CSCI3160: Special Exercise Set 9

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Problem 1. Prove: all the SCCs of a directed simple graph are mutually disjoint.

Problem 2. Let $G = (V, E)$ be a directed simple graph and G^{scc} be the SCC graph defined in our lecture. Let S_1 and S_2 be two SCCs of G. Prove: if S_1 cannot reach S_2 in G^{scc}, then no vertex of S_1 can reach any vertex of S_2 in G.

Problem 3. Prove: G and G^{rev} have the same SCCs.

Problem 4. Prove: There cannot exist two SCCs S_1 and S_2 such that $S_1 \neq S_2$ but $S_1 \cap S_2 \neq \emptyset$.

Problem 5. In the lecture, we proved that if an SCC S_1 has an edge to an SCC S_2 in G^{scc}, then $label(S_1) > label(S_2)$. Strengthen the claim into the following: if an SCC S_1 has a *path* to an SCC S_2 in G^{scc}, then $label(S_1) > label(S_2)$.

Problem 6. Prof. Goofy proposes his own SCC algorithm:

- Step 1: Perform DFS on the input graph G and compute a label for each vertex (just like Step 1 of our algorithm).
- Step 2: Perform another DFS on G (note: not on G^{rev}) subject to the following rules:
  - Start the first DFS from the vertex with the smallest label.
  - Whenever a restart is needed, do so from the white vertex with the smallest label.

Give a counterexample to prove that Prof. Goofy is wrong.