Exercises for CSCI5010

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Problem 1. Let P be a set of points in $\mathbb{R}^d$. Consider the following algorithm for finding an approximation of $\text{diam}(P)$ (i.e., the diameter of P).

algorithm
1. pick an arbitrary point $p \in P$
2. identify a point $q \in P$ maximizing $dist(p, q)$ (the Euclidean distance between p and q)
3. return $dist(p, q)$

Prove: The algorithm returns a $\frac{1}{2}$-approximate answer, i.e., $\frac{1}{2} \cdot \text{diam}(P) \leq dist(p, q) \leq \text{diam}(P)$.

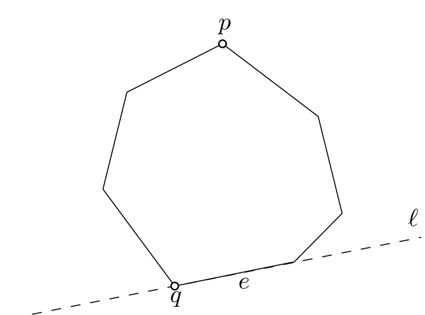
Problem 2. Let P be a set of points in $\mathbb{R}^d$. Describe how to find in $O(n \log n + n/\epsilon^d)$ time a value Δ satisfying $\text{diam}(P) \leq \Delta \leq (1 + \epsilon) \cdot \text{diam}(P)$ for any $0 < \epsilon < 1$.

Problem 3*. Let P be a set of 2D points. Let $p, q \in P$ be two points such that $dist(p, q) = \text{diam}(P)$. Prove: p and q must be vertices of the convex hull of P.

Hint: Use the fact that if a point p is not a vertex of the convex hull, then every line ℓ passing p has the property that there are points of P falling on both sides of ℓ.

Problem 4*. Let G be a convex polygon. Define p and q as the two vertices of G maximizing $dist(p, q)$. Prove: G has an edge e such that

- one of p and q is a vertex of e;
- if p is a vertex of e, then q is a vertex of G having the maximum distance to the line ℓ passing e.



Hint: First find two parallel lines passing p and q, respectively, and enclosing G in between. Then, rotate these lines.

Remark: Problems 3 and 4 suggest that the (precise) diameter of a 2D point set P can be found in $O(n \log n)$ time.

Problem 5 (reading exercise). Prove the correctness of the algorithm discussed in the lecture for computing a t-spanner graph for a set of points in $\mathbb{R}^d$.

Hint: If you do not want to think, read Pg 128-129 of the lecture notes. However, an inductive argument similar to the one for solving Problem 4 of Exercise List 6 suffices.

Problem 6 (reading exercise). Prove the correctness of the algorithm discussed in the lecture for computing a $(1+\epsilon)$-approximate EMST for a set of points in $\mathbb{R}^d$.

Hint: If you do not want to think, read Pg 129-130 of the lecture notes.