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TESTS OF GAUSSIAN TEMPORAL FACTOR LOADINGS IN FINANCIAL APT

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ABSTRACT

The main difficulty of financial APT analysis concerns identifying unambiguously the hidden statistical factors. Lack of effective techniques to retrieve the true factors often leads to inappropriate interpretation of the underlying factor structure. In literature, PCA and MLFA, assuming multivariate Gaussian distributions, and ICA, assuming non-Gaussian distributions, are used to extract factors and determine the corresponding factor loadings. Recently, a new technique called TFA is proposed in [1, 2] which seeks to solve the problem of rotation indeterminacy encountered in conventional factor analysis. In this paper we will focus on statistical tests and inference on the APT temporal factor loadings recovered by TFA.

1. INTRODUCTION

Arbitrage Pricing Theory (APT) as an asset pricing model has attracted considerable interest from practitioners in the finance field since it was proposed by Ross [3] in 1976. APT assumes returns being generated under an *exact factor structure* in which the residual component of returns not explained by the factors is uncorrelated among securities. As a result, various efforts have been devoted to recover the factors and their loadings via Maximum Likelihood Factor Analysis (MLFA). However, MLFA is well-known for the rotation indeterminacy. Despite Roll and Ross [4] point out that factor rotation has no effect on the rejection region for APT, such indeterminacy and lack of unique identification create great obstacles towards finding a reasonable and appropriate economic interpretation to the results of APT analysis.

In view of the lack of uniqueness in MLFA solution, Chamberlain and Rothschild [**?**] make an effort to relax the original exact factor structure imposed on APT and propose the so-called *approximate factor structure* with an aim to simplify the complication involved in factor analysis. The major tradeoff with the original model is that the residual component being no longer uncorrelated. In the same paper, they also prove that if k eigenvalues of the population covariance matrix increase without bound as the number of securities in the population increases, then the elements of the corresponding k eigenvectors of the covariance matrix can be used as factor sensitivities. Connor and Korajczyk [6] further show that this conclusion holds for the sample covariance matrix as well.

However, Shukla and Trzcinka [7] criticize this approach on the ground that eigenvectors being used instead of statistical factor loadings in the returns-generating model for a large economy, possibly with infinitely many assets, does not necessarily imply that they can be used in a cross-sectional model of security pricing in finite economies. The reason is that PCA is very different from factor analysis in context of finite number of securities. PCA is more constrained than factor analysis for the application in APT because it tends to overlook idiosyncratic risks.

In recent years, Back and Weigend [8] apply Independent Component Analysis (ICA) to multivariate financial time series to recover statistically independent non-Gaussian distributed factors. It is well-known that ICA will exploit higher-order statistics of the distribution to overcome the problem of rotation indeterminacy. Although it is quite reasonable and realistic to assume non-Gaussian distribution of factors, the critical weakness of applying ICA for this specific task arises from treating the noise or idiosyncratic risks component as negligible. Obviously, this frequently conflicts with the truth from both a theoretical and empirical point of view.

Recently, the development of Temporal Baysian Ying-Yang (TBYY) Theory proposed in [1] leads to the inception of a new factor analytic technique called Temporal Factor Analysis (TFA). TFA can be seen as an extension to MLFA with the distinct strength to overcome rotation indeterminacy as well as to provide an appropriate answer to the number of hidden factors via its automatic model selection ability. As a result, it may serve as an alternative for traditional APT analysis. In [**?**], we have applied TFA to determine the factor number in real APT analysis using the same set of data. In this paper, we will further our research on the Gaussian factor loadings which is partially based on the findings in [**?**].

The rest of the paper is divided into five sections. Section 2 reviews the original APT by Ross on which our analysis will be based. In Section 3, we will discuss the potential benefits of applying the temporal factor model in the APT analysis. Experiments and statistical tests results will be presented in Section 4 & 5. Section 6 will be devoted to concluding remarks.

2. THE ARBITRAGE PRICING THEORY

The APT begins with the assumption that the $n\times 1$ vector of asset returns, $\tilde{R}_t$, is generated by a linear stochastic process with k factors:

$$\tilde{R}_t = \bar{R} + Af_t + e_t \tag{1}$$

where f_t is the $k\times 1$ vector of realizations of k common factors, A is the $n\times k$ matrix of factor weights or loadings, and e_t is a $n\times 1$ vector of asset-specific risks. It is assumed that f_t and e_t have zero expected values so that $\bar{R}$ is the $n\times 1$ vector of mean returns. It is usually assumed that $E(f_i f_j) = 0$, $E(e_i e_j) = 0$ so that if V is the variance-covariance matrix of returns. It may be written as:

$$V = AA' + \Sigma \tag{2}$$

where $E(ee') = \Sigma$.

