Exercises for CSCI5010

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Problem 1. Let P be a set of n points in $\mathbb{R}^2$. A *slab* is the region between two parallel lines (inclusive of the two lines). The *perpendicular width* of a slab is the (perpendicular) distance between its boundary lines. Suppose that parallel lines ℓ_1 and ℓ_2 define a width the smallest perpendicular width among all the slabs enclosing all the points of P. Prove: either ℓ_1 or ℓ_2 passes two points of P.

Problem 2*. Let P be a set of n points in $\mathbb{R}^2$. Describe an algorithm to find a slab with the minimum perpendicular width that encloses all the points of P. Your algorithm should run in $O(n \log n)$ time.

Hint: Duality and Problem 1 helps.

Problem 3. Let L be a set of n non-vertical lines in $\mathbb{R}^2$ where no two lines are parallel. Explain how to compute in $O(n)$ time an axis-parallel rectangle that contains all the $\binom{n}{2}$ intersect points of those lines.

Problem 4*. Let P be a set of n points in $\mathbb{R}^2$, and $k \leq n$ be an integer Describe an algorithm to find a slab with the minimum perpendicular width that encloses precisely k points of P. Your algorithm should run in $O(n^2 \log n)$ time.

Hint: Think in the direction of Problem 1.

Problem 5*. Let P be a set of n points in $\mathbb{R}^2$. Describe an algorithm to find the smallest-area triangle whose vertices are from P. Your algorithm should finish in $O(n^2 \log n)$ time.

Hint: Revisit Problem 6 of the previous exercise list.