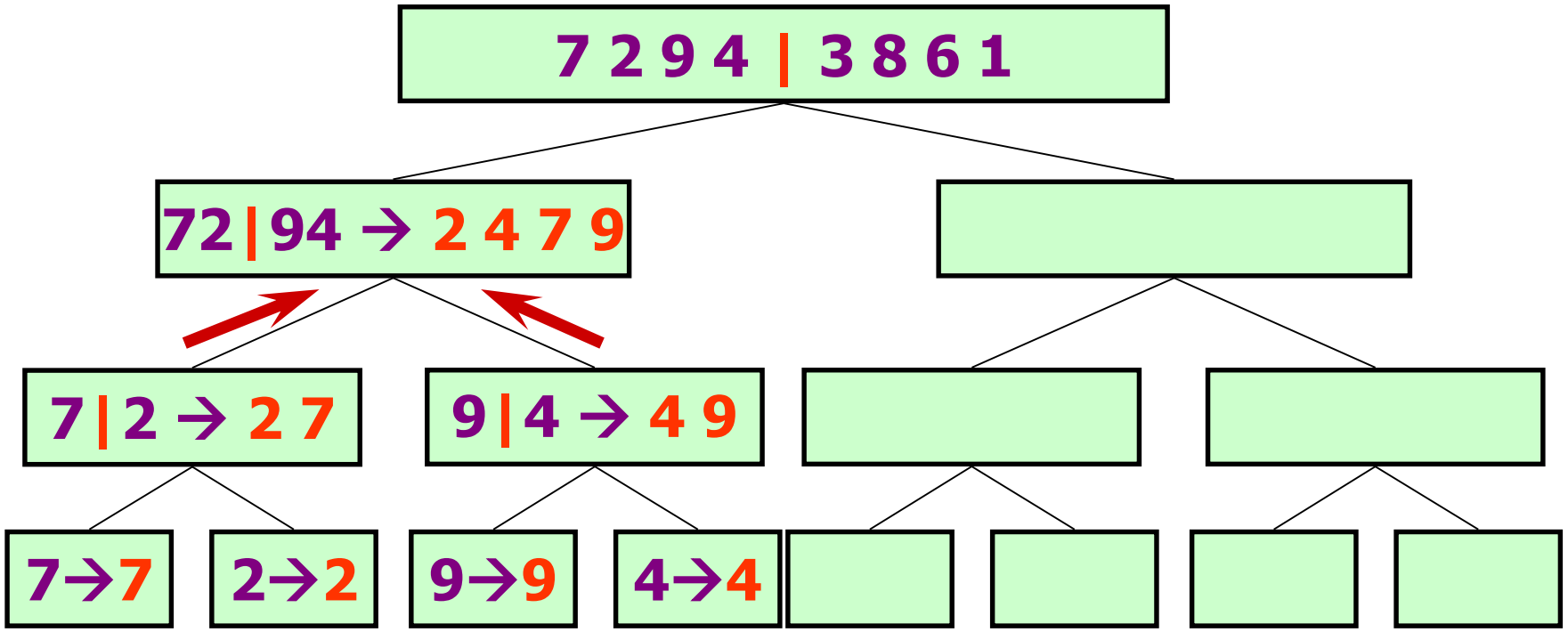
Merge Sort - example

Recursive call, base case, …, base case, merge



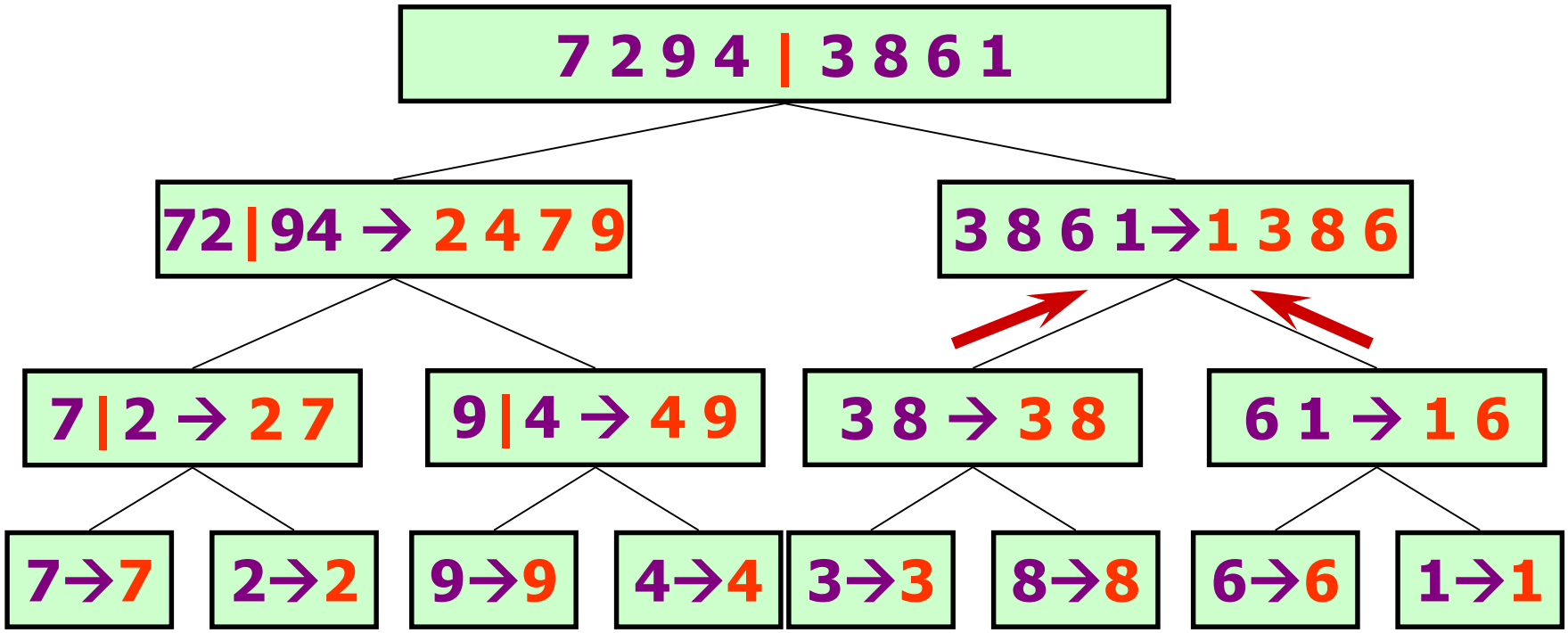
Merge Sort - example

Merge



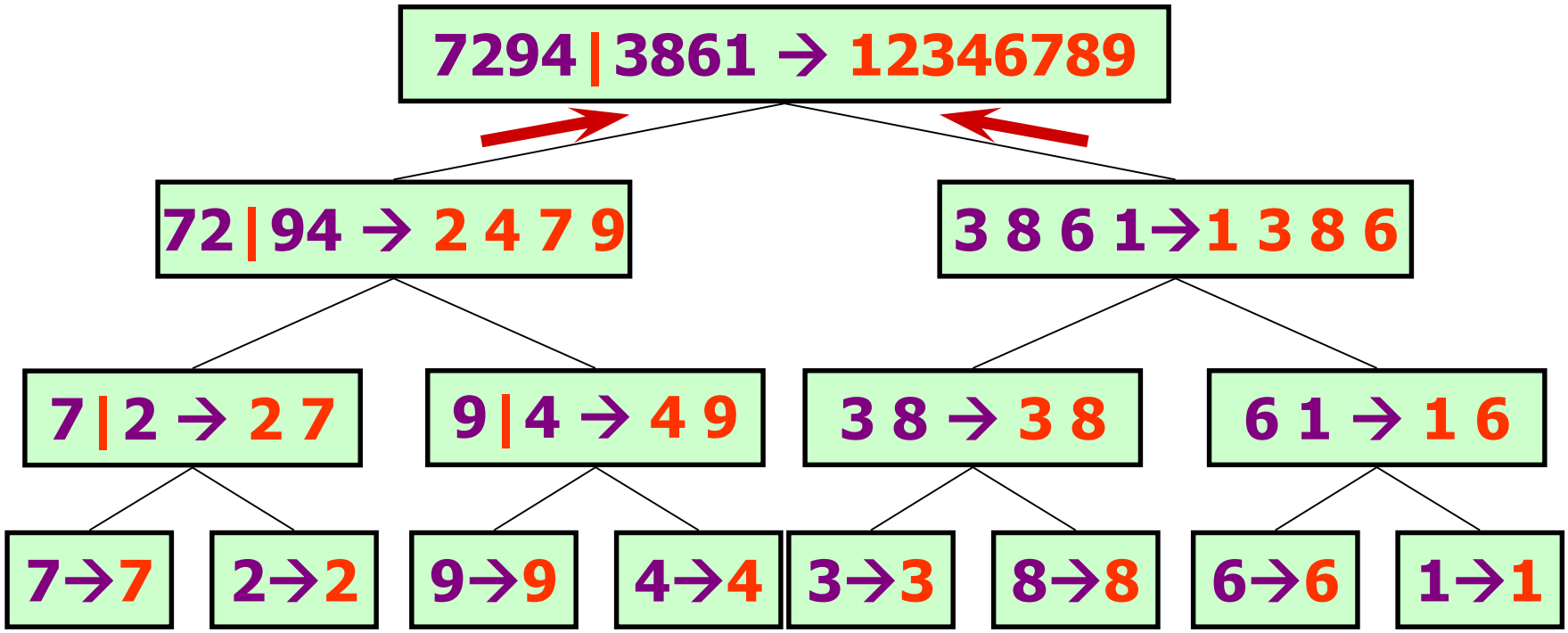
Merge Sort - example

Recursive call, …, merge, merge



Merge Sort - example

Merge



Merge Sort - analysis

- A stable sorting algorithm
- Worst case run time: $O\ (n \log n)$.
- Need $O\ (n)$ extra space

Merge Sort – sample code

```
void mergesort(int low, int high)
{
    if (low<high)
    {
        int middle=(low+high)/2;
        mergesort(low, middle);
        mergesort(middle+1,  high);
        merge(low, middle, high);
    }
 }
```

Merge Sort – sample code (cont.)

```
void merge(int low, int middle, int high)
{
    int i, j, k;
    // copy both halves of a to auxiliary array b
    for (i=low; i<=high; i++)
        b[i]=a[i];
    i=low; j=middle+1; k=low;
    // copy back next-greatest element at each time
    while (i<=middle && j<=high)
        if (b[i]<=b[j])    a[k++]=b[i++];
        else               a[k++]=b[j++];
    // copy back remaining elements of first half (if any)
    while (i<=middle)
        a[k++]=b[i++];
}
```

Questions in previous exam

- Given the sequence (5,2,8,4,1,6,3,7), sort the sequence using Mergesort.
- Illustrate the result after each pass.
- How many comparisons have you performed?

Questions in previous exam

- Show all the inversion pairs in the list (4,1,5,2,3)?
- List them out in an organized manner
- i.e., sort them in an ascending order by the first element and then by the second element

Questions in previous exam

- Given an array with n elements to sort,
- Write down the best and worst time complexity when using Bubble sort, Selection sort, Insertion sort and Merge sort in terms of Big-O notation