## CMSC5724: Exercise List 9

Answer Problems 1-2 based on the following dataset:


Problem 1. Recall that, in discussing hierarchical clustering, we introduced 3 distance metrics on two sets of points: min, max, and mean. Let $S_{1}=\{a, c\}$ and $S_{2}=\{b, d\}$. What is the distance between $S_{1}$ and $S_{2}$ under those three metrics, respectively (assuming that the distance of two points is calculated by Euclidean distance)?

Problem 2. Show the dendrogram returned by the Agglomerative algorithm under the min and max metrics, respectively.

Problem 3. Suppose that we use $d_{\text {min }}$ to define the similarity of two clusters $C_{1}, C_{2}$. Give an algorithm to compute the dendrogram on $n$ points in $O\left(n^{2} \log n\right)$ time.

Problem 4. Suppose that we use $d_{\text {mean }}$ to define the similarity of two clusters $C_{1}, C_{2}$. As discussed in the lecture, $d_{\text {mean }}\left(C_{1}, C_{2}\right)=\frac{1}{\left|C_{1}\right|\left|C_{2}\right|} \sum_{\left(p_{1}, p_{2}\right) \in C_{1} \times C_{2}} \operatorname{dist}\left(p_{1}, p_{2}\right)$. Give an algorithm to compute the dendrogram on $n$ points in $O\left(n^{2} \log n\right)$ time.

Problem 5. Consider the set $P$ of points below:


Set $\epsilon=1$ and minpts $=3$. Show the clusters output by DBSCAN, assuming that the distance metric is Euclidean distance.

Problem 6. Given a pair of parameters $\epsilon$ and minpts, describe an algorithm to compute the DBSCAN clusters in $O\left(n^{2}\right)$ time, assuming that the distance metric is Euclidean distance, and that the dimensionality of the data space is a constant.