The model addresses how expected returns behave in a market with no arbitrage opportunities and predicts that an asset's expected return is linearly related to the factor loadings or

$$\bar{R} = R_f + Ap \tag{3}$$

where R_f is a $n\times 1$ vector of constants representing the risk-free return, and p is $k\times 1$ vector of risk premiums.

3. TEMPORAL FACTOR ANALYSIS

3.1. An Overview of TFA

Suppose the relationship between a state $y_t \in \Re^k$ and an observation $x_t \in \Re^d$ are described by the first-order state-space equations as follows:

$$\begin{aligned} y_t &= By_{t-1} + \varepsilon_t, & (4)\\ x_t &= Ay_t + e_t, \qquad t = 1,2,\ldots,N. & (5)\end{aligned}$$

where ε_t and e_t are mutually independent zero-mean white noises with $E(\varepsilon_i\varepsilon_j) = \Sigma_\varepsilon\delta_{ij}$, $E(e_i e_j) = \Sigma_e\delta_{ij}$, $E(\varepsilon_i e_j) = 0$, and δ_{ij} is the Kronecker delta function:

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j,\\ 0, & \text{otherwise.}\end{cases} \tag{6}$$

We call ε_t driving noise upon the fact that it drives the source process over time. Similarly, e_t is called measurement noise because it happens to be there during measurement. The above model is generally referred to as the TFA model.

In the context of APT analysis, eq.(1) can be obtained from eq.(5) by substituting $(\tilde{R}_t - \bar{R})$ for x_t and f_t for y_t. The only difference between the APT model and the TFA model is the added eq.(4) for modelling temporal relation of each factor. The added equation represents the factor series $\mathbf{y} = \{y_t\}_{t=1}^T$ in a multi-channel auto-regressive process, driven by an i.i.d. noise series $\{\varepsilon_t\}_{t=1}^T$ that are independent of both y_{t-1} and e_t. Specifically, it is assumed that ε_t is Gaussian distributed. Moreover, TFA in [1, 2] is defined such that the k sources $y_t^{(1)}, y_t^{(2)}, \ldots, y_t^{(k)}$ in this state-space model are statistically independent, i.e.,

$$\begin{aligned} p(y_t \mid \mathbf{y_{t-1}}) &= \prod_{j=1}^{k} p(y_t^{(j)} \mid y_{t-1}^{(j)}),\\ p(\mathbf{y_0}) &= \prod_{j=1}^{k} p(y_0^{(j)}), \end{aligned} \tag{7}$$

where y_0 is the initial state, and $y_t = [y_t^{(1)}, y_t^{(2)}, \ldots, y_t^{(k)}]^T$. The objective of TFA is to estimate the sequence of y_t's with unknown model parameters $\Theta = \{A, B, \Sigma_\varepsilon, \Sigma_e\}$ through available observations. For eq.(7), this constraint implies B is diagonal and ε_t is mutually independent in components.

In implementation, an adaptive algorithm has been suggested. At each time unit, factor loadings are estimated by cross-sectional regression and factor scores are estimated by maximum likelihood learning. A simplified version of the algorithm in [2] is shown below.

Assume $G(\varepsilon_t|0, I)$ and $G(e_t|0, \Sigma)$.

- **Step 1** Fix A, B and Σ, estimate the hidden factors y_t by

$$\begin{aligned} \hat{y}_t &= [I + A^T\Sigma^{-1}A]^{-1}(A^T\Sigma^{-1}\bar{x}_t + B\hat{y}_{t-1}),\\ \varepsilon_t &= \hat{y}_t - B\hat{y}_{t-1},\\ e_t &= \bar{x}_t - A\hat{y}_t, \end{aligned} \tag{8}$$

- **Step 2** Fix, $\hat{y}_t$, update B, A and Σ_e by gradient descent method as follows:

$$\begin{aligned} B^{new} &= B^{old} + \eta diag[\varepsilon_t\hat{y}_{t-1}^T],\\ A^{new} &= A^{old} + \eta e_t\hat{y}_t^T,\\ \Sigma^{new} &= (1-\eta)\Sigma^{old} + \eta e_t e_t^T. \end{aligned} \tag{9}$$

