CSCI3160: Quiz 1

Name: Student ID

Problem 1 (40%). Consider an array storing $n = 9$ integers: $A = (70, 30, 40, 10, 80, 90, 50, 60, 20)$. Recall that, in the k-selection algorithm, we randomly select a pivot p from A and recurse into a subproblem if the subproblem has size at most $2n/3$. However, we declare "failure" if the subproblem has size larger than $2n/3$. Let us set $k = 5$ (i.e., the goal of k-selection is to find the 5th smallest element in A). Assuming that the pivot p equals 40, answer the following questions:

1. What is the *rank* value of p? Remember that the rank is defined as the number of integers in A that are less than or equal to p.
2. After p has been obtained, how do you compute the rank of p in $O(n)$ time? Your description must work on a *general* array A, rather than only the one in the problem statement.
3. Does the algorithm declare "failure" after selecting $p = 40$? You must explain your answer.

Solution.

1. The rank is 4.
2. Initialize a counter $c = 0$. For each $i \in [1, n]$, compare $A[i]$ to p and increase the counter c if $A[i] \le p$.
3. Since $k = 5$ and p has rank 4, we will need to recurse into a subproblem whose input is $(50, 60, 70, 80, 90)$. The subproblem has a size 5, which is less than $2n/3 = 6$. Therefore, no failure is declared.

Problem 2 (40%). Prof. Goofy proposes the following algorithm to calculate the multiplication of two $n \times n$ matrices A and B, where n is a power of 2. He divides each of A and B into 4 submatrices of order $n/2$:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Then, he computes the multiplication as follows:

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Note that he needs to solve 8 subproblems. Prove: the running time of his algorithm is $O(n^3)$.

Solution. Let $f(n)$ be the running time of Prof. Goofy's algorithm. The function satisfies $f(n) \le 8f(n/2) + O(n^2)$ and $f(1) = O(1)$. Solving the recurrence gives $f(n) = O(n^3)$.

Problem 3 (20%). Run the activity-selection algorithm taught in the class on the following input:

$$S = \{[1, 10], [2, 22], [3, 23], [20, 30], [25, 45], [40, 50], [47, 62], [48, 63], [60, 70]\}.$$

What is the set of intervals returned?

Solution. $[1, 10]$, $[20, 30]$, $[40, 50]$, $[60, 70]$