Exercises for CSCI5010

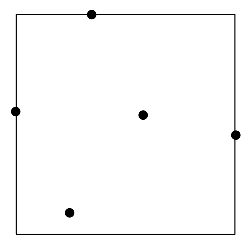
Prepared by Yufei Tao

Problem 1. Prove that our 2D MEB problem (i.e., the "no point known" variant) runs in $O(n)$ expected time.

Problem 2*. Give an algorithm to solve the 3D MEB problem in $O(n)$ expected time.

Hint: Obviously, you should first extend the "geometric facts" in Section 1 of our lecture notes to 3D space.

Problem 3. Let P be a set of n points in $\mathbb{R}^2$. Find an axis-parallel square of the smallest size to cover the whole P. The figure below shows such a square on an example of $n = 5$.

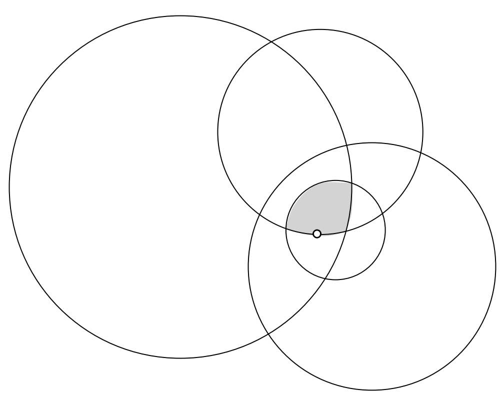


- Show that this problem can be cast as a linear programming problem with a constant dimensionality.
- Give a <u>deterministic</u> algorithm to solve the problem in $O(n)$ time.

Problem 4*. Let P be a set of n points in $\mathbb{R}^2$. Give a deterministic algorithm to find in $O(n)$ time an enclosing ball of P with radius at most $\sqrt{2}r^*$, where r^* is the radius of the MEB of P.

Hint: Use the result from Problem 3.

Problem 5. Let S be a set of n discs in $\mathbb{R}^2$. Design an $O(n)$ expected time algorithm to find the lowest point that is inside all the discs (or report nothing if there are no such points). Your algorithm may assume that no three circles pass the sample point in $\mathbb{R}^2$.



In the example above, your algorithm should output the white point.