3.2. Grounds and Benefits for Using TFA in APT Analysis

Firstly, we believe that temporal factors have Gaussian distributions. There is a consensus that the noisy component in most econometric and statistical models being Gaussian distributed. The rationale comes from the central limit theorem which implies that the compounding of a large number of unknown distributions will be approximately normal. Secondly, we believe that factors recovered must be independent of each other. Although economic factors are seldom independent, it is helpful to discover statistically independent factors for the purpose of analysis because the restriction of independence will rule out many possible solutions which contain redundant elements. Furthermore, economic interpretation of factors recovered can be easily achieved by appropriate combination of those independent factors. Thirdly, we believe there is significant temporal effects between factors. Eq.(4) of the TFA model is nothing more than an AR(1) time series model. The reason why an AR model of order more than 1 is not required can be attributed to the weak form of Efficient Market Hypothesis (EMH). Given the assumption of the weak form EMH is valid, stock price today is conditionally independent of all previous prices given the price of yesterday.

Compared with MLFA, TFA has at least two distinct benefits. First, with the independence assumption in the derivation, the recovered factors are assured to be statistically independent. Second, it has been shown in [2] that taking into account temporal relation effectively removes rotation indeterminacy. As a result, the solution given by TFA is unique. Theorem 3 in [1] illustrates this point. Moreover, it should be noted that MLFA is a special case of the model with $B = 0$ in eq.(4).

3.3. Testability of TFA

The TFA model retains virtually all statistical properties of the original APT model. It is simply an extension of the APT model because it additionally includes temporal relation between factors in the APT model. Apart from that, there is no difference. Since the relationship between y_t and y_{t-1} described by the added equation is also linear, the entire TFA model is a linear model with both the driving noise ε_t and the measurement noise e_t assumed to be Gaussian distributed. Moreover, as both the returns and factors are stationary and the factors is assumed to be uncorrelated with idiosyncratic risks, we can safely conclude that the model is testable, just like APT.

4. EXPERIMENTAL ILLUSTRATIONS VIA HYPOTHETICAL DATA

In this experiment, we will assume artificial data being generated by a TFA model and demonstrate the ability of the TFA algorithm to identify the hidden states or factors. As discussed above, PCA, ICA and MLFA have been used in APT analysis in the literature. Thus we will also compare the results of TFA with them.

In this experiment, we let the observations $\{x_t\}_{t=1}^{T}$ be generated by:

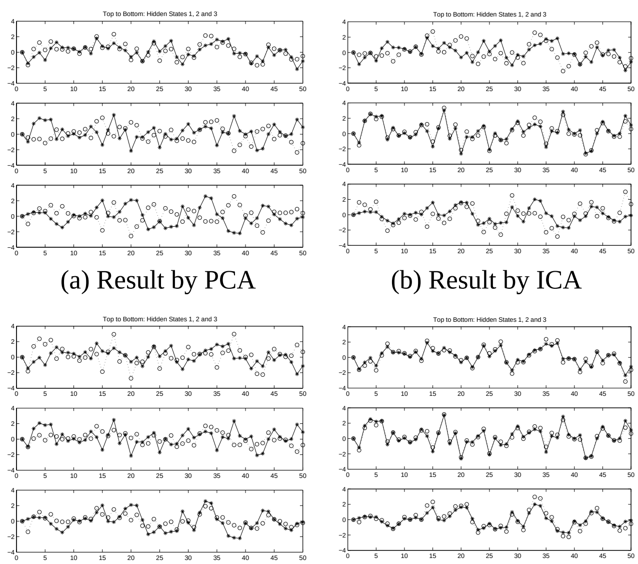
$$
\begin{aligned}
y_t &= \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & 0.6 \end{pmatrix} y_{t-1} + \varepsilon_{\mathbf{t}}, \\
x_t &= \begin{pmatrix} 1.2 & 0.8 & 0.4 \\ 0.4 & -1.1 & 0.6 \\ 1.5 & 0.5 & 1.0 \end{pmatrix} y_t + e_t, \quad 1 \le t \le N
\end{aligned}
\tag{10}
$$

where ε_t and e_t are distributed with $G(\varepsilon|0, I)$ and $G(e_t|0, .05I)$ respectively.

