CSCI3160: Special Exercise Set 6

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Problem 1. Define function $f(x)$ — where $x \geq 0$ is an integer — as follows:

- $f(0) = 0$
- $f(1) = 1$
- $f(x) = f(x-1) + f(x-2)$.

Give an algorithm to calculate $f(n)$ in $O(n)$ time (you can assume that $f(x)$ fits in a word for all $x \leq n$).

Problem 2. Let A be an array of n integers. Consider the following recursive function which is defined for any i, j satisfying $1 \leq i \leq j \leq n$:

$$f(i,j) = \begin{cases} 0 & \text{if } i = j \\ A[i] \cdot A[j] + \min_{k=i+1}^{j-1} f(i,k) + f(k,j) & \text{if } i \neq j \end{cases}$$

Design an algorithm to calculate $f(1, n)$ in $O(n^3)$ time.

Problem 3. In Lecture Notes 8, we defined function $f(i, j)$ based on strings $x = \texttt{ABC}$ and $y = \texttt{BDCA}$. Calculate $f(i, j)$ for all possible i and j.

Problem 4. In the rod-cutting problem, suppose that $n = 5$ and the price array P is $(2, 6, 7, 9, 10)$. What is the maximum revenue achievable?

Problem 5 (Textbook Problem 15.1-3). Consider a modification of the rod-cutting problem in which, in addition to a price $P[i]$ for each length $i \in [1, n]$, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the segments minus the total cost of making the cuts. Give a dynamic-programming algorithm to solve this modified problem in $O(n^2)$ time.