Written Assignment 1 Solution

1.1.1 Let $S = \sum_{i=0}^{\infty} i/4^i$.
$3S = 4S - S = \sum_{i=0}^{\infty}(i+1)/4^i - \sum_{i=0}^{\infty} i/4^i$
$= \sum_{i=0}^{\infty} 1/4^i = 1/(1-1/4) = 4/3$. Therefore, $S = 4/9$.

1.1.4 $\sum_{i=1}^{n} i = n(n+1)/2$

1.1.5 If $a \neq 1$, $\sum_{i=1}^{n} a^i = a(a^n - 1)/(a-1)$, otherwise $\sum_{i=1}^{n} a^i = n$.

1.2.1 $T(1) = 1$
$T(2) = aT(1) + 2b = a + 2b$
$T(3) = aT(2) + 3b = a^2 + 2ab + 3b$
$T(4) = aT(3) + 4b = a^3 + 2a^2b + 3ab + 4b$
$T(n) = a^{n-1} + b\sum_{i=2}^{n} ia^{n-i}$ (You may verify by induction)

1.2.2 Suppose n is a power of 2. (You can make this assumption for homeworks and in exam. Also you may assume that the logarithm is in base 2)
$T(1) = 1$
$T(2) = T(1) + 2 \cdot 1b$
$T(4) = T(2) + 4 \cdot 2b$
$T(8) = T(4) + 8 \cdot 3b$
$T(n) = 1 + \sum_{i=1}^{\log n} i2^i$

1.3.3 $\sum_{i=1}^{n}(2i-1) = 2\sum_{i=1}^{n} i - n = 2n(n+1)/2 - n = n^2$
You may also prove by induction if you don't want to use the formula for arithmetic series.

1.3.4 For $n \geq 3$, $2n^2 = n^2 + n^2 \geq n^2 + 2n + 1$. Therefore, there exist c and n_0, such that $\forall n \geq n_0, (n+1) \leq cn^2$, i.e. $(n+1)^2 = O(n^2)$.

1.4

	10	100log n	$5.6n$	$n \log n$	$0.25n^2$	$0.1n^3$	$0.001 \cdot 2^n$
big-O	$O(1)$	$O(\log n)$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(n^3)$	$O(2^n)$

	10	$100 \log n$	$5.6n$	$n \log n$	$0.25n^2$	$0.1n^3$	$0.001 \cdot 2^n$
10	-	-	-	-	-	-	-
$100 \log n$	2	-	-	-	-	-	-
$5.6n$	2	125	-	-	-	-	-
$n \log n$	5	101	49	-	-	-	-
$0.25n^2$	7	48	23	17	-	-	-
$0.1n^3$	5	16	8	5	3	-	-
$0.001 \cdot 2^n$	14	19	17	16	16	20	-

1.5.3 $f(n) = \sum_{i=1}^{n} \sum j = i^n \sum_{k=1}^{j} 1 = \sum_{i=1}^{n} \sum j = i^n j = \sum_{i=1}^{n} \frac{(n+i)(n-i+1)}{2} = \frac{1}{6}n(n+1)(2n+1) = O(n^3)$.

1.5.4 $f(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=1}^{in} 1 = \sum_{i=1}^{n} \sum_{j=i}^{n} in = \sum_{i=1}^{n} in(n-i+1) = \frac{1}{6}n^2(n+1)(n+2) = O(n^4)$.

1.8.1 Please calculate and show the characteristic points(e.g. discontinuous point) of the curves. To make it easier to plot and read, you can use non-uniform broken axis.

1.8.2

n	10	20	30	50	70	100
time A	10	20	30	$1.25 \cdot 10^5$	$3.43 \cdot 10^5$	10^6
time B	10	20	900	2500	$3.43 \cdot 10^5$	10^6
space A	10	20	45	75	105	150
space B	50	100	150	25	35	50

1.8.3

$$\bar{C}_A = \frac{1}{100} \sum_{n=1} 100 C_A(n) = 2.404 \cdot 10^5$$

$$\bar{C}_B = \frac{1}{100} \sum_{n=1} 100 C_B(n) = 1.981 \cdot 10^5$$

B is better, by $4.23 \cdot 10^4$.

1.8.4 Use A for $1 \leq n < 50$ and use B for $50 \leq n \leq 100$.

1.8.5

$$\bar{C}_{hyb} = \frac{1}{100} \sum_{n=1} 100 C_{hyb}(n) = 1.976 \cdot 10^5$$

The hybird algorithm is better than A by $4.28 \cdot 10^4$, and is better than B by $5 \cdot 10^2$.

(Other answers are OK as long as they are consistent with 1.8.4)