The performance of each algorithm is measured by the Mean Square Error(MSE) between the estimated state $\hat{y}_t^{(j)}$ and the true state $y_t^{(j)}$. The formula to compute MSE is given by:

$$
\mathrm{MSE}(y^{(j)}, \hat{y}^{(j)}) = \frac{1}{N} \sum_{t=1}^{N} (y_t^{(j)} - \hat{y}_t^{(j)})^2, \tag{11}
$$

Snapshots of the results by PCA, ICA, MLFA and TFA are shown respectively in Fig. 1(a)-(d).



(a) Result by PCA (b) Result by ICA

(c) Result by MLFA (d) Result by TFA

Fig. 1. Snapshots: '*' denotes true value; 'o' denotes estimated value.

The respective MSE values by PCA, ICA, MLFA and TFA algorithm are as shown in Tab. 1 for comparison.

Based on the average MSE values, we can see that TFA is good at preserving the original waveform than its counterparts. The relatively small MSE values of ICA can be explained by its ability to exploit higher than second order statistical information in non-Gaussian distributed hidden factors. It should be noted that although the underlying

Table 1. Comparison of MSE values by PCA, ICA, MLFA and TFA

Hidden State	PCA	ICA	MLFA	TFA
1	0.4151	1.1143	1.5089	0.0314
2	2.3006	0.1028	1.6965	0.0649
3	3.5759	1.3436	0.3502	0.1222
Average	2.0972	0.8536	1.1852	0.0728

distributions of the temporal factors are Gaussian when taking into account the temporal relation, they are in fact non-Gaussian distributed if we treat all data being coming from a particular distribution with no temporal relation. In other words, the temporal information that TFA seeks to utilize can be conceived to partially contribute to the higher order statistical information in the overall distribution that ICA makes use of to assist in the de-mixing process. As a result, ICA performs better than MLFA and PCA. The relatively large MSE values of MLFA and PCA are attributed to rotation indeterminacy owing to their inability to extract higher than second order statistical information from the factors.

5. HYPOTHESIS TESTS ON APT

Since TFA outperforms its counterparts, we will use TFA to extract factors and loadings in real APT analysis. After that, we will test the individual and joint significance of the factor loadings. Previously, the test of significance of individual factor loadings is not viable because of rotation indeterminacy. Consequently it is not possible to test directly whether a given factor is priced. However, the analysis by TFA makes it possible to apply t-test on individual factor loadings as it successfully overcomes this constraint. Before going further, certain considerations will precede our analysis.

5.1. Data Considerations

We have carried out our analysis using past stock price and return data of Hong Kong. Daily closing prices of 86 actively trading stocks covering the period from January 1, 1998 to December 31, 1999 are used. The number of trading days throughout this period is 522. These stocks can be subdivided into three main categories according to different indices they constitute. Of the 86 equities, 30 of them belongs to the Hang Seng Index (HSI) constituents, 32 are Hang Seng China-Affiliated Corporations Index (HSCCI) constituents and the remaining 24 are Hang Seng China Enterprises Index (HSCEI) constituents. We do not adopt random sampling in the stock selection process so as to avoid the small-firm effect. This is because there are lots of small-sized listed companies in Hong Kong, many even very inactive. Obviously this kind of stocks is not representative of the whole market and including them will adversely affect the validity of our analysis and tests.

Regarding the observation frequency, we has considered using either daily or monthly equity returns. The potential benefit associated with the use of daily data in the estimation of variances and covariances is enormous. This is because the more frequent the observation, the more precise the parameter estimates. Moreover, for equal number of daily and monthly data points, the period spanned by daily data will be much shorter. As a result, our analysis will be just focused on a small period and is therefore less susceptible to large fluctuations such as structural break.

5.2. Data Preprocessing

Before carrying out the analysis, the stock prices must be converted to stationary stock returns. The transformation applied can be described in four steps as shown below.

- **Step 1** Transform the raw prices to returns by $R_t = \frac{p_t - p_{t-1}}{p_{t-1}}$.
- **Step 2** Calculate the mean return $\bar{R}$ by $\frac{1}{N}\sum_{t=1}^{N} R_t$.
- **Step 3** Subtract $\bar{R}$ from R_t to get the zero-mean return.
- **Step 4** Let the result of above transformation be the adjusted return $\widetilde{R}_t$.

