Exercises for CSCI5010

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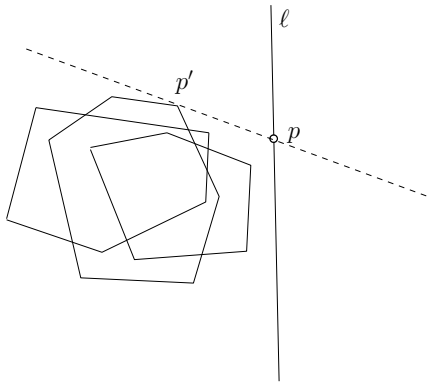
Problem 1* (General Binary Search). Let A be an array of n real values. If we start from some position and then look at these values in a cyclic manner, we see a pattern where the values initially increase monotonically and then decrease monotonically. For example, $A = (10, 20, 30, 25, 15, 0, 5)$ has the property: by inspecting the values in the order "0, 5, 10, 20, 30, 25, 15", we observe the pattern mentioned earlier. On the other hand, $A = (5, 20, 30, 25, 15, 0, 10)$ does not have the property. Design an algorithm to find the maximum value in A in $O(\log n)$ time.

Problem 2 (Gift Wrap). Let $P_1, ..., P_m$ be m arbitrary convex polygons, each of which has no more than k vertices. Let ℓ be a line in the plane such that all of $P_1, ..., P_m$ fall on the left side of ℓ. Now, fix a point p on ℓ. We want to turn ℓ counterclockwise with p as the pivot, and stop as soon as ℓ hits a vertex of any polygon (e.g., in the figure below, the answer is p'). Design an algorithm to find in $O(m \log k)$ time the first vertex hit.



Problem 3. Let S be a set of n points in $\mathbb{R}^2$. You are given an integer $\hat{k}$ that is guaranteed to be larger than or equal to the number of vertices on the convex hull of S. Give an algorithm that computes the convex hull in $O(n \log \hat{k})$ time.

(Hint: Arbitrarily divide S into groups of size $\hat{k}$ and apply the result of Problem 2.)

Problem 4. Design an algorithm to compute the convex hull of n 2D points in $O(n \log k)$ time, where k is the number of points on the convex hull.