5.3. Hypothesis and Test Statistics

In our setting, the factor structure of each security will be represented by a distinct multiple linear regression equation. Test statistics will be computed on a security basis. Consider the j^{th} cross-sectional regressions of the form

$$\tilde{R}_t^{(j)} = \bar{R}^{(j)} + a_1^{(j)} y_{1t}^{(j)} + a_2^{(j)} y_{2t}^{(j)} + \cdots + a_k^{(j)} y_{kt}^{(j)} \quad (12)$$

where $1 \leq j \leq p$ and p is the total number of securities. F-test will be used for testing the joint significance of factor loadings. The null hypothesis is that all factor loadings are simultaneously zero. This implies that the alternate hypothesis is

H_1 : There exist nonzero constants $a_1, a_2, \cdots, a_k$ such that

$$\tilde{R}_t^{(j)} - \bar{R}^{(j)} = a_1^{(j)} y_{1t}^{(j)} + a_2^{(j)} y_{2t}^{(j)} + \cdots + a_k^{(j)} y_{kt}^{(j)}$$

On the other hand, t-test will be used for testing the significance of individual risk premium coefficient. The degrees of freedom is k and $N-k-1$ for F-test while the degrees of freedom for t-test is $N-k-1$. We will examine the results of both tests at levels of significance of both 5% and 10%.

5.4. Number of Factors

Determining the appropriate number of factors is crucial to APT analysis. Thanks to the automatic model selection ability of TFA, we can choose the correct number of factors by enumerating k incrementally and then select an appropriate k via the following cost function:

$$\min_k J(k) = \frac{1}{2}[k \ln(2\pi) + k + \ln |\Sigma|] \qquad (13)$$

As one of our findings in [**?**] and shown in Fig. 2, the number of factors for the total of 86 securities is five and that for the 30 HSI constituents is 4. Consequently, we will preset the value of k accordingly before applying TFA to discover the temporal factors.

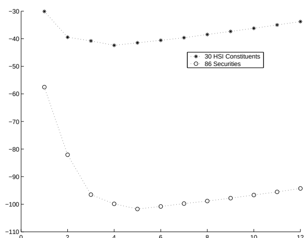


Fig. 2. $J(k)$ for real 30 HSI constituents and the total of 86 securities.

5.5. Empirical Results

After the discovery of Gaussian temporal factors, we subsequently go on to test the individual and joint significance of factor loadings. We have performed the investigation using two groups of securities. Group 1 consists of all 86 securities from all three indices while Group 2 is composed of 30 HSI constituents.

5.5.1. Results of Group 1 Securities

Partial results of 86 securities are shown in Tab. 2. Factors affecting these 86 securities are considered market wide factors. We find that the factor loadings are jointly significant at both 5% and 10% level of significance. Apart from few exceptions, most factor loadings are individually significant at such level of significance. The results are shown in Tab. 3.

In the absence of factor rotation, we can reasonably expect all factors found to be priced. From the results of t-test shown in Tab. 3, only less than 15% of 86 securities are insignificant at $\alpha = 5\%$ while less than 5% of total securities are insignificant at $\alpha = 10\%$. The results also provide support for our findings that the number of market wide factors is most probably 5.

Table 2. Partial Results Showing p-Values of t-test and F-test using 86 Securities

	p-Value of t-test					p-Value of
#	a_1	a_2	a_3	a_4	a_5	F-test
5	0.0000	0.0015	0.0000	0.0000	0.0964	0.0000
10	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0963	0.0000	0.0171	0.0000	0.0000	0.0000
20	0.0205	0.0023	0.0601	0.0000	0.0683	0.0000
25	0.0505	0.0278	0.0000	0.0000	0.0000	0.0000
30	0.0241	0.0818	0.0060	0.0000	0.0000	0.0000
35	0.0361	0.0366	0.0570	0.0000	0.0000	0.0000
40	0.0232	0.0352	0.0414	0.0000	0.0000	0.0000
45	0.0000	0.0507	0.0153	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
55	0.0000	0.0000	0.0288	0.0002	0.0000	0.0000
60	0.0000	0.0049	0.0728	0.0027	0.0000	0.0000
65	0.0000	0.0355	0.1560	0.0482	0.0000	0.0000
70	0.0887	0.0437	0.0000	0.0000	0.0000	0.0000
75	0.0000	0.1994	0.0476	0.0232	0.0000	0.0000
80	0.0000	0.0403	0.0966	0.0268	0.0000	0.0000
85	0.0000	0.0198	0.0000	0.0000	0.0000	0.0000

Table 3. Number and Percentage of Total Securities (86) Not Significant at 5% and 10% Level of Significance

α	a_1	a_2	a_3	a_4	a_5
5%	12	11	12	6	4
	13.95%	12.79%	13.95%	6.98%	4.65%
10%	2	3	1	1	1
	2.33%	3.49%	1.16%	1.16%	1.16%

5.5.2. Results of Group 2 Securities

Results are shown in Tab. 4 and 5 respectively. The results are very similar to the results of Group 1. Major difference arises from using 4 factors instead of 5 for HSI consituents.

5.6. Significance of the Results to Finance

It has been a common consensus in the finance literature [4, **?**] that the number of factors is between three and five. However, the assertion has no theoretical support yet and empirical results have not provided a concrete answer to the problem owing to the various difficulties and indeterminacies in financial APT analysis discussed above. Consequently, the determination of factor number is purely based on heuristic approaches. Intuitively, we do not expect the factor number to change with the number of securities used under a stable market structure, nor do we expect the number of factors to be one because both theoretically and empirically the APT model is superior to the one-factor CAPM model. Interestingly, the factor number determined via the

Table 4. Results Showing p-Values of t-test and F-test on 30 HSI Constituents

	p-Value of t-test				p-Value of
#	a_1	a_2	a_3	a_4	F-test
1	0.0001	0.0047	0.0013	0.0000	0.0000
2	0.0000	0.0000	0.0143	0.0774	0.0000
3	0.0098	0.0624	0.0000	0.0000	0.0000
4	0.1443	0.0003	0.0000	0.0000	0.0000
5	0.0097	0.0487	0.0002	0.0000	0.0000
6	0.0000	0.0000	0.0029	0.0093	0.0000
7	0.0336	0.0445	0.0007	0.0000	0.0000
8	0.0255	0.0554	0.0000	0.0000	0.0000
9	0.0146	0.0000	0.0000	0.0000	0.0000
10	0.0947	0.0739	0.0001	0.0000	0.0000
11	0.0063	0.0148	0.0001	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0820	0.0001	0.0442	0.0000	0.0000
14	0.0048	0.0000	0.0000	0.0379	0.0000
15	0.0082	0.0231	0.0828	0.0000	0.0000
16	0.0000	0.0000	0.0000	0.0000	0.0000
17	0.0000	0.0000	0.0983	0.0657	0.0000
18	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0887	0.0011	0.0064	0.0000	0.0000
20	0.0000	0.0174	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.0000	0.0000	0.0000	0.0000	0.0000
23	0.0000	0.0300	0.0005	0.0000	0.0000
24	0.0000	0.0036	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0683	0.0000
27	0.0041	0.0461	0.0058	0.0000	0.0000
28	0.0000	0.0002	0.0534	0.0000	0.0000
29	0.0000	0.0696	0.0000	0.0000	0.0000
30	0.0000	0.0978	0.1084	0.0000	0.0000

cost function $J(k)$ and the TFA technique in [**?**] is found to be consistent with the assertion. Moreover, the results of tests on factor loadings shown in this paper provide assurance to the factor number determined in [**?**]. All these results lead us to suspect that actual priced factors may be a linear or nonlinear mixture of the independent Gaussian factors discovered via TFA. Of course, further research is required to discover the hidden relationship.

6. CONCLUSION

In this paper we have compared the effectiveness of different techniques to preserve waveforms in the context of factor analysis and blind source separation. Our results show that TFA can not only overcome rotation indeterminacy, but also provide a solution to determine the number of factors via its automatic model selection ability. These make it a desired candidate for the task of real APT analysis. Our experimental results using real stock data and the subsequent statistical tests demonstrate that the factors discovered are most likely priced and this also provide support for the number of factors determined using $J(k)$ in [**?**].

Table 5. Number and Percentage of Total Securities (30) Not Significant at 5% and 10% Level of Significance

α	a_1	a_2	a_3	a_4
5%	4	5	4	3
	13.33%	16.67%	13.33%	10%
10%	1	0	1	0
	3.33%	0%	3.33%	0